

1. Which of the following numbers is the greatest?

- A. 2022^{2023} B. 2023^{2022} C. 2021^{2024} D. 2024^{2021} E. 2012^{2025}

2. Define the binary operation $\$$ by $a\$b = ab + 2a - 3b$. Find $x\$(2\$x)$.

- A. $x^2 - 9x + 12$ B. $x^2 + 12x + 9$ C. $-x^2 + 12x - 9$ D. $x^2 - 12x - 9$ E. $-x^2 + 9x - 12$

3. The equation $2a^4 + b^3 + c^2 = 2022$ has a unique solution where a , b , and c are all positive integers. Find $a + b + c$ for this solution.

- A. 37 B. 39 C. 41 D. 43 E. 45

4. A shirt whose regular price is \$77 is put on sale at a $p\%$ discount. After the sale, the price increased by $q\%$ from the sale price back to the original price of \$77. Find q in terms of p .

- A. $\frac{100}{100-p}$ B. p C. $\frac{100p}{100-p}$ D. $\frac{100p}{1-p}$ E. $\frac{p}{1-p}$

5. Find $x + y$ if $x - y = 29$ and $\sqrt{x} + \sqrt{y} = 29$.

- A. 421 B. 481 C. 841 D. 870 E. 1684

6. Consider the function $g(x) = ax^4 + bx^3 + cx^2 + dx + e$ whose graph is obtained by shifting the graph of the function $f(x) = 2x^4 + 4x^3 + 26x^2 - 60x + 81$ three units to the left. Find $a + b + c + d + e$.

- A. 305 B. 840 C. 855 D. 995 E. 1025

7. George writes down a 3-digit number N with three different nonzero digits and then rearranges the digits to form another 3-digit number M with no digit in its original place. If $M + N = 1092$, find $|M - N|$.

- A. 162 B. 378 C. 432 D. 612 E. 738

8. Let $P(x) = Ax^5 + Bx^4 + Cx^3 + Cx^2 + Bx + A$ be a fifth-degree polynomial with integer coefficients where $A > 0$ and the greatest common factor of A , B , and C is 1. If $\sqrt{7}$ is a zero of $P(x)$, find $A + B + C$.

- A. -50 B. -36 C. 0 D. 36 E. 50

9. There are four cards on a table, and they each have a letter on one side and an integer on the other. The sides that are face-up are E, K, 4, and 7. Someone makes the following claim: If there is a vowel (A, E, I, O, or U) on one side of a card, then there is an even number on the other. Joe then turns over x cards, and is able to verify that this claim is indeed true. What is the minimum possible value of x ?

- A. 0 B. 1 C. 2 D. 3 E. 4

10. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says, "Z is the knave." Y says, "X is the knight." Z says, "I am the spy." Which of the following correctly identifies all three people?

- | | | | | |
|------------------|------------------|------------------|------------------|------------------|
| A. | B. | C. | D. | E. |
| X is the knave. | X is the spy. | X is the knight. | X is the knight. | X is the knave. |
| Y is the knight. | Y is the knave. | Y is the knave. | Y is the spy. | Y is the spy. |
| Z is the spy. | Z is the knight. | Z is the spy. | Z is the knave. | Z is the knight. |

11. Suppose there are three light bulbs that are turned off. When a button is pushed, a random bulb is selected and then changed: if the bulb was off when the button was pushed, the bulb turns on, and if the bulb was on when the button was pushed, the bulb turns off. What is the probability that there is at least one bulb on after four button pushes?

- A. $7/27$ B. $4/9$ C. $5/9$ D. $20/27$ E. $8/9$

12. Let N be the smallest integer greater than 100 with the following property: N , $N + 1$, and $N + 2$ are each the sum of the square of a positive integer and the cube of a positive integer. That is, $N = a^3 + b^2$, $N + 1 = c^3 + d^2$, $N + 2 = e^3 + f^2$ where a, b, c, d, e, f are positive integers (not necessarily distinct). Find $a + b + c + d + e + f$.

- A. 30 B. 31 C. 32 D. 33 E. 34

13. A point is chosen at random from the interior of a square with side length 16. Find the probability that this point is at least $\sqrt{2}$ units from both diagonals.

- A. $9/16$ B. $5/8$ C. $3/5$ D. $3/4$ E. $4/9$

14. Given the system $\begin{cases} x^3 + y = 1739 \\ x + y^3 = 1343 \end{cases}$, find $x + y$ if x and y are both positive integers.

- A. 20 B. 21 C. 22 D. 23 E. 24

15. An analog clock's minute hand is rotating too quickly at a constant rate. Suppose the minute hand starts at 12, and after ten real minutes, it points to 4. If two additional real minutes pass (for a total of twelve real minutes), how many possible positions are there for the minute hand?

- A. 1 B. 3 C. 4 D. 5 E. 7

16. $\{a_n\}$ is a sequence defined by $a_n = f(n)$ where $f(n)$ is a 3rd degree polynomial function. If the first four terms are $a_0 = 1$, $a_1 = 2$, $a_2 = 3$, and $a_3 = 5$, find a_{12} .

- A. 144 B. 177 C. 233 D. 300 E. 377

17. $\triangle PQR$ has area A . If m is the measure of the angle at P and n is the measure of the angle at Q, what is the length of side PQ?

- A. $\sqrt{2A(\tan m + \tan n)}$ B. $\sqrt{A(\sin m + \sin n)}$ C. $\sqrt{A(\cos m + \cos n)}$ D. $\sqrt{A(\tan m + \tan n)}$ E. $\sqrt{2A(\cot m + \cot n)}$

18. A circle with center P is inscribed in $\triangle ABC$ with right angle A. The circle is tangent to all 3 sides of ABC. Segment BP is extended until it intersects AC at point M. If the length of leg AB is 1050 and the length of leg AC is 1728, the length of segment AM can be written in lowest terms as p/q where p and q are relatively prime positive integers. Determine $p + q$.

- A. 425 B. 473 C. 475 D. 4725 E. 4733

19. Over the weekend, Omar, Paula, Quentin, Rosa, Sam, and Thieu each packed several gift bags; they packed 91, 92, 93, 94, 95, and 96 bags, respectively. Each of them packed k times as many bags on Saturday as on Sunday, where k was a different whole number between 1 and 6 inclusive for each of them. Who packed three times as many bags on Saturday as on Sunday?

- A. Paula B. Quentin C. Rosa D. Sam E. Thieu

20. For a natural number n , the radical of n , denoted $rad(n)$, is defined to be the largest square-free factor of n . For example, $rad(8) = 2$ and $rad(252) = 42$. Let the factoradical of n , denoted $rad!(n)$, be defined recursively via $rad!(1) = 1$ and $rad!(n) = rad(n) \cdot rad!(n-1)$. For example, $rad!(9) = 15120$. What is the highest integer power of 2022 that divides $rad!(2022)$?

- A. 1 B. 2 C. 4 D. 6 E. 8