

1. In U.S. restaurants it is traditional for diners to give a tip to their server, where the tip is a percentage of the bill before taxes. If the tax is 8%, two diners had a bill of \$45 before tax and tip, and the final bill was \$56.70, their tip was

- A. 15% B. 18% C. 20% D. 22% E. 25%

2. Let $p|q|r_b$ represent the number in base b with b^2 digit p , b digit q , and units digit r . If $p|q|r_9 = q|r|p_6$, find $p + q + r$. Note that leading digits cannot equal 0.

- A. 10 B. 11 C. 12 D. 13 E. 14

3. Let (a, b, c) be a solution to the equation $a^5 + b^3 + c^2 = 2025$. If a, b , and c are all odd positive integers, find $a + b + c$.

- A. 15 B. 43 C. 45 D. 47 E. 49

4. Knights (Kt) always tell the truth, knaves (Kv) always lie, and spies (Sp) sometimes lie and sometimes tell the truth. Al says, "I am an only child," Bo says, "Al is either an only child or a knight or both," and Cy says, "Al is either an only child or a knave or both." If exactly one of the three is a knight, one a knave, and one a spy, which of the following represents (Al, Bo, Cy)?

- A. (Kt, Sp, Kv) B. (Kv, Kt, Sp) C. (Sp, Kv, Kt) D. (Sp, Kt, Kv) E. (Kv, Sp, Kt)

5. In the diagram at the right, each letter represents a different digit from 1 to 9. In each row and column, the numbers on the end equal the result of performing the operations in that row from left to right or that column from top to bottom without regard to order of operations. For example, in the middle row s and t are added and the result is multiplied by u to get 45. Find $v + t + r$.

- A. 10 B. 11 C. 12 D. 13 E. 14

p	+	q	+	r	14
-		+		+	
s	+	t	\times	u	45
+		\times		\times	
v	-	w	\times	x	14
7		12		56	

6. Find the solution set of the inequality $\frac{(x-2)}{(x+3)} \geq 6$.

- A. $(-3, -2]$ B. $[-4, -3)$ C. $(-\infty, -3) \cup [2, +\infty)$ D. $(-\infty, -4] \cup (-3, +\infty)$ E. $(-\infty, -3) \cup (4, +\infty)$

7. A *palindromic polynomial* is a polynomial whose coefficients when written in standard form read the same from left to right as from right to left (for example, $4x^3 - 3x^2 - 3x + 4$). If $P(x)$ is a fourth-degree palindromic polynomial with $P(-1) = 18$, $P(0) = 2$, and $P(1) = -2$, find $P(3)$.

- A. -22 B. -4 C. 32 D. 50 E. 65

8. For $P(x)$ in the previous problem, find the sum of the real solutions of the equation $P(x) = 0$.

- A. 3/2 B. 5/2 C. 7/2 D. 9/2 E. 11/2

9. Square $ABCD$ shares a base with rectangle $AEFD$. If $AE + EF$ equals the length of the circular arc from A to C centered at D , find the measure of $\angle AFE$ to the nearest one-tenth of a degree.

- A. 29.7° B. 30.1° C. 30.5° D. 30.9° E. 31.3°

10. Find the sum of all real solutions of the equation $\log_6(2x - 3) + \log_6(2x + 1) + \log_6(2x - 2) = 2$.

- A. 5/2 B. 7/2 C. 1 D. 2 E. 3

11. For any positive integer N , $\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \dots + \frac{1}{\log_{20} N} =$

- A. 210 B. $\log_{20}(N!)$ C. 20! D. $\log_N(20!)$ E. $\log_N 210$

12. Let $f(n)$ be defined on the positive integers with $f(1) = 5$ and $f(n) = n + f(n - 1)$ for $n \geq 2$. Find $f(100)$.
- A. 5004 B. 5005 C. 5050 D. 5054 E. 5055
13. For the relation $R = \left\{ (x, y) : \frac{x}{y} = xy, x, y \in \mathbb{R} \right\}$, find the range of R (that is, the set of all y -values which are members of at least one ordered pair in R).
- A. $\{1\}$ B. $\{1, -1\}$ C. $[-1, 1]$ D. $(-\infty, +\infty)$ E. $(-\infty, 0) \cup (0, +\infty)$
14. Let $f(n)$ be the function on the positive integers defined as follows: $f(1) = 0$; for p a prime number, $f(p) = p^2$; for a composite number ab , $f(ab) = af(b) + bf(a)$. Find $f(42)$.
- A. 62 B. 144 C. 504 D. 744 E. 1764
15. The n th triangular number T_n is the sum of the first n positive integers; for example, $T_3 = 1 + 2 + 3 = 6$. For how many different values of n does there exist a value k for which $T_n - T_k = 50$?
- A. 0 B. 1 C. 2 D. 3 E. 4
16. Find the number of arithmetic series with nonnegative integer terms whose sum is 50.
- A. 65 B. 70 C. 73 D. 74 E. 75
17. For any two positive integers a and b , let $S(a, b) = a + b$ and $P(a, b) = ab$, that is, $S(a, b)$ is their sum and $P(a, b)$ is their product. Find the smallest integer $N > 100$ for which there are no values of a and b with $N = S(a, b) + P(a, b)$.
- A. 101 B. 102 C. 103 D. 104 E. 105
18. Equilateral $\triangle ABC$ has side length 5. It is subdivided into 25 congruent equilateral triangles each with side length 1. Find the number of equilateral triangles of any size made up of one or more of the small triangles.
- A. 42 B. 45 C. 47 D. 48 E. 49
19. Becca and Carlos generate a single sequence of H's and T's by alternately flipping a fair coin. They continue to flip until either the sequence TTT appears, in which case Becca wins, or the sequence HTT appears, in which case Carlos wins. Find the probability that Becca wins.
- A. $1/8$ B. $3/8$ C. $1/2$ D. $5/8$ E. $7/8$
20. A residential street has ten houses along its east side. Every day on which mail is delivered, exactly one of the following situations occurs: (1) Of every four consecutive houses (1^{st} through 4^{th} , 2^{nd} through 5^{th} , 3^{rd} through 6^{th} , etc) exactly one receives mail. (2) Of every four consecutive houses exactly two receive mail. In how many different ways can mail be delivered to these houses?
- A. 10 B. 120 C. 210 D. 330 E. 540