Test #2	AMATYC Stude	nt Mathematics Lea	ague	Winter 2018	
1. When each is g Memphis, TN on a in Memphis on the A. 20/9	iven in Fahrenheit a particular day is e same day if each B. 36/5	, the sum of the hi 68. What is the su is given in Celsius C. 20	gh and low temper m of the high and s? <i>Hint</i> : F = (9/5)C D. 772/9	ratures for low temperatures C + 32 E. 772/5	
2. Four rings of di ascending order (s empty posts. You (taking the top rin	fferent sizes are st smallest on top). T are able to move on g from one post ar	acked on a post, in There are two other one ring at a time nd moving it to and	other		
post), but you may	y never place a lar	ger ring on a small	ler ring. What is th	ne minimum	
number of moves A. 12	required to move t B. 14	he entire stack to C. 15	the middle post? D. 16	E. 17	
3. Find the sum of	f all of the real solu	utions to $ 4 - 3 - $	2 - 1 - x = 0.		
A2	B1	C. 0	D. 1	E. 2	
4. A regular pentagon is rotated 36° around its center to produce a second pentagon. The area of the intersection of the two pentagons is what fraction of the area of the original pentagon, to the nearest whole percent?					
11. 0070	D . 0970	C. 9070	D . 9170	E . 92/0	
5. The area of the the lines $3x + 4u =$	four-sided region i	in the first quadra	nt bounded by the	<i>x</i> -axis, <i>y</i> -axis, and	
A. 33/76	B. 2/5	C. 11/19	D. $1/2$	E. 21/38	
6. Each letter in the equation $\sqrt{AMATYC} = MYM$ represents a distinct non-zero decimal digit. Find T					
A. 3	B. 4	C. 5	D. 6	E. 7	
7. The line <i>y</i> = <i>mx</i> A. 5/12	a + b is tangent to t B. 5/2	he circle $(x + 1)^2 +$ C. 7/2	$(y-1)^2 = 25$ at (3, D. 20/3	4). Find <i>m</i> + <i>b</i> . E. 35/4	
8. Compute the following (where $i = \sqrt{-1}$): $\sum_{i=1}^{2018} (i^n + i^{-n})$					
A. 2	B2	C. 0 $^{n=1}$	D. i - 2	E. 2 - i	
9. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says "Z is a Knight.", Y says "Z is not a Knight.", and Z says "I am a Knave." Which of the following correctly identifies all three people?					
A.	B. Via the sur-	C.	D. Via the lociet	E.	

А.	D.	U.	D.	E.
X is the spy.	X is the spy.	X is the knight.	X is the knight.	X is the knave.
Y is the knight.	Y is the knave.	Y is the knave.	Y is the spy.	Y is the knight.
Z is the knave.	Z is the knight.	Z is the spy.	Z is the knave.	Z is the spy.

10. Suppose	$\log_A B = \log_B A$ and A	> B > 1. Which	n of the following	is true?
A. $AB = 1$	B. $AB = e$	C. $AB = e^2$	D. $AB = 10$	E. This is not possible.

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11. Let <i>M</i> be the g Let <i>N</i> be the great A. 19	greatest integer less test integer that div B. 21	s than 30 such than rides $c^4 - c^2$ for all C. 26	nt <i>M</i> !(<i>M</i> +1)!/2 is a p integers <i>c</i> > 1. Fir D. 29	perfect square. nd <i>M</i> + <i>N</i> . E. 36	
12. A store carries left and 5 plain ba someone wanted to bagels would be p A. 293,710	s 11 different types agels left, but still l to purchase a doze ossible (assuming B. 594,880	of bagels. Late of nad several dozen n bagels then, how that bagels of the C. 594,946	ne day, they had or of each of the other v many different co same type are indi D. 646,184	nly 3 onion bagels r types. If ombinations of 12 stinguishable)? E. 646,646	
13. Kara repeatedly flips a fair coin (the probabiliy of flipping heads and the probability of flipping tails are both $\frac{1}{2}$), and stops when she flips two consecutive heads. What is the expected number of flips?					
A. 3	B. 4	C. 5	D. 6	E. 7	
14. S is a set of four distinct real numbers, with greatest element z. If one adds each possible pair of elements of S, the results are (in ascending order): 2, 3, 4, 5, x, y. Find the sum of all possible values of z.					
A. 17/2	B. 26/3	C. 9	D. 28/3	E. 10	
15. How many of the following are both a well-defined function on \mathbb{R} and also equivalent to the identity function $f(x) = x$ on \mathbb{R} ? $g(x) = \ln e^x$, $h(x) = e^{\ln(x)}$, $k(x) = \sqrt{x^2}$, $m(x) = \sqrt[3]{x^3}$,					
$h(x) = 1 + \frac{1}{x+1}, p(x)$	R^{2} = $\sin(a(csinx), q)$	(all x),	$T(x) = \pm x , s(x) = ($	E 5	
16. The graph of t vertex of this para A. 0	the equation $y = x^2$ abola occurs at the B. 1/8	-2kx + k is a para point (<i>a</i> , <i>b</i>), find t C. 1/4	bola for any real n he greatest possibl D. 1/2	umber k. If the e value of b. E. 1	
17. The three digit decimal number <i>abc</i> is equivalent to the three digit number <i>cba</i> in hexadecmial (base 16). Find $a + b + c$.					
A. 9	B. 11	C. 13	D. 15	E. 17	
18. A collection of identical spheres can be formed into a "square" pyramid (a pyramid with a base (bottom layer) made up of $n \times n$ spheres whose next layer is made up of $(n-1) \times (n-1)$ spheres, continuing this way up to the top layer of 1 sphere). The same collection of spheres can also be formed into a single-layer $k \times k$ "square" where $k < 100$. Find the largest possible value of k for such a collection of spheres and record it on your answer sheet.					

AMATYC Student Mathematics League

Winter 2018

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19. Consider a set of positive integers less than 100 such that no two elements have a sum of 100. Let *M* be the maximum number of distinct elements that such a set can contain. Let *C* be the coefficient of x^5 in the expansion of $(x - 0.5)^8$. Find |C - M|. A. 42 B. 43 C. 44 D. 56 E. 57

20. Suppose that $f(x) = \frac{x^3 + x^2 + cx + d}{x + 2}$ is equivalent to $g(x) = ax^2 + bx + 4$ on its domain. Find f(3). A. 38/5 B. 9 C. 10 D. 54/5 E. 12