1. A grocery store se opened). Kara wants she would be able to A. 14	lls oranges only in sma to purchase exactly <i>N</i> o buy any amount greate B. 27	ll bags of 4 oranges an oranges, but she canno or than <i>N</i> (assuming an C. 39	d large bags of 15 oran t since she can only bu unlimited supply of fu D. 41	nges (bags cannot be y whole bags. However, unds and oranges). Find <i>N</i> . E. 43			
2. Find the sum of th A2	e coefficients of the po B1	lynomial obtained by t C. 1	fully expanding $(x - 2)^5$ D. 2	5. E. 4			
3. A child wishes to color each of the six states on a map as shown. What is the minimum number of different color crayons she will need to use if each state must be a different color than all states to which it is adjacent?							
A. 2	B. 3	C. 4	D. 5	E. 6			
4. On the February 14, 1963 episode of <i>The Twilight Zone</i> entitled "From Agnes — With Love," Wally Cox asks Agnes (the world's first supercomputer), "What is the smallest prime greater than the 17 th root of nine trillion, three hundred fifty-five million, one hundred twenty-six thousand, six hundred six?" Agnes responds, "Five." What is the correct answer?							
A. 2	B. 3	C. 5	D. 7	E. 11			
5. Consider the system $\begin{cases} x^2 + y^2 + z^2 = 120\\ xy + yz - xz = 52 \end{cases}$. For each solution (x, y, z) with all integer values, compute							
A. 32	B. 64	C. 98	D. 128	E. 192			
6. The vertices of $\triangle ABC$ are labeled with the positive integers <i>a</i> , <i>b</i> , and <i>c</i> (in that order). The midpoint of \overline{AB} is labeled 6, the midpoint of \overline{BC} is labeled 7, and the midpoint of \overline{AC} is labeled 5. Suppose the sum of the three numbers along each side (the two vertices and the midpoint) is the same value for all three sides. Find the smallest possible value of this common sum.							
7	1 41 11 41 4		1 1 20201 1				
7. The number 2022 $2022! = a^m b^n c^p q x$	where q is a natural nun	hber not divisible by a	b, or c. Determine m^{-1}	+ n + p.			
A. 2026	B. 3018	C. 3026	D. 4024	E. 4032			
8. The variables <i>a</i> , <i>b</i> , <i>c</i> , <i>d</i> , <i>e</i> , and <i>f</i> are each assigned a distinct value from the set {1, 2, 3, 4, 5, 6} (in other words, different variables get different values). Find the largest integer less than or equal to the maximum possible value of the expression $ab + \frac{c}{2} - \frac{e}{2}$.							
A. 30	d f B. 31	C. 32	D. 33	E. 34			
9. Container A holds a 60% acid solution and container B holds an 80% acid solution. The two containers together hold a total of 300 liters of solutions. A tech removes x liters of solution from each container, then adds the solution removed from Container A to Container B, and vice versa. After thoroughly mixing the solutions in each container, container A holds a 64% acid solution and container B holds a 78% acid solution. Find x. A. 10 B. 20 C. 30 D. 40 E. 50							
10. Consider only the points on the graph of $y = x^2$ which have integer coordinates with $-10 \le x \le 10$. Suppose each point with $x < 0$ is connected to each point with $x > 0$ via a line segment. What is the sum of all distinct <i>y</i> -coordinates of the <i>y</i> -intercepts of all such line segments?							
A. 1472	B. 1508	C. 1512	D. 1548	E. 1705			

AMATYC Student Mathematics League

Winter/Spring 2022

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Test #2

A. 1472	B. 1508	C. 1512	D. 1548	E. 1705

11. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says, "I am the knight." Y says, "X isn't lying." Z says, "I am the spy." Which of the following correctly identifies all three people?							
X is the knave	X is the sny	X is the knight	X is the knight	X is the knave			
Y is the knight	Y is the knave	Y is the knave	Y is the sny	Y is the sny			
Z is the sny	Z is the knight	\overline{Z} is the spy	Z is the knave	Z is the knight			
2 is the spy.	Z is the kinght.	Z is the spy.		Z is the kinght.			
12. If $(f, 0)$ is the focus of the parabola given by $y^2 = 12x$, and (h, k) is the center of the hyperbola given by $x^2 - y^2 - 6x - 8y = 8$, find $f + h + k$.							
A2	B1	C. 0	D. 1	E. 2			
13. For how many <i>a</i> 6 A. 0	= (0, 1) will $sin(x) + a sB. 2$	$\sec(x) = \cos(x) + a \csc(x)$ C. 4	(x) have at least two di D. 6	stinct solutions on $(0, 2\pi)$? E. Infinitely many			
14. The equation a^4 +	$-2h^2 + c^2 = 2022$ has st	ix solutions where a. b	c are positive integer	s. The values of a and c			
are both even for all s	vix solutions. Find the c	only value of <i>b</i> that and	ears in two different s	olutions			
	D 5	C_{11}		E 20			
A. 1	D . <i>J</i>	C. 11	D. 19	L. 29			
15. A rectangular tabl dimensions 28" by 17 tabletop consists of ec A. 11.5%	letop is made from 100 .5". What percentage (r dge tiles (that is, tiles th B. 12.2%	0 square tiles of the sa rounded to the nearest nat have one or two ed C. 12%	me size with no space tenth of a percent) of t ges along the outside e D. 12.6%	between them. It has the total area of the edge)? E. 13%			
			11 1 1 10				
16. (1) Is the binary of (1)	peration $a * b = a + b$	-ab commutative for	all real numbers <i>a</i> , <i>b</i> ?				
(11) Is the binary of	peration $a * b = a + b$	-ab associative for al	l real numbers <i>a</i> , <i>b</i> ?				
A. (1) Y es (11) Y es	B. (1) Y es (11) No	C. (1) No (11) Yes	D. (1)No (11)No E. I	mpossible to determine			
17. The position of the hands of a clock are recorded at a moment in time between 4:25:00 and 5:00:00. Call this time HR:MIN:SEC. 33 minutes and 25 seconds later, the angle measured clockwise from the hour hand to the minute hand is 10 times greater than it was when the positions were first recorded. To the nearest second, what is $MIN + SEC^2$							
A. 57	B. 61	C. 65	D. 67	E. 77			
18. A basketball team plays eight games in a tournament against evenly matched teams (so we may assume that the probability of the team winning each of the eight games is $\frac{1}{2}$). Find the probability that the team wins at least three games in a row or loses at least three games in a row at least once during the tournament. A. $\frac{31}{64}$ B. $\frac{35}{64}$ C. $\frac{39}{64}$ D. $\frac{43}{64}$ E. $\frac{47}{64}$							
19. Nine players are r	plaving Texas Hold 'Er	n using a standard dec	k of 52 cards (2-10. J.	O. K. A in each of 4			
suits) Fach player is	dealt two cards, then fi	ve cards are dealt face	un for all players to sh	are Suppose loe is dealt			
two hearts. What is th	a probability to the ne	$\frac{1}{2}$ areast $\frac{9}{2}$ that he will m	ap for an players to si	ch means that there will			
two nearts. What is the probability, to the heatest 70, that he will make a heart mush (which means that there will							
be at least three more	nearts dealt face up)?		D (0)	F F 0/			
A. 3%	B. 4%	C. 5%	D. 6%	E. 7%			
20. Al and Bob are at opposite ends of a diameter of a silo in the shape of a tall right circular cylinder with radius 150 ft. Al is due west of Bob. Al begins walking along the edge of the silo at 6 ft per second at the same moment that Bob begins to walk due east at the same speed. The value closest to the time in seconds when Al first can see Bob is:							
A. 46	B. 47	C. 48	D. 49	E. 50			