

Scientific Measurements: Significant Figures and Statistical Analysis

Purpose

The purpose of this activity is to get familiar with the approximate precision of the equipment in your laboratory. Specifically, you will be expected to learn how to correctly

- record measurements with an appropriate number of significant figures,
- manipulate measured values when performing basic mathematical operations (+/-/×/÷),
- distinguish between accuracy and precision, and
- report and interpret an average and standard deviation for a set of data.

Background

If you measure your weight at home on your bathroom scale, you may get a reading of 135 lb. At a doctor's office 30 minutes later, the nurse measures your weight to be 145 lb. The scale at the doctor's office is probably more **accurate** than the one you have at home. Perhaps it was calibrated, whereas yours wasn't.

On the other hand, suppose you are at home and your new digital scale reads 134.8 lb. You step off, and step back on. It reads 134.6 lb. Stepping on and off the balance a few times leads to slightly different values each time. As long as the values tend to be around a certain value, being slightly off is not "wrong". However, the closer the values are to each other, the higher **precision** has been achieved.

As a scientist, gaining an understanding of the accuracy and precision of measurements is important. Here are some highlights:

- Every measurement contains some degree of error. No measurement is ever exact.
- In cases where a "true value" is not provided, or you are to determine a value experimentally, the mean (average) will be your best value for a measurement.
- The percent difference is used to compare an experimental value with a "true" value. The smaller the percent different difference, the greater the accuracy.
- The standard deviation (stdev or SD) and the relative standard deviation (rel. stdev or RSD) are used to discuss precision. The smaller the standard deviation, the higher the precision.

Accuracy vs. Precision

Precision and accuracy are terms that are often misused – they are not interchangeable and have very different meanings in a scientific context.

Accuracy is a measure of the correctness of a measurement. For example, the density of zinc at 25 °C is 7.14g/mL. Experimentally, you might determine the density of a piece of zinc to be 7.27 g/mL, but another student in the class may calculate the density of zinc to be 6.56 g/mL (assuming 25 °C). Since your answer is closer to the agreed upon value, your measurement is more accurate.

How is accuracy measured? We express how accurate our results are as a % error, sometimes also called a % difference. You can compare the extent of error in your experimental readings by using the following formula:

$$\% \text{ error} = \left| \frac{\text{experimental} - \text{true}}{\text{true}} \right| \times 100\%$$

If you use the values given above, you will find that your zinc density gives you a % error of $|(7.27-7.14)/7.14| \times 100 = 1.82\%$, while your classmate got a % error of $|(6.56-7.14)/7.14| \times 100 = 8.12\%$. This means you were more accurate. **For many experiments, an error of 10% is an acceptable range for accuracy of your results**, and a higher error might indicate that you might have problems with your technique, reagents, or equipment (though the acceptable % error varies with the type of experiment you may do – some experiments are very sensitive to error and may result in a % error that is reasonably higher than others).

Often there is no “true” value to compare to in an experiment as we had above and therefore you cannot comment on a measurement’s accuracy. In these cases you will do several trials or multiple experiments and the average (or mean) will be taken to be your “true” value.

To calculate a mean, you sum all the values of your trials and divide by the number of trials... something you have probably done many times. Let’s translate this into statistics lingo, where \bar{x} is the mean, n = number of trials, capital sigma (Σ) means “sum”, and x is the value obtained for each trial, from 1 through n .

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{1}{n} (x_1 + \cdots + x_n)$$

Precision is a measure of the reproducibility of a measurement. Imagine you repeated the zinc density experiment a second time and this time measured a value of 7.26 g/mL, and a third time it was 7.29 g/mL. Your value is very close to your first experiment. You could say that your precision is quite good but how do you quantify it?

How is precision measured? Conceptually, you can see that the less the values deviate (or differ) from each other, the higher the precision. We will use a statistical measure called a standard deviation.

To calculate a standard deviation (σ), you take each trial value, x_i , subtract it from the mean, \bar{x} , and square it to get a variance. Then you add all of these variances for every trial to get a sum of variances. Then you divide by $N-1$ where N = number of trials. Then you take a square root. In statistical lingo, it looks like this:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

With more than just a few trials, this calculation can be tedious and is much easier to do with your calculator or Excel – ask your instructor or refer to your calculator manual for help (or a Google search). If you calculate a standard deviation for your density values 7.27 g/mL, 7.26 g/mL, and 7.29 g/mL, you will need the average (7.27 g/mL). The standard deviation is:

$$\sigma = \sqrt{\left[\frac{(7.27-7.27)^2 + (7.26-7.27)^2 + (7.29-7.27)^2}{3-1} \right]} = \mathbf{0.02 \text{ g/mL}}$$

How do I report my experimental results? For your density experiment, you would report both the mean and the standard deviation in this format: $\bar{x} \pm \sigma$. Therefore, you would report 7.27 ± 0.02

g/mL as your final result. Comparing another student's results, 7.18 ± 0.15 g/mL, you could say their result was more accurate (7.18 g/mL is closer to 7.14 g/mL, the true value of the density of zinc) but their precision (± 0.15 g/mL) was not as good as yours (± 0.02 g/mL). Note that a standard deviation is reported to the same number of decimal places as the measurement.

What does the standard deviation mean? There is a lot to understand about statistics to answer this question that fall outside the scope of this course. For simplicity, we can say that your measurement of 7.27 ± 0.02 g/mL means your collected measurements should fall within the range 7.25 g/mL – 7.29 g/mL most of the time. Since the range is rather small, we say the precision/reproducibility is good.

What is a “good” standard deviation? How small is “small”, and how large is “large”? To answer this question, you can calculate a relative standard deviation, which like % error, gives you a value relative to the mean and is expressed in %.

$$\text{Relative standard deviation (RSD)} = \left(\frac{\sigma}{\text{mean}} \right) \times 100\%$$

In our zinc density example, the standard deviation was 0.02 g/mL and the mean was 7.27 g/mL. This means the RSD is $(0.02 / 7.27) \times 100 = 0.3\%$. For our purposes, we will consider an RSD of 10% to be rather small. The result has high precision (<10% RSD).

NOTE: The 10% guideline for % error and RSD are just guidelines. Please do not start an experiment over if your results do not follow the guidelines. Always ask your instructor before discarding results. Do not start an experiment over without permission from your instructor due to time constraints.

So far we've taken you through a lot of statistics and calculations. In real life, you don't normally measure something three times and always take an average and standard deviation. That would be very tedious! For example, when you weigh yourself, you rarely get on the balance three times and do the calculations. Something about the balance can tell you how good the measurement is, and the level of precision in the measurements obtained with only one trial.

In addition to reproducibility, precision also deals with the closeness or fineness with which a measurement may be made. What does that mean? Think about measuring your weight. If you measure your weight at home on your bathroom scale, you may get a reading of 135.5 lb. At a doctor's office that day the nurse measures your weight to be 135.39 lb. The value 135.39 has digits out to the hundredths place whereas the value 135.5 only has digits out to the tenths place; therefore, the scale at the doctor's office has a higher precision than the one you have at home.

Engineers and scientists represent the precision of a piece of equipment by writing a “ \pm error” after the measurement. If trials were done, you might calculate the standard deviation and use that value for the \pm error. But if you are taking one measurement, you might estimate what that \pm error value is based on the graduations of the measuring tool. For example, the first scale's measurement could be written as 135.5 ± 0.1 lb. This shows that there is an uncertainty in the last digit of the measurement. The doctor's scale measurement would be written as 135.39 ± 0.01 lb. (For our purposes, we will assume the unit of “1” for the decimal place of the last written digit.)

Chemists use significant figures (sig figs) to indicate the precision without having to record the \pm error of the equipment used to make the measurements. There is an implied understanding that the last recorded digit contains some level of error. Instead of writing the \pm error explicitly, we use significant figures to communicate the precision of the measuring device.

Significant figures include all the known values of a measurement plus one guess. Let's go back to the scale example. When you stand on the bathroom scale, the needle might point between 135 lb and 136 lb (as in figure 1 below). You know that you weigh more than 135 lb but less than 136 lb. The correct way to report this value is to report the known values (135) plus a guess (135.4 ← the 4 is the guess).

In general, estimate the value to **one decimal place** more than the level of graduation marks (or increments or divisions).
(Or, in other words, 1/10 of the smallest division you can see on the scale!)

In the example above, the graduation is every 1 lb. Therefore, the measurement is reported to the 0.1 lb (135.4 lb has one decimal place).

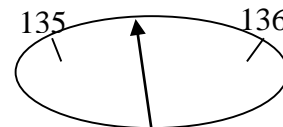


Figure 1 – Scale showing a mass between 135 and 136 lb. Note the lack of markings between 135 and 136.

Every measurement you make must include the proper number of significant figures!! You can determine this by looking at the graduations on your device. The last value is always a guess made by you. Don't hesitate; just guess. This makes the last digit the uncertain digit.

Are Significant Figures Important?

Many students ask if significant figures are important. Read the following fable and then draw your own conclusions: *A student once needed a cube of metal that had to have a mass of 83 grams. He knew the density of this metal was 8.67 g/mL, which told him the cube's volume. Believing significant figures were invented just to make life difficult for chemistry students and had no practical use in the real world, he calculated the volume of the cube as 9.573 mL. He thus determined that the edge of the cube had to be 2.097 cm. He took his plans to the machine shop where his friend had the same type of work done the previous year. The shop foreman said, "Yes, we can make this according to your specifications - but it will be expensive."*

"That's OK," replied the student. "It's important." He knew his friend has paid \$35, and he had been given \$50 out of the school's research budget to get the job done.

He returned the next day, expecting the job to be done. "Sorry," said the foreman. "We're still working on it. Try next week." Finally the day came, and our friend got his cube. It looked very, very smooth and shiny and beautiful in its velvet case. Seeing it, our hero had a premonition of disaster and became a bit nervous. But he summoned up enough courage to ask for the bill. "\$500, and cheap at the price. We had a terrific job getting it right -- had to make three before we got one right."

"But--but--my friend paid only \$35 for the same thing!"

"No. He wanted a cube 2.1 cm on an edge, and your specifications called for 2.097. We had yours roughed out to 2.1 that very afternoon, but it was the precision grinding and lapping to get it down to 2.097 which took so long and cost the big money. The first one we made was 2.089 on one edge when we got finished, so we had to scrap it. The second was closer, but still not what you specified. That's why the three tries."

"Oh!"¹ So, what do you think? Are sig figs important for communicating information about precision? Accuracy? Both?

Procedure:

Part 1: Recording measurements to the correct number of significant figures.

Part A Volume: Take a 10 mL graduated cylinder and record its graduations. Add 5 to 6 mL of water, and record the volume to the exact number of significant figures and units.

Part B Length: Take a small plastic ruler and a meter stick and record their graduations by cm and mm. Record the length of a notecard using the small plastic ruler and the meter stick and express the value in centimeters, millimeters and meters.

Part C Mass: Obtain 3 different pennies and record their mass to the correct number of decimal places and units.

Part 2: Significant figures in calculations

Take a 150 mL Erlenmeyer flask and fill it up to the 150 mL mark with tap water. Record the volume to the correct number of significant figures. Pour 8 to 9 mL of water from this flask into a 10 mL graduated cylinder, then 23 to 24 mL into a 25 mL cylinder, then 48 to 49 mL into a 50 mL cylinder, and pour the remaining water into a 100 mL cylinder. Record the volume in each cylinder to the appropriate number of significant figures. Add the volumes to get the total volume. Compare the volume obtained from the reading on the Erlenmeyer flask to the sum of the volumes using various graduated cylinders.

Part 3: Accuracy versus Precision

Obtain a 100 mL beaker and add 30-40 mL of water. Note the water level reading to the appropriate number of significant figures. Take a 50 mL cylinder and record the mass of the empty cylinder. Pour the water from the beaker to a 50 mL graduated cylinder and record the volume. Record the mass of the cylinder with the water. Calculate the mass of the water. Determine the true volume of water, using the density of water (0.9982 g/mL at 20°C). Calculate the percent error for the volume recorded using the beaker and the graduated cylinder.

Part 4: Average and standard deviation

Take a 50 mL beaker and record its mass. Measure 1 mL of distilled water using a 5 mL pipette and a green pipette pump and transfer it to the beaker. Record the mass of the beaker and water. Repeat the above steps by adding 1 mL of water each time (and recording the mass), until you have added 5 mL of water. Calculate the density of water to the correct number of significant figures. The true density of water is 0.9982 g/mL at 20°C. Using your data, calculate the average density, standard deviation, relative deviation, percent error and answer the follow up questions.

¹<http://dbhs.wvusd.k12.ca.us/webdocs/SigFigs/SigFigsFable.html>

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Report Sheets

Scientific Measurements

Name _____ Section _____

Lab partner _____

Instructions: Please turn in these sheets only. (Detach the previous pages.)

Grading Criteria - You will earn/lose points based on the following:

- Number of significant figures used, based on the measurement tool
- Including units for all measured values
- Explanations where required
- Use of a **pen** for recorded data
- Do not white out or erase data, use a strikethrough

Part 1: Recording measurements to the correct number of significant figures.

Part A: Volume

Graduation of a 10-mL cylinder _____ mL

Recorded volume _____ mL

Part B: Length

Smallest graduation of a plastic ruler _____ cm _____ mm

Length of notecard using plastic ruler _____ cm _____ mm

Smallest graduation of a meter stick _____ cm _____ mm

Length of notecard using meter stick _____ m

Part C: Mass

Penny	Mass(g)
1	
2	
3	

This balance reports to the _____ (ones, tenths, etc.) decimal place.

Part 2: Significant Figures in Calculations

Part A: Volume

Volume of water in Erlenmeyer flask _____ mL

Graduated Cylinder	Volume (mL)
10-mL	
25-mL	
50-mL	
100-mL	

Calculated total volume _____ mL

Part 3: Accuracy vs. Precision**Part A: Beaker**

Graduation of a 100-mL beaker _____

Recorded volume (your experimental value) _____

Part B: Graduated Cylinder

Mass of empty 50-mL graduated cylinder _____

Recorded volume (your experimental value) _____

Mass of graduated cylinder and water _____

Mass of water
(Show your work.) _____

True volume of water* _____

(Show your work. Calculate the true volume of water using the density of water, 0.9982 g/mL at 20 °C, and the mass calculated above.)

Use this true volume for both percent error calculations below.*Part C: Percent Error**% error for the 100-mL beaker
(Show your work.) _____% error for the 50-mL graduated cylinder
(Show your work.) _____**Part 4: Average and Standard Deviation**

Mass of empty 50-mL beaker: _____

Volume of water (mL)	Mass of beaker + water (g)	Mass of water (g)	Density of water (g/mL)
1.00			
2.00			
3.00			
4.00			
5.00			

Average density of water

(Show your work.)

Standard deviation density of water

(Show your work.)

Relative standard deviation density of water

(Show your work.)

Percent error density of water

The true density of water is 0.9982 g/mL at 20 °C.

(Show your work.)

Follow up Questions**Part 1: Recording measurements to the correct number of significant figures.****Part B: Length**

- 1.) When you converted your measurement from centimeters to millimeters, should there be more, less, or the same number of significant figures? **Explain.** (Hint: Did you use a new measuring device?)

- 2.) Based on you data, did your measurement have a different precision when you use a meter stick as compared to the plastic ruler? State yes or no, and explain.

Part C: Mass

- 1.) If the mass of an object was measured on the balance in our lab and a classmate said that the reading was 120.1 g, would this be correct? **No, it would be incorrect because the balance measures more sig figs than that!** Unlike in math class where trailing zeros are often neglected, zeros may need to be included at the end of a number to indicate the precision of the device.
- The correctly reported mass should be = _____ g instead of 120.1 g
 - What is the implied error range for the following measurements:
 $120.10 \text{ g} \pm \text{_____g}$ vs. $120.1 \text{ g} \pm \text{_____g}$
 - Is there a difference in reporting 120.1 g versus 120.10 g? Yes / No
 - Which measurement above implies a greater level of precision? **Explain** briefly.
-

Part 2: Significant Figures in Calculations**Volume**

- Did your 150-mL Erlenmeyer flask really hold 150 mL? Circle: Yes or No
- Based on your answer above, would you say an Erlenmeyer flask is better suited for measuring volumes or for holding and mixing solutions? Circle: Measuring or Holding/Mixing
- If the density of water is 0.9982 g/mL at 20 °C, what is the mass of water that was in the Erlenmeyer flask? Use your calculated total volume of water measured using the graduated cylinders. Show your work.

Part 3: Accuracy vs. Precision**Part A: Beaker**

- What was the recorded volume? _____
- Which digit contains the uncertain value? Circle the digit above that was estimated.
- How many significant figures can you report based on the graduations? _____

Part B: Graduated Cylinder.

- What was the recorded volume? _____
- Which digit contains the uncertain value? Circle the digit above that was estimated.
- How many significant figures can you report based on the graduations? _____

- 4.) Examine the beaker and the graduated cylinder. Look at your recorded measurements. Which glassware will give a more **precise** measurement? **Explain.**

- 5.) Based on the calculated % error values, which is more **accurate**, the volume measurement using the beaker or the graduated cylinder? (It is expected that the volume measurement using the beaker should be less accurate than with the graduated cylinder. If your results do not agree with this, repeat the experiment.)

- 6.) Many times you'll come across a new piece of lab equipment and need to determine its precision. What would you look at to determine the precision of a piece of glassware?

Part 4: Average and Standard Deviation

- 1) Comment on the **accuracy** of your calculated average density of water using your calculated **percent error**.

- 2) Comment on the **precision** of your data for the density of water using the **standard deviation** and **relative standard deviation**.

Additional Questions

- 1) Suppose a 10-mL pipet has an uncertainty of about ± 0.01 mL. If it is filled with to the mark (10-mL), the volume should be reported as (include the necessary decimal places):

_____ mL

- 2) If your 100-mL graduated cylinder (the same one used in this lab experiment) is filled to 35.0 mL with pure ethanol whose density is 0.79 g/mL at 20.0°C, calculate the mass of ethanol. Show your work.

_____ g

- 3) Mark the glassware's scale so that it can **report to** the thousandths place. "Report to" means to record digits to this decimal place, including your estimated digit.

HINT: A common **mistake** is to draw devices graduated to the thousandths places, don't do this.



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NAME _____ SECTION _____

Pre-Lab Assignment—To be completed BEFORE lab!

Read the background section of this experiment. Answer the following questions.

1) A student performed an analysis of a sample for its calcium content and obtained the following results:

14.92 g, 14.91 g, 14.88 g, 14.92 g

The actual amount of calcium in the sample is 20.90 g. For this lab, the instructor asked students to obtain results within 5% error and RSD and to redo the experiment if that was not achieved.

(a) Report the student's results for the average and the standard deviation of the measured masses. Show your calculations below. Do the calculations by hand, and if you have a calculator or Excel you can use it to check your answer.

_____ g \pm _____ g
(average) (standard deviation)

(b) Calculate the percent error, using the average mass, for this student's experiment.

(c) Calculate the relative standard deviation for this student's experiment.

(d) Briefly comment on the accuracy vs. the precision obtained by the student. Does the student need to redo the experiment?

2) (a) Provide a measurement for each of the following pieces of glassware filled to the level indicated by the arrow. Use the correct number of significant figures in your answer. Always use units. **Remember to estimate one digit more than the level of graduation.**

a. _____

b. _____

c. _____

(See note below*)

*Volumes in burets are read from top to bottom.

(b) Which gives the most precision, (a.) the graduated cylinder, (b.) the buret, or (c.) the beaker?

(c) How can you tell by simply looking at the glassware? **Explain briefly.**

3) (a) Label this beaker with values so that it can **report** volume to the **tens (not tenths)** place. (“Report to” means to record digits to this decimal place, including your estimated digit.)

(b) Give a sample measurement that could be obtained from this beaker. Use mL for units. Use significant figures.

