Test #2 AMATYC Student Mathematics League Winter 2017

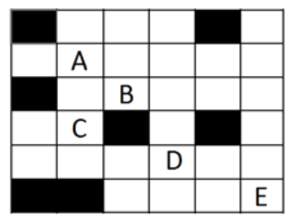
1. The triangles ΔABC and ΔDEF are not isosceles, not congruent, and have integer-length sides. If they have the same perimeter, what is the smallest such perimeter they could share? A. 10 B. 11 C. 12 D. 13 E. 14

2. At the intergalactic trading station, several different currencies are used. Today, 15 gleeks = 11 zorks, 7 gleeks = 3 zeffs, and 5 zeffs = 2 gems. A certain merchant lists prices in gleeks, but only gives change in zorks. Can someone afford to buy an item that costs 45 gleeks if s/he has 8 gems? If so, how much change will be given (to the nearest tenth)?
A. No B. Yes, 3.1 zorks C. Yes, 1.2 zorks D. Yes, 0.3 zorks E. Yes, 1.8 zorks

3. Suppose a and b are integers such that (a, b) is a solution of $a^2 + b^2 + 2ab + 16a + 16b = 36$. Let c be the average of a and b. Find the sum of all possible values of c.
A. 2 B. -8 C. -18 D. -2 E. 8

4. If cars hold 5 passengers and charge for $29 a trip to the airport, and vans hold 7 passengers and charge $41, find the minimum cost to transport 49 people to the airport.
A. $290 B. $285 C. $287 D. $280 E. $282

5. In the grid (made up of 1×1 squares) on the right, which of the squares A, B, C, D, or E, when shaded, will allow the unshaded squares to be covered by exactly 14 dominos (1×2 rectangles) with no overlaps or gaps?



6. The graph of $x^2 + xy + x + 3y = 6$ is
A. an ellipse B. a parabola C. a hyperbola D. 2 parallel lines E. 2 intersecting lines

7. Let a and b be positive integers such that (a, b) is a solution to $\sqrt[3]{a + 4\sqrt{b}} + \sqrt[3]{a - 4\sqrt{b}} = 3$.
Find the smallest possible value of a+b. A. 14 B. 18 C. 22 D. 26 E. 30

8. The matrix A = $\begin{bmatrix} a & 8 \\ -3 & b \end{bmatrix}$ is its own inverse (that is, A times A equals the identity matrix).
Find $|a - b|$. A. 4 B. 6 C. 8 D. 10 E. 12

9. Let $f(x) = x^2 + bx + c$. If f(4) = f(2) + 11, find f(4) - f(0).
A. -6 B. -8 C. 8 D. 10 E. 14

10. Three people (X, Y, Z) are in a room with you. One is a Knight (Knights always tell the truth), one is a Knave (Knaves always lie), and the other is a Spy (Spies may either lie or tell the truth), but you don't know who is which. Each person makes exactly one statement. Which of the following sets of three statements is NOT possible?

A.	B.	C.	D.	E.
X: I am a Knight	X: I am not a Spy	X: I am a Spy	X: I am a Knight	X: I am not a Knave
Y: I am a Knave	Y: I am not a Spy	Y: I am a Spy	Y: I am a Knave	Y: I am not a Knave
Z: X is a Spy	Z: X is not a Knight	Z: I am a Knight	Z: X is a Knight	Z: I am not a Knave

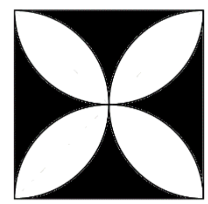
11. For a positive integer n, let S(n) be the sum of the first n positive integers (for example, S(5) = 15). For how many positive integers, n, less than 2017, will all digits of S(n) be 1s?
A. 0 B. 1 C. 2 D. 3 E. 4

12. Ed filled 2/3 of his radiator with antifreeze and then added 4 more quarts (a gallon) of antifreeze. After draining half the antifreeze, he needed 11 quarts of antifreeze to fill the radiator to capacity. How many gallons of antifreeze can the radiator hold?
A. 4.65 B. 4.875 C. 18.6 D. 19.5 E. 78

13. On a game show, the final contestant each day can win $1,000,000 by correctly guessing an integer between 1 and 100 inclusive (which is chosen randomly each day). Before guessing the contestant can ask one yes/no question of his or her choice. Monday's contestant asked "Is the number 57?" and Tuesday's contestant asked "Is the number greater than 50?". Let P(M) be the probability of Monday's contestant winning and let P(T) be the probability of Tuesday's contestants winning (assume each contestant properly uses the information gained from the question). Which of the following is true?
A. $\frac{P(M)}{P(T)} \leq .1$ B. $0.1 < \frac{P(M)}{P(T)} < 0.9$ C. $0.9 \leq \frac{P(M)}{P(T)} \leq 1.1$ D. $1.1 < \frac{P(M)}{P(T)} < 2$ E. $\frac{P(M)}{P(T)} \geq 2$

14. Let P(x) be a degree 5 polynomial with rational coefficients and P(0) = -53,040. Suppose x =12, x =3+5i, and x=4-7i are zeros of P(x). In which interval does the coefficient of x^3 lie?
A. $(-\infty,-500]$ B. $(-500,-100]$ C. $(-100,100)$ D. $[100, 500)$ E. $[500,\infty)$

15. A company designed a new logo by constructing semicircles inside of a unit square (side length = 1) as shown on the right. Which of the following is closest to the area of the shaded region?
A. 0.4 B. 0.45 C. 0.5 D. 0.55 E. 0.6



16. How many positive integers less than 1000 are divisible by exactly one of 7 or 11?
A. 196 B. 208 C. 220 D. 232 E. 244

17. A neon light is failing. When the switch is flipped, it lights for a second, then goes off for a second; lights for a second, then goes off for 2 seconds; lights for a second, then goes off for 3 seconds, etc. Exactly two minutes after the switch is flipped, how long (in seconds) will it stay off before it goes on again?
A. 12 B. 13 C. 14 D. 15 E.16

18. Let N be the smallest positive integer such that ALL N-digit numbers of the form aa…a are divisible by 7. Let M be the smallest positive integer such that 10^M does NOT have a factorization ab in which neither factor has any 0 digits. Find M+N.
A. 18 B. 17 C. 16 D. 15 E. 14

19. A triangle has vertices A(0,0), B(3,0), and C(3,4). If the triangle is rotated counterclockwise around the origin until C lies on the positive y-axis, find the area of the intersection of the region bounded by the original triangle and the region bounded by the rotated triangle. A. 21/16 B. 25/16 C. 29/16 D. 35/16 E. 75/16

20. Consider a game where a player bets $X and then flips a *biased* coin where the probability of flipping heads is 0.4. If the result is heads, she wins $X; if it is tails, she loses $X. Suppose she starts with $25 and her first bet is $5. Every time she wins, she will bet double what she won on the next flip. Whenever she loses, she will bet $5 on the following flip. If she has $100 or more at any point, she will quit. What is the probability (rounded to the nearest thousandth) that she will quit with $100 or more in 7 flips or less?
A. 0.026 B. 0.035 C. 0.038 D. 0.052 E. 0.070