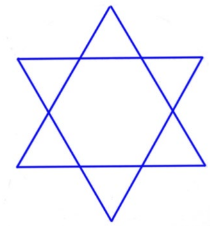
1. Alice and Bo are standing at diametrically opposite points on a circular running track of length 2000 ft. Alice starts walking clockwise at a constant rate. Bo (who walks faster) starts walking at a constant rate at the same moment as Alice. If Bo walks counterclockwise, they will meet for the first time 2 minutes later; if Bo instead walks clockwise, they will meet 10 minutes later. Find how fast Alice walks in ft/min.
A. 100 B. 120 C. 150 D. 200 E. 300

2. A number a is randomly selected from (0,1), a number b is randomly selected from (0, 2), and a number c is randomly selected from (0, 3). What is the probability that $a > b > c$?
A. 1/6 B. 1/12 C. 1/24 D. 1/36 E. 1/72



3. A star is formed by extending the edges of a regular hexagon (with side length s) as shown. What is the total area of the entire star (the hexagon and the six triangles)?
A. $\sqrt{3}s^2/2$ B. $3\sqrt{3}s^2/2$ C. $3\sqrt{3}s^2$ D. $9s^2/2$ E. $9s^2$

4. How many real ordered pairs (x, y) with $|x| \le 2, |y| \le 2$ are solutions to $x^2 + 2x\sin(xy) + 1 = 0$?
A. 0 B. 1 C. 2 D. 3 E. Infinitely many

5. Find the sum of all real solutions to $x^2 + ax = bx + a + 7$ if $a = \begin{cases} 1 & x \le 3 \\ 2x-3 & 3 < x \end{cases}$ and $b = \begin{cases} x+2 & x < 7 \\ 3 & 7 \le x \end{cases}$
A. -8 B. -4 C. 0 D. 4 E. 8

6. If positive integers a and b are solutions to $\frac{1}{a} = \frac{1}{5} - \frac{1}{b}$, how many distinct values of a are possible?
A. 0 B. 1 C. 2 D. 3 E. 4

7. The number 2021 has exactly two distinct prime factors, a and b. The number 2021! can be factored $2021! = a^m b^n q$ where q is a natural number not divisible by a or b. Determine $|m - n|$.
A. 1 B. 2 C. 3 D. 4 E. 5

8. In competitive soccer, a team that wins a match is awarded 3 points and the losing teams receive 0 points. In case of a draw (tie), both teams receive 1 point. A six team round robin (each team plays one match against each of the other five teams) is played, and Liverpool finishes with more points than any other team. What is the fewest number of points that Liverpool could have?
A. 4 B. 5 C. 6 D. 7 E. 8

9. Suppose a,b,c are nonzero real numbers. If $ax^2 + bx + c = 0$ has exactly one real solution and $f(x) = \frac{x^2}{bx+a}$ has an oblique asymptote (a.k.a. slant asymptote) with a y-intercept of (0, 7), find c.
A. -28 B. -1/28 C. -1/4 D. 1/28 E. 4

10. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says, "Y is the spy." Y says, "Z is the spy." Z says, "Y is the spy." Which of the following correctly identifies all three people?

A.	B.	C.	D.	E.
X is the knave.	X is the spy.	X is the knight.	X is the knight	X is the knave.
Y is the knight.	Y is the knave.	Y is the knave.	Y is the spy.	Y is the spy.
Z is the spy.	Z is the knight.	Z is the spy.	Z is the knave.	Z is the knight.

11. Square ABCD is inscribed in circle O (that is, A, B, C, and D all lie on the circle) and its area is a. Square EFGH is inscribed in a semicircle of circle O (that is, E and F lie on a diameter and G and H lie on the circle). What is the area of square EFGH?
A. $a/5$ B. $2a/5$ C. $a/3$ D. $a/2$ E. $3a/5$

12. Consider the graph of the following function in polar coordinates: $r = \frac{2}{1+cos\theta}$. This graph can also be represented in Cartesian coordinates in the form Ax + By + Cxy + Dx^2 + Ey^2 = 1. Find |A – B| + |C| + |D – E|.

A. 1 B. 5/4 C. 5/2 D. 5 E. 10

13. If 1 BTC = $47,686, 1 ETH = 1584 ADA, and 23,000 ADA = 1 BTC, how many ETH (rounded to the nearest hundredth) can we buy with $5,000?

A. 0.66 B. 1.52 C. 6.57 D. 15.22 E. 65.69

14. Suppose that a is real, $i = \sqrt{-1}$, and the real part of $\frac{3-5i}{a-7i}$ is $\frac{19}{25}$. Find the sum of all possible values of a.

A. 1 B. 56/19 C. 75/19 D. 56/17 E. 75/17

15. Let $f(x) = \log_b(x)$ and $g(x) = x^2 - 4x + 4$. If $f(g(x)) = g(f(x)) = 0$ has exactly one real solution and $b > 1$, find b.

A. $\sqrt{2}$ B. $\sqrt{3}$ C. 2 D. $\sqrt{5}$ E. $2\sqrt{3}$

16. How many six-digit (0-9) passcodes are possible if repeats are allowed and any passcode containing at least one 5 must also have at least one 8? (note: passcodes may begin with 0)

A. 730,703 B. 818, 810 C. 850,005 D. 909,009 E. 990,000

17. Find the point in time between 4 and 5 o'clock when the hour-hand and minute-hand of a clock will point in exactly opposite directions. Express your answer in the form HR:MIN:SEC where SEC is rounded to the nearest second. What is MIN + SEC?

A. 44 B. 54 C. 66 D. 81 E. 87

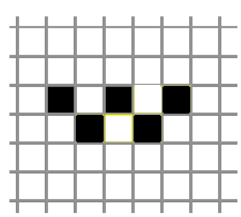
18. Four people are to be seated in six chairs at a round dinner table. How many different seating arrangements are possible if, for each person, we are concerned only about which one or two other people occupy the two chairs beside them, with no distinction between left side and right side nor orientation (for example, if two arrangements both result in A sitting next to B only, B sitting next to both A and C, C sitting next to B only, and D sitting next to no one, then they are not to be considered as different arrangements).

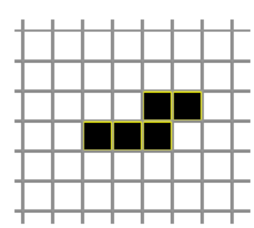
A. 18 B. 21 C. 24 D. 27 E. 30

19. Nine players are playing Texas Hold'em using a standard deck of 52 cards (2-10, J, Q, K, A in each of 4 suits). Each player is dealt two cards, and one player is dealt "pocket kings" (two of the four kings in the deck). What is the probability (to the nearest %) that another player is holding the only better hand, "pocket aces"?

A. 0% B. 1% C. 2% D. 3% E. 4%

20. John Conway's *Game of Life* is played on an infinite square grid where some squares are populated with live cells (the black squares) to begin. Each turn, the grid changes based on the following rules:
(1) A live cell with 0 or 1 live neighbors (live cells in the 8 adjacent squares, including diagonals) or greater than 3 live neighbors dies, while each live cell with 2-3 live neighbors lives.





(2) Each unpopulated cell with exactly three live neighbors becomes populated with a live cell.
Two different starting states with 5 live cells are given. One will reach a steady state with live cells after M turns (M is the smallest value for which the grid is the same after M and M+1 turns), and the other will have no live cells for the first time after N turns. Find $M + N$.

A. 7 B. 8 C. 9 D. 10 E. 11