

Critical Thinking

Mark Storey
Bellevue College

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Chapter 1: Thinking Critically about the Logic of Arguments

Logic and critical thinking together make up the systematic study of reasoning, and reasoning is what we do when we draw a conclusion on the basis of other claims. In other words, reasoning is used when you infer one claim on the basis of another. For example, if you see a great deal of snow falling from the sky outside your bedroom window one morning, you can reasonably conclude that it's probably cold outside. Or, if you see a man smiling broadly, you can reasonably conclude that he is at least somewhat happy. In both cases, you are reasoning from evidence to a conclusion.

We use reasoning all the time, but sometimes we make a mess out of it. Whether a line of reasoning is good or not is definitely more than “just a matter of opinion.” Surely the reasoning in the following arguments is not compelling:

- * My four-year-old niece says that the planet Mars is smaller than Jupiter. It must thereby be the case that Mars is smaller than Jupiter.
- * Some women are baseball fans. And some mothers are baseball fans. Thus, all women are mothers.
- * An earthquake occurred in San Francisco five minutes after the senator's speech there. Thus that senator's voice causes natural disasters.

But the reasoning in the next set of arguments is better, yes?

- * All bears are mammals. Grizzlies are bears. Thus grizzlies are mammals.
- * If Jimmy Carter was the U.S. President, then he was a politician. Carter was indeed the U.S. President. Thus, Carter was a politician.
- * It has rained in Seattle, Washington every year for the past 100 years. Thus it will probably rain there next year.

Some examples of reasoning are clearly better than others. The study of logic and critical thinking are designed to make us better at recognizing good from bad lines of argumentation.

An *argument* consists of one or more statements, called *premises*, offered as reason to believe that a further statement, called the *conclusion*, is true. Technically speaking, premises and conclusions should be made up of *statements*. A statement is a sentence that declares something to be true or false. They are thus sometimes called *declarative sentences*. A *sentence* is a grammatically correct string of words, and there are many kinds of sentences other than statements. Questions (e.g., “What is your name?”), commands (e.g., “Turn to page three”), and exclamations (e.g., “Ouch!”) are all grammatically correct sentences that are not statements. They are not statements because it makes no sense to say they are true or false. (“What is your name?” “That's *true!*” This would be a ridiculous mini-conversation.) Statements will always be true or false, never both, and never neither. We may disagree on whether a given statement is true (e.g., “God exists”), or we may not be able to determine whether a statement is true or false

(e.g., “There is a mountain on Pluto exactly 1000 meters tall, plus or minus 2 centimeters”), yet the statement is objectively true or false (but not both) nonetheless.

In this course, the words “statement” and “sentence” can—in many contexts—be used interchangeably. This is so because all statements are sentences (although not all sentences are statements). So we can refer to “Bellevue is in Washington” as both a statement (because it declares something to be true) and a sentence (because it is a grammatically correct sequence of words conveying a meaning).

An argument can have any number of premises, but technically speaking there is one conclusion per argument. Thus, an argument splits into two distinct parts:

1. One or more *premises* offer evidence for the truth of the conclusion.
2. The *conclusion* is supported by the premise or premises.

Here is an argument:

All dogs are mammals.
No mammals are birds.
Thus, no dogs are birds.

The conclusion seems well supported by the two premises. However, things are not so good in the following argument:

Some cats are animals.
Some animals are fish.
Hence, some cats are not fish.

In both examples above, the arguments contained two premises and one conclusion, but in the second argument immediately above, the premises by themselves do not offer good reason to believe the conclusion—even if though the premises are true!

Sometimes the conclusion of an argument can be used as a premise of a following argument, making a chain of arguments. Still, to be precise, each argument or specific line of inference contains one and only one conclusion, although each may contain varying number of premises. For instance:

1. All dogs are mammals.
2. All mammals are animals.
3. Thus, all dogs are animals.
4. Scooby-Doo is a dog.
5. Thus, Scooby-Doo is an animal.
6. No animals are plants.
7. All trees are plants.
8. Thus, Scooby-Doo is not a tree.

Whew! Here the first argument in the chain has lines 1 and 2 as premises, and has line 3 as its conclusion. The second argument then uses line 3 as a premise and uses it with line 4 to conclude in line 5 that Scooby-Doo is a dog. The third argument then uses line 5 as a premise, hooks it up with lines 6 and 7, and uses the trio together to infer line 8 as the final conclusion.

****Practice Problems: Types of Sentences**

Are the following statements or not?

1. George Carlin is presently president of the USA.
2. Chocolate is a popular flavor of ice cream in the USA.
3. Sally Brown, come on down!
4. Washington State is south of Oregon.
5. Bob believes that Washington State is south of Oregon.
6. College students are morally obliged to believe that Washington State is south of Oregon.
7. Who in Oregon is rooting for the Huskies?
8. It is prudent for Duck fans not to wear green when going to a Husky game in Seattle.
9. Green is an Oregon Ducks color, while purple is a Washington Huskies color.
10. The Huskies are my favorite college football team!
11. Go Cougars!
12. The Ducks will never win the Apple Cup.
13. Huskies
14. Ducks vs. Cougars
15. The Ducks will play the Cougars tonight.
16. Slap a ham on Omaha, pals!
17. Dennis and Edna sinned.
18. Rats live on no evil star.
19. Tarzan raised Desi Arnaz' rat.
20. Go deliver a dare, vile dog.

Answers:

- | | | | |
|--------------------|--------------------|---------------------|---------------------|
| 1. statement | 6. statement | 11. not a statement | 16. not a statement |
| 2. statement | 7. not a statement | 12. statement | 17. statement |
| 3. not a statement | 8. statement | 13. not a statement | 18. statement |
| 4. statement | 9. statement | 14. not a statement | 19. statement |
| 5. statement | 10. statement | 15. statement | 20. not a statement |

Indicator Words

Before determining whether an argument is good or bad, we need to recognize its structure. We need, that is, to know which claims are premises and which one is the conclusion. Indicator words or phrases can help us out here.

A *conclusion indicator* is a word or phrase that, when used in the context of an argument, signals that a conclusion is about to be given or was just given. In the two examples above, “Thus” and

“Hence” were used as indicators to signal the presence of the conclusion. The following are some of the commonly used conclusion indicator words and phrases:

Therefore	In conclusion	Hence	entails that
Thus	Accordingly	Ergo	We may infer
So	It follows that	We can conclude that	implies that

A *premise indicator* is a word or phrase that, when used in the context of an argument, signals that a premise is about to be given or was just given. Here are some examples:

Because	Since	If	Provided that
For the reason that	for	Given that	Assuming that
Due to the fact that is reason to believe that	may be inferred from	Inasmuch as supports the claim that	is evidence for

If you want to make your reasoning as clear as possible when you present arguments, use indicator words to signal your premises and conclusions. Your audience (e.g., a teacher grading your essay) will appreciate it, and your reasoning will be easier to follow than it otherwise might be.

Note, though, that some indicator words have multiple uses. The premise indicator “if,” for instance, is often used in other ways. For example, in the sentence, “If Yogi is a bear, then Yogi is an animal,” the word “if” is used as part of a single complex statement called a conditional (i.e., an “if... then...” statement). Also, the conclusion indicator “so” can be used in many ways, such as, “I am so happy I’m studying logic!” Indicator words can be helpful, but we must still be careful in recognizing how they function in a sentence.

****Practice Problems: Indicator Words**

For each argument, (a) state any premise or conclusion indicators, and (b) state the conclusion.

1. Since Tuan is a student, it follows that he studies regularly.
2. Sarah is a mother, because she has given birth to a child.
3. All dogs are mammals, and all mammals are animals; thus all dogs are animals.
4. Given that Kim is the country’s president, that Kim is a politician may be inferred from the fact that all presidents of countries are politicians.
5. The ground is wet during a heavy rain. Consequently, due to the fact that it’s raining now, the ground now is wet.
6. Provided that two is greater than one, and three is greater than two, it follows that three is greater than one.
7. Tran is happy. Hence Tran is happy.
8. Simón Bolívar was born in Venezuela. Bolívar was a military hero in South America. This implies that a military hero was born in Venezuela.
9. According to Socrates, people will do what they believe is in their best interests. Thus, since the good is in people’s best interest, it behooves philosophers to explain the good to people.
10. Given that all dogs are mammals, and because no mammals are birds, it must be concluded that no dogs are fish.

11. Assuming that Senator Sunny Shine likes to swim, and inasmuch as today is warm and the sun is out, it follows that Sunny Shine is swimming in her backyard pool.
12. Shine and her husband don't want to unduly offend their neighbors, and their neighbors are not fans of skinny-dipping. We may infer that Shine and her husband don't skinny-dip when their neighbors are watching.
13. Pastor Bustle is opposed to all skinny-dipping. Bustle is opposed to some of the Shines' activities, due to the fact that Bustle knows that the Shines like to skinny-dip.
14. If Bustle climbs a ladder to look over the fence at the Shines, then Bustle will probably fall and twist his ankle. Bustle does indeed climb a ladder to look over the fence at the Shines. This entails that Bustle will probably fall and twist his ankle.
15. Either Bustle gets away with voyeurism or the police fail to charge him with a misdemeanor. Since the police do fail to charge Bustle with a misdemeanor, Bustle accordingly gets away with voyeurism.

Answers:

1. (a) Since; it follows that; (b) he studies regularly
2. (a) because; (b) Sarah is a mother
3. (a) thus; (b) all dogs are animals
4. (a) Given that; may be inferred from; (b) Kim is a politician
5. (a) Consequently; due to the fact that; (b) the ground is now wet
6. (a) Provided that; it follows that; (b) three is greater than one.
7. (a) Hence; (b) Tran is happy (the second instance of the claim)
8. (a) This implies that; (b) a military hero was born in Venezuela
9. (a) Thus; since; (b) it behooves philosophers to explain the good to people
10. (a) Given that; because; it must be concluded that; (b) no dogs are fish
11. (a) Assuming that; inasmuch as; it follows that; (b) Sunny Shine is swimming in her backyard pool.
12. (a) We may infer that; (b) Shine and her husband don't skinny-dip when their neighbors are watching
13. (a) due to the fact that; (b) Bustle is opposed to some of the Shines' activities
14. (a) This entails that; (b) Bustle will probably fall and twist his ankle
15. (a) Since; accordingly; (b) Bustle accordingly gets away with voyeurism

Distinguishing Arguments from Non-arguments

We probably will not understand what an argument is unless we can tell the difference between an argument and a non-argument. This is why the ability to distinguish arguments from things that are not arguments is an important skill in logic.

An *argument* is someone's reasoning expressed in the format of a language. When an argument is given, one or more reasons are being offered for a conclusion. However, there are many things we do with language besides reason. We use language to describe things, to explain things, to express our feelings, to give orders, to ask questions, to tell stories, to give advice, to offer reports, to babble incoherently, and on and on. None of these activities involves logical argumentation; none constitutes giving an argument. Essentially, in the case of a non-argument,

someone is *not* trying to prove a point—someone is *not* offering a reason to believe a claim that is being advanced, someone is not offering evidence *for a conclusion*, while in the case of an argument, someone is offering reasons in support of a conclusion, reasons to believe that a claim is true.

****Practice Problems: Arguments and Non-arguments**

In each case, does the passage present an argument or a non-argument?

1. Elizabeth and Marty went together to school on Tuesday, got in a minor automobile accident, and were late for their biology class. Their teacher was giving a test that day, and the two students were not there to take it.
2. Elizabeth and Marty left their house to go to school on Tuesday, but on the way decided to spend the day at the movie theater instead. Their biology teacher was giving a test that day, and the two students were not there to take it. That is why they received a poor grade for their coursework that week.
3. Elizabeth and Marty, you two are crazy! You should not have gone to the movies Tuesday, especially when you had a test in your biology class. You should go to school each day classes are in session.
4. Elizabeth and Marty went together to school every day this week and studied the material covered in class. Students who attend class regularly and study regularly usually do well in class. Thus Elizabeth and Marty probably did well in class this week.
5. Some students do not attend class regularly. For instance, Elizabeth and Marty went together to school on Tuesday, but decided to return home to play Grand Theft Auto all day. Such behavior is indicative of poor study habits.
6. Maria studies every night for her chemistry class, and works very precisely in her chemistry lab work. She also attends class each day and takes complete notes. We can conclude that Maria will likely do well in her chemistry class.
7. Both Mahatma Gandhi and Sri Aurobindo were philosophically minded, both were male, both were from India, and both wrote commentaries on the *Bhagavad Gita*. Gandhi fought against British occupation of India. Thus probably Aurobindo did, too.
8. Rene Descartes had trouble seeing the relations between things in Nature, focused on breaking “problems” into smaller parts, and missed viewing systems holistically. Thus he has been deemed a “mechanistic” philosopher.
9. Fatima likes pizza. Julio likes football. Takashi likes reading *The Tale of Genji*.
10. Sunzi wrote *The Art of War*, and *The Art of War* was written by a Chinese philosopher. Sunzi must therefore be a Chinese philosopher.

Answers:

1. Non-argument. It’s merely a report of the day’s events with no inference.
2. Non-argument. It’s a causal explanation of the students’ poor grades with no inference.
3. Non-argument. It’s a combination of opinion and advice, but with no inference.
4. Argument. There are a series of claims serving as premises leading to a conclusion (note the indicator word “thus”).
5. Non-argument. It’s merely an illustration of the opening claim with no inference.
6. Argument. Note the use of the conclusion indicator, “We can conclude that.”

7. Argument. This is an argument from analogy.
8. Non-argument. We find the word “thus” (which is often a conclusion indicator), but here it is pointing to the effect of a causal relation. That is, the final statement is explained by the previous ones, but there is no inference intended here.
9. Non-argument. It’s just an unconnected string of claims.
10. Argument. The two claims in the first sentence offer reason to believe the claim in the final sentence. Also, “therefore” is functioning as a conclusion indicator.

Chapter 2: Deduction and Induction

Logicians divide all arguments into two broad categories: deductive arguments and inductive arguments. Every argument falls into one of these two categories. Of course people offering arguments often do not fully understand what they are doing. That is, they may be unclear how powerful their arguments could be, assuming they are arguing well. Still, once we understand what their arguments are, it's usually not difficult to determine if each is best understood as deductive or inductive.

Deductive Arguments

A *deductive* argument claims (explicitly or implicitly) that if the premises all are true, then the conclusion *must* be true. Deductive arguments thus aim to establish their conclusions with complete certainty in such a way that the conclusion is guaranteed to be true if the premises all are true. Note that the argument may fail in its aim; what makes the argument deductive is that it is the kind of argument that—if it were successful—would have the premises absolutely guarantee the conclusion to be true.

The following four arguments are all deductive:

- * All bats are cute animals. No cute animals are mean. So, certainly, no bats are mean.
- * Joan is Lauren's mother. Therefore, Joan must be older than Lauren.
- * Nobody knows Ned. Therefore, it must be that Ned does not know himself.
- * Some cats are pets. Thus, some pets must be cats.

Inductive Arguments

An *inductive* argument claims (explicitly or implicitly) that if the premises all are true, the conclusion is thereby *probably*, or *likely*, true, although not absolutely guaranteed. Inductive arguments thus aim to establish their conclusions with probability, or likelihood, but not with complete certainty. An inductive argument does not attempt to guarantee that its conclusion is true; however, it aims to show that we have good reasons to accept the conclusion as true nevertheless. "Probably," in this context, means greater than 50 percent chance. Admittedly, quantifying the likelihood of a conclusion being true is not always easy, but "*better* than a 50 percent chance" gets the idea across. Note that the inductive argument may fail in its aim; what makes the argument inductive is that it is the kind of argument that—done well—can give *good* but less than 100 percent conclusive reason to believe the conclusion.

Examples of induction are found in everyday life, where people use less than guaranteed reasoning to go about their daily business. You've never (or rarely) been poisoned at the school cafeteria, so you feel safe eating there later today. Your past experiences do not absolutely guarantee that you won't get sick eating there today, but you will eat there nonetheless...and be perfectly rational in doing so. You'll be using a strong form of inductive reasoning. The following four arguments are also inductive:

- * Pat and Jan are lifelong best friends. So Jan probably knows Pat's parents.
- * It has been sunny for ten days in a row, and there are no clouds in the sky. So probably it will be sunny tomorrow.
- * Joe hasn't had a drink in ten years. So it is likely he won't drink at the party tonight.
- * Most dogs are loving animals. Fido is a dog. Therefore Fido is probably a loving animal.

Deductive or Inductive?

Sometimes it is difficult to tell whether a person's argument is deductive or inductive because the reasoning is not clearly stated. When trying to decide whether an argument is deductive or inductive, a good rule of thumb is to ask yourself: Is the arguer aiming to show that the conclusion is *guaranteed* to be true, or is he or she aiming only to show that the conclusion is *likely* to be true, i.e., is probably true but less than certain? If the arguer's reasoning is expressed clearly, then there will often be words or other clues indicating which type of argument (deductive or inductive) is intended. If a deductive argument is intended, then the conclusion may be introduced with words or phrases indicating necessity or certainty, such as:

It is certain that
 Absolutely
 Undeniably, it must be that
 For sure
 It is necessarily true that

However, if an inductive argument is intended, then the conclusion may be introduced with words or phrases indicating probability, such as:

The most reasonable conclusion is
 Probably
 It is likely that
 It is reasonable to suppose that
 I'll bet that

Some common phrases are found in both deductive and inductive arguments, and thus do not help much in determining which kind of argument you are dealing with. "It must be the case that" is an example of a phrase you often find attached to both premises and the conclusion, of both deductive and inductive arguments.

Notice, though, that whereas deductive arguments have an air of certainty, confidence, and conclusiveness, inductive arguments have an air of uncertainty and incompleteness. While a deductive argument claims its conclusion *must* be true, and with certainty, an inductive argument claims only that if its premises all are true then its conclusion is probable although not completely certain.

If everyone were honest or understood inductive and deductive indicator words, and if everyone offering an argument used such vocabulary accurately, we'd have a fairly easy time determining if arguments are intended as inductive or deductive. But none of this is the case, unfortunately. Too many people don't understand or care about the difference between induction and deduction, or try to overstate the strength of their inductive position by couching their arguments in deductive indicator words. We need additional hints to be confident that we are dealing with a deductive (or inductive) argument.

Argument Patterns

Since so few people use indicator words at all or correctly, it's of great use to become familiar with patterns of argument that usually are associated with deduction or induction. These tips are not fail-safe, as occasionally odd counterexamples can be imagined. Still, the patterns provide good reason for saying that a given argument is deductive, or inductive. Examples of such patterns include the following.

Deductive patterns:

* Arguments based on math

Since we know basic math truths (e.g., $2+2=4$) with our highest degree of certitude, any argument *based* (not merely *about*) math will have a similar degree of certitude.

E.g., Joe passed two touchdowns in the first half of the football game, and he passed one touchdown in the second half of the game. Thus Joe passed three touchdowns in that game.

E.g., This Euclidian triangle has one right interior angle and one 10 degree interior angle. Thus the third interior angle must be 80 degrees.

Note: The following argument is not *based* on math, it is *about* math; it is *not* deductive: "All my math teachers say that two plus two equals four. Thus two plus two equals four."

* Arguments based on definitions

If an inference is based on the definition of a word, then (even if the definition is mistaken) the argument is best understood as deductive.

E.g., Sara is a physician. Thus Sara is [by definition] a doctor.

E.g., This shirt is damp. Thus this shirt is moist.

E.g., Bob has no hair on his head. Therefore Bob is bald.

* Categorical syllogisms

A syllogism has two premises, and a categorical syllogism is a two-premise argument in which each premise and the conclusion begin with the words "All," "No," or "Some," in any combination.

E.g., All dogs are hairy things, and all hairy things are mammals. Thus all dogs are mammals.

E.g., No dogs are cats. Some cats are white animals. Thus some dogs are not white animals.

* Conditional syllogisms

A "conditional syllogism" is a two-premise argument in which at least one premise is an "if..., then..." statement, known as a conditional, or implication.

E.g., If Yogi is a bear, then Yogi is an animal. Yogi is a bear. Thus Yogi is an animal.
E.g., If it rains today, then the picnic is canceled. If the picnic is canceled, then I'll be sad.
Hence, if it rains today, then I'll be sad.

* Disjunctive syllogisms

A disjunction is an "...or..." statement, so a disjunctive syllogism is a two-premise argument in which one of the premises is an "...or..." statement (with or without an "either").

E.g., Either Bob is logical or Sue is happy. But Bob is not logical. Therefore Sue is happy.

Inductive patterns:

* Predictions

If the conclusion is a prediction (about the future), then it is likely an inductive argument, since we cannot know the future with total certainty.

E.g., It has rained every day for the past five weeks. Thus it will probably rain tomorrow, too.

* Arguments from analogy

An argument from analogy compares two things or two groups of things, notes many relevant similarities between the two, and concludes that probably what is known to be true of one will be true of the other.

E.g., I've eaten at Dick's Drive-In 20 times this past month and enjoyed my dining experience each time. Dick's has not changed recently in any relevant way, so I will likely enjoy my dining experience at Dick's when I go there later today.

E.g., After studying only one night beforehand, I've aced the last three logic quizzes in this class. So even though I've studied only one night for this class's upcoming logic quiz, I'll probably ace it, too.

* Appeal to authority

An appeal to authority argues that because an expert says such-and-such, then such-and-such is likely to be true. As long as the expert is truly expert (i.e., authoritative) in his, her, their, or its field, then the argument can be quite strong.

E.g., Every biology instructor will say that mice are mammals. Thus we have good reason to believe that mice are mammals. (A strong argument, because biology instructors are very knowledgeable on such matters.)

E.g., My chemistry teacher says that God does not exist. We can conclude that God does not exist. (A weak argument, because (a) chemistry teachers qua chemistry teachers are not experts on God's existence, and (b) it makes little sense to consider anyone an expert on God's existence. The same would be the case if the teacher had said that God *does* exist. If the chemistry teacher came up with a good *argument* against (or for) God's existence, then perhaps that *argument* might give us good reason to disbelieve (or believe) in God.

E.g., The Washington State highway sign we just passed said that we're entering the city of Seattle. Thus we are probably entering the city of Seattle. (A strong argument, as state highway signs are usually reliable regarding city names.)

* Generalizations

Think of polls as common examples of generalizations. You determine that something is true of a portion of a group, and conclude that the same thing must be true of the group as a whole. E.g., Twenty percent of my Bellevue College logic students are fans of Miles Davis. Thus twenty percent of Bellevue College's students are fans of Miles Davis. E.g., Most Roman Catholics living in North Bend, Washington believe in a Christian concept of God. Thus most people in the world believe in a Christian concept of God.

* Causal arguments

A causal argument appeals to well-known causal relations to argue from cause to effect or from effect to cause. Police detectives do this when they see a crime scene with effects (e.g., a dead body with a knife in its back) and try to argue towards the cause (e.g., who did it, why, when, how, etc.). Doctors and auto mechanics trying to figure out the cause of a medical or mechanical problem argue the same way.

E.g., I placed a jug of water in my freezer last night. Thus that jug of water is now likely frozen. (The argument here is from cause to effect.)

E.g., The instructor just passed back our logic tests, and Maria is holding hers and smiling. I conclude that she must have received a high score on that test. (The argument here is from effect to cause.)

An argument can fit more than one pattern, and that's not a problem. For instance

My trustworthy wife said that she will buy some bread this evening. Thus she will likely buy some bread this evening.

This argument is an appeal to authority and its conclusion is a prediction. It's quite clear that it's best understood as inductive. The indicator word "likely" makes its inductive nature all the more clear.

If an argument clearly fits both an inductive and a deductive pattern, then it's generally safe to figure the argument is deductive. This is because if an argument is constructed so that if the premises are true the conclusion can be guaranteed to be true, it will also be the case that the conclusion is *likely* true. If a conclusion is sure to be true, then it's obviously going to be probably true. But only deductive arguments have premises that absolutely guarantee the conclusion; so if an argument can do that, it's deductive. For example:

If a blizzard dropped four feet of snow on the city park last night, then today's picnic will be canceled. A blizzard did indeed drop four feet of snow on the city park last night. Thus, today's picnic will be canceled.

The argument fits both the conditional syllogism pattern for deduction and the prediction pattern of induction. But *if* the premises are true, they guarantee that the conclusion will be true; and only a deductive argument can do that. So we're justified in interpreting this as a deductive argument.

****Practice Problems: Deduction and Induction**

For each argument, state whether it is deductive or inductive. Some contain deduction or induction indicator words; others do not.

1. Some dogs are mammals. Some mammals are animals. Thus some dogs are animals.
2. Either Thomas Aquinas was a writer or he was an astronaut. But he was not a writer. Thus he definitely was an astronaut.
3. Nearly all geometry teachers are serious when they say that triangles have three sides. Thus, on that basis, we can say that triangles have three sides.
4. Some diseases can spread easily from one person to another by skin contact. Thus it is guaranteed that some diseases are contagious.
5. No human has ever lived for 2000 years. Thus the current prime minister of England will not live for 2000 years.
6. The official sign posted at the edge of our campus says that this is Catatonic State University. Therefore, this must be Bellevue College.
7. If Michael Jackson was president of the United States, then he was a politician. Michael Jackson was a politician. Thus Michael Jackson was president of the United States.
8. The U.S. military dropped many bombs on Iraq while fighting there. Bombs almost always explode, destroying things near them. Thus the U.S. military probably destroyed things in Iraq.
9. If Lady Gaga [the female singer] is an adult man, then Lady Gaga is a male. But Lady Gaga is not an adult man. Thus Lady Gaga is not a male.
10. German philosopher Georg Hegel was a space alien. Thus Georg Hegel was a space alien.
11. No dogs are cats. No cats are mice. Thus it is guaranteed that some cats are not mice.
12. Large, naturally occurring icebergs have yet to be found in the middle of the Sahara Desert. Thus it is likely that no such ice berg will be found there next year.
13. The Atlantic Ocean lies between Africa and South America. Africa is immediately east of the Atlantic Ocean. Thus South America is certainly west of the Atlantic Ocean.
14. The sign placed by officials on the Statue of Liberty in New York City says that it was made by Peruvian artists. Thus the Statue of Liberty was likely made by Peruvian artists.
15. Every rock-n-roll musician says that we should all eat corn for dinner every night. Thus we should eat corn for dinner every night.
16. The label on this bottle says, "Poison." Therefore the bottle must contain poison.
17. Three is larger than two, and two is larger than one. Thus one is less than three.
18. Most people in our society are opposed to murder. Thus murder is probably wrong.
19. God has all perfections. Existence is a perfection. Thus it is necessary that God has the perfection of existence. Thus it is necessary that God exists.
20. There is much evil in the world. And since God is supposed to be all-good and all-powerful, God would not want or need to allow evil. Thus God likely does not exist.

Answers:

- | | | | |
|--------------|---------------|---------------|---------------|
| 1. Deductive | 6. Inductive | 11. Deductive | 16. Inductive |
| 2. Deductive | 7. Deductive | 12. Inductive | 17. Deductive |
| 3. Inductive | 8. Inductive | 13. Deductive | 18. Inductive |
| 4. Deductive | 9. Deductive | 14. Inductive | 19. Deductive |
| 5. Inductive | 10. Deductive | 15. Inductive | 20. Inductive |

Chapter 3: Evaluating Deductive Arguments

Valid vs. Invalid

Every argument in the universe needs to “pass” two tests; the arguments must be *logically* good and *factually* good. We are speaking loosely at this point, but all deductive and inductive arguments must meet the same basic pair of demands: it must be the case that (a) its premises give good reason to believe the conclusion, and (b) the premises are actually true. The first concern pertains to the relation the premises have to the conclusion, and the *actual* truth or falsity of the premises is often irrelevant. The second concern pertains to the facts of the matter and to whether the claims of the premises correspond accurately to the world. Figuring out if an argument is *logically* good or not often involves a hypothetical thought experiment in which you don’t really care if the premises are actually true or not. Figuring out if the argument is *factually* good forces you to step out of the hypothetical thought experiment and rely on your knowledge of the real world. We’ll begin by focusing our attention on the first concern.

As we have seen, a deductive argument is any argument claiming either explicitly or implicitly that if the premises all are true, then the conclusion must be true. Deductive arguments are evaluated as either “valid” or “invalid.” A deductive argument is valid when it is indeed the case that if the premises are true then the conclusion must be true, and a deductive argument is invalid when it is not the case that if the premises are true then the conclusion must be true.

To determine if a deductive argument is valid or invalid, ask yourself a question: Is it logically impossible for the premises to be true and at the same time and from the same perspective for the conclusion to be false? If “Yes,” then the argument is valid. If “No,” then the argument is invalid.

The distinction between valid and invalid arguments will become clearer after you’ve examined some examples. The following deductive arguments are all valid. Notice that it is *impossible* for the premises to be true and the conclusion false.

- * Every square has four sides. This figure is a square. Therefore this figure must have four sides.
- * Tom is older than Bob and Bob is older than Ed. So Tom must be older than Ed.
- * Some cats are pets. Thus, it must be that some pets are cats.
- * Alfredo is Sue’s (biological) father. Therefore, Alfredo must be older than Sue, because fathers are always older than their biological children.

The following deductive arguments are invalid. Notice that it is *possible* for the premises to be true and the conclusion false.

- * Javon is older than Betty. Therefore, Javon must be taller than Betty.
- * All members of the XYZ club are senior citizens. Thus it must be that all senior citizens are members of the XYZ club.
- * All members of the Hells Angels live in California. Joe lives in California. Therefore, it is certain that Joe is a member of the Hells Angels.

* If the sun is out, then Vu is swimming. Vu *is* swimming. So it must be that the sun is out.

Each argument above is deductive because the claim in each case is that the conclusion must be true if the premises are true. However, some are valid deductive arguments and some are invalid, because some succeed in showing that their conclusions *must* be true if their premises are true, and some do not. That is, for the invalid arguments, it is logically possible for the premises to be true and the conclusion to be false. The premises are thus not guaranteeing the conclusion.

We have been speaking of “logical possibilities.” The word “possibility” can mean different things in different contexts. In the context of logic, anything counts as logically possible as long as it does not imply a contradiction. A contradiction is a claim that says a statement is both true and false at the same time and from the same perspective. For instance, “Bob is six feet tall, and it is false that Bob is six feet tall.” Both statements can’t be true; they contradict each other. As long as a statement does not contradict itself or imply a contradiction, it is *logically possible* to be true, or more loosely, *logically possible*.

Sometimes in ordinary conversation we speak of possibility in a different sense. For instance, “It’s impossible for a person to swim across the Atlantic Ocean in under two minutes.” It’s *logically possible* for such a feat to take place, as it implies no contradiction, but it’s *physically impossible* certainly, given the world’s laws of physics and the nature of our bodies. The friction of such movement through so much water would rip the flesh off anyone’s body. Logicians speak of *logical impossibility*, though, and are less concerned usually with what we can loosely call *physical impossibility*.

The following statements are logically possible because although they are extremely unlikely they do not contradict themselves or imply a contradiction.

* Bob was over 60 years old when he entered the Olympic track and field event for the USA, yet he won the gold medal in the 50-meter sprint.

* A living Tyrannosaurus Rex will destroy Bellevue College’s cafeteria in 2015.

In logic, any statement of the form “P and it is not the case that P” (where P is a statement) counts as a logical contradiction. Here are three examples of logical contradictions:

* Ahmad is a sibling of Jules; however Jules is not a sibling of Ahmad.

* Sara now has exactly two coins in her right hand, and she now has precisely an even number of coins in her right hand.

* Mexico is north of the USA and it is not the case Mexico is north of the USA.

In each case above, the statement begins saying one thing, and then ends up saying that what it said at the beginning is false. Yet no statement can be true and false at the same time from the same perspective. Contradictions thereby are always false.

The notion of logical possibility is important in critical thinking, as it's a key concept in assessing the logical success of many arguments. Specifically, it's crucial in considering an argument's *validity*.

Valid vs. Invalid

Suppose you are looking at a deductive argument trying to decide whether it is valid or invalid. How do you decide? Again, ask yourself a hypothetical question: Is it logically impossible for the premises to be true *and* at the same time the conclusion be false?

If you answer “Yes”—in other words—if the conclusion must be true if the premises are true, then the argument is *valid*. However, if your answer is “No” because the conclusion might be false even if the premises are true, then the argument is *invalid*. For example, suppose the Smiths are a big family living in Lynnwood, Washington:

All the Smiths are Catholics.

All Catholics live in Italy.

So, all the Smiths must live in Italy.

Is it *impossible* the premises could be true *and* the conclusion false? Yes! If the premises were true, the conclusion would *certainly* be true. This argument is therefore valid. It is *logically* good. The structure of the argument is such that *if* the premises were true (and they are not, but for now that's irrelevant) the conclusion would be guaranteed to be true. We see this once we agree to do the thought experiment of asking about the *possibility* of the premises being true while the conclusion is false. Of course, given what we know about the Smiths (i.e., that they live in Lynnwood, Washington) the second premise is clearly false (and maybe the first premise, too), so the argument is *factually* bad, but we'll get to that concern momentarily. For right now we are concerned only with the logical structure—or “bones”—of the argument. We'll look at issues pertaining to the facts, or “truth value,” of the premises in a bit.

More Examples...

The following are additional examples of valid arguments:

This apple is red.

Thus, this apple is colored.

I have exactly two coins in my right hand.

Hence, I have an even number of coins in my right hand

If it snows, then the roof will be white.

If it rains, then the roof will be wet.

It will either snow or rain.

So, definitely, either the roof will be white or it will be wet.

If we go to Texas, then we will go to Dallas.
If we go to Dallas, then we will visit the JFK museum.
Therefore, surely if we go to Texas, then we will visit the JFK museum.

If the air freezes, then the pond will freeze.
The air will freeze.
Then the pond will certainly freeze.

Either the box is in Portland or the box is in Seattle.
The box is not in Seattle.
Consequently, it must be in Portland.

These are examples of invalid arguments:

This apple is colored.
Thus, this apple is red.

I have an even number of coins in my right hand.
Hence, I have exactly two coins in my right hand.

Isaac is Susan's cousin.
Rita is Susan's cousin.
Therefore, Rita and Isaac are necessarily cousins, too.

Every time it rains my car gets wet.
My car is wet.
So, it certainly must be raining.

All cats are mammals.
Dogs are mammals.
So, dogs are guaranteed to be cats.

We need to note that things can get a little weird sometimes. Again, a valid argument is one for which it is impossible for the premises to be true and the conclusion false. In evaluating the validity of an argument we'd normally just ask if the premises provide enough information to absolutely guarantee the conclusion. And this approach works for nearly every normal deductive argument. But what about the following arguments?

- #1 Frank Sinatra was a singer.
Thus, Frank Sinatra was a singer.
- #2 Two plus two equals five.
Thus, Bo Diddle is currently the U.S. president.
- #3 Two plus two equals five.

Thus, Frank Sinatra was a singer.

#4 The Beatles were from England.
Thus, one plus one equals two.

#5 The Beatles were from Peru.
Thus, one plus one equals two.

Weird arguments, yes? The first seems nearly useless (and it is), but it's both valid and sound. It's valid because it's impossible for the premise to be true (and it is) and at the same time from the same perspective the conclusion be false. But what about the other four arguments? The premises don't seem to have anything to do with the conclusions. Still, they are all valid because in each case it's impossible for the premise to be true and the conclusion false. In #2 and 3, the premise of each is mathematically certain; the claim is necessarily true...it's impossible to be false. And with #4 and 5, each conclusion is necessarily true. So, for #2-5, each argument either has a premise that can't possibly be true or a conclusion that can't possibly be false, so none of them are such that it is possible for the premises to be true and the conclusion false.

This may seem counter-intuitive until the definition of validity sinks in, but keep in mind that we are only talking about the logical structure of arguments right now. Argument #1 may be trivial, since the premise more or less just proves itself. Arguments #2, 3, and 5 have false premises, so those arguments will be rejected due to their factual problems. Argument #4 is actually okay, although seemingly strange. It's just that the conclusion is necessarily true, so we can appeal to anything—or nothing—to see that it's true. We'll look at necessary truths (e.g., tautologies) in more detail later in the course.

Valid \neq True
Invalid \neq False

It is time to fine-tune our use of English. When we call an argument valid we are *not* saying that the argument is “true.” First of all, as our terms have been defined, there is no such thing as a “true argument.” There are true premises, true conclusions (as well as false ones, of course), but there is no such thing as a “true argument.” Only statements can be true or false. To talk about a “true argument” would be like referring to a “red idea.” You can have an idea of redness, but ideas themselves are not colored. It's what philosophers call a “category mistake.” Secondly, in logic, the word “valid” does *not* mean true. More specifically, to call an argument valid is *not* to say that the premises are true, and it is also *not* to say that the conclusion is true. An argument may have false premises and still be valid. It is not required that an argument have true premises in order to be valid. For instance, here is a valid argument with false premises and a false conclusion:

All dogs are birds.
All birds are fish.
Hence, all dogs are fish.

Furthermore, an argument may have true premises and even a true conclusion and yet be *invalid!* It is not required that an argument have false premises in order to be invalid. For instance:

All poodles are dogs.
All poodles are mammals.
It follows that all dogs are mammals.

This argument is invalid because if the premises were true then the conclusion would still not be guaranteed to be true. The premises do not supply enough information for us to be sure of the conclusion, even though we happen to know that the conclusion is actually true. For all the premises tell us, there might be some dogs that are not mammals.

So keep the following points in mind:

- * In some cases an argument is valid yet it has false premises and a false conclusion.
- * In some cases an argument is invalid even though it has true premises and a true conclusion.

In a valid argument, the premises are related to the conclusion in such a way that *if* the premises were true, then the conclusion would have to be true as well. Validity is thus a hypothetical relationship between two things. Even if the premises and conclusion are all false, they can still be related *hypothetically* in such a way that *if* the premises were to be true, the conclusion *would have to be* true.

Let's apply this to one more example. Consider the following deductive argument:

All cats are fish.
All fish are purple.
Therefore, it must be that all cats are purple.

Notice that each premise is clearly false; and the conclusion is false as well. But: *If* the premises were to be true (hypothetically), *then* the conclusion would have to be true. The conclusion *would* be guaranteed. If the premises were to be true (hypothetically), then the conclusion would have to be true. The conclusion would be certain. Therefore, the argument is valid, even though it has false premises and a false conclusion.

****Practice Problems: Valid and Invalid Arguments**

For each of the following deductive arguments, determine whether it is valid or invalid.

1. Some dogs are mammals. Some dogs are poodles. Thus, some mammals are poodles.
2. Either the former U.S. president George W. Bush was not a professional baseball player or he was not a famous rock singer. But he was a famous rock singer. Thus, George W. Bush was not a professional baseball player.
3. The word 'wet' has three letters in it. Thus, the word 'wet' has an odd number of letters in it.
4. If Mahatma Gandhi was a woman, then Mahatma Gandhi was a female. But Mahatma Gandhi was not a woman. Therefore, Mahatma Gandhi was not a female.

5. Nine is greater than four, and four is greater than six. Thus, nine is greater than four.
6. René Descartes is now the U.S. President. Thus, René Descartes is now the U.S. President.
7. All cats are tigers. No tigers are fish. Thus, no cats are fish.
8. Snow-covered landscapes are chilly. Hence, snow-covered landscapes are cold.
9. This geometric figure is a square. Therefore, this geometric figure has three sides.
10. If Bertrand Russell wrote a book on advanced logic, then he was a logician. Bertrand Russell did indeed write a book on advanced logic. And either he was not a logician or he was a ballet star. Thus, Bertrand Russell was a ballet star.
11. Five is greater than two. Two is greater than ten. Thus, five is greater than ten.
12. Elephants fly. Elephants are animals. Thus, some animals fly.
13. Elias is a gzworg. Thus, Elias is a gzworg.
14. If Malcolm X was a National Hockey League star, then Malcolm X was a professional athlete. Malcolm X was not a professional athlete. Thus, Malcolm X was not a National Hockey League star.
15. Some dogs are German shepherds. Thus, some dogs are not German shepherds.
16. June is a tyrewkj. Thus, it is false that June is a tyrewkj.
17. Senator Garcia is a bachelor. Thus, Senator Garcia is an unmarried adult male.
18. No dogs are cats. No cats are elephants. Thus, no dogs are elephants.
19. Most people in our country believe that murder is morally wrong. Thus, it is certain that murder in our country is morally wrong.
20. If we were meant to be nude, we'd all be born that way. We *were* indeed all born nude. Thus, we were meant to be nude.
21. Squares have three sides. Thus, apples are fruit.
22. Squares have four sides. It follows that apples are not fruit.
23. Squares have three sides. Hence, two plus two equals four.
24. Squares have four sides. Therefore, two plus two equals six.
25. Some apples are red. We can conclude that two plus two equals four.
26. Squares have ten sides. Thus, squares have ten sides.
27. The majority of the people in our culture believe that white people should get preferential treatment. Thus white people in our culture should get preferential treatment.
28. Our moral beliefs are largely produced by the upbringing we had in our culture. Thus there is nothing objectively true or false about our moral beliefs.
29. People disagree about what's morally or right and wrong. Thus there is no objective basis from which to determine whether and action is morally right or wrong.
30. No one has yet proved that there is an objective basis for determining whether an action is morally right or wrong. Thus, there is no objective basis for determining whether an action is morally right or wrong.
31. All the arguments for the moral theory known as Cultural Relativism are logically bad. Thus, we can be certain that the moral theory known as Cultural Relativism is false.
32. The morality of a culture is determined solely by its social norms. The norms of the USA are at present at least partially racist. Thus it is morally obligatory for the people of the USA to be at least partially racist.
33. The moral obligation of a culture is determined solely by its social norms. Anyone who fights against social norms is going against what's morally obligatory. Martin Luther King, Jr. fought against social norms in Birmingham, Alabama in the early 1960s. Thus we should say today that

Martin Luther King, Jr. was morally wrong then to do so.

34. If social norms determine what is morally obligatory for a culture (and all cultures obviously follow their norms), then what is morally obligatory for a culture can be discovered merely by determining its norms. The norms of our culture at present are anti-gay. Thus we should be anti-gay.

35. Our cultural norms determine what is morally obligatory for us. But every culture at any time is (obviously) acting according to its norms. Thus at any given time, a culture is morally perfect in every respect and cannot improve morally. Thus the USA is at present morally perfect regarding race relations, and cannot improve morally in that matter.

Answers:

- | | | |
|------------|-------------|-------------|
| 1. Invalid | 14. Valid | 27. Invalid |
| 2. Valid | 15. Invalid | 28. Invalid |
| 3. Valid | 16. Invalid | 29. Invalid |
| 4. Invalid | 17. Valid | 30. Invalid |
| 5. Valid | 18. Invalid | 31. Invalid |
| 6. Valid | 19. Invalid | 32. Valid |
| 7. Valid | 20. Invalid | 33. Valid |
| 8. Valid | 21. Valid | 34. Valid |
| 9. Invalid | 22. Invalid | 35. Valid |
| 10. Valid | 23. Valid | |
| 11. Valid | 24. Invalid | |
| 12. Valid | 25. Valid | |
| 13. Valid | 26. Valid | |

Chapter 4: Evaluating Inductive Arguments

Strong vs. Weak

As we have seen, an inductive argument is any argument claiming either explicitly or implicitly that if the premises all are true then the conclusion is probably true though not certain. In logic we evaluate inductive arguments as either “strong” or “weak.” An inductive argument is strong when it is indeed the case that if the premises are true then the conclusion is probably true though not certain, and an inductive argument is weak when it is not the case that if the premises are true then the conclusion is true.

To determine whether an inductive argument is strong or weak, ask yourself a hypothetical question: *If* the premises were true (and this need only be asked hypothetically as a thought experiment), then would they provide enough information to make it likely that the conclusion is true? If “Yes,” then the argument is strong. If “No,” then the argument is weak.

The distinction between strong and weak inductive arguments will become clearer after you have examined some examples. The following are all strong inductive arguments.

- * Without any exceptions, Ed has eaten a Dick’s burger for lunch every day for the past two years. Today is an ordinary day. Therefore, Ed will probably eat a Dick’s burger for lunch today, although it is not certain.
- * In all of recorded history it has never snowed in San Diego in the month of August. So it probably won’t snow next August in San Diego.
- * No human being has ever run a one-minute mile. Ed has never done anything athletic. Thus, it is unlikely Ed will run a one-minute mile today when he goes to the track for the first time.
- * It is 100 degrees outside and the temperature is rising. Ice cream melts at 34 degrees. Therefore, if I leave my ice cream cone directly out in the hot Sun right now, it will probably melt in less than an hour.

The following are all weak inductive arguments.

- * It has been raining for two days in a row. So it will probably be raining every day next month.
- * Jan is from Minnesota. Bob Dylan grew up in Minnesota. Therefore, Jan probably likes Bob Dylan’s music.
- * Phat is an artist. Therefore, Phat very likely has an MFA degree from San Francisco State.
- * We interviewed 10 people in front of a Catholic church in Renton, Washington after Mass last Sunday and 9 of them said they were Catholic. It follows that probably 90 percent of all Americans are Catholic.

Notice that each of the eight arguments above is inductive because the claim in each case is that the conclusion is probably true if the premises are true. However, some are strong arguments and some are weak because some succeed in showing that their conclusions probably are true if their premises are true, and some do not.

Ask yourself: *If* the premises were true, would they provide enough information to make it likely (i.e., better than 50 percent chance) that the conclusion is true? Yes: The argument is strong. No: The argument is weak.

****Practice Problems: Strong and Weak Arguments**

For each of the following inductive arguments, state whether it is strong or weak.

1. Serious biologists will tell you that mice are mammals. Thus mice are mammals.
2. It has rained every day in the Darién Gap for the past twenty-five years. Thus it will probably rain in the Darién Gap tomorrow.
3. People try on shoes before buying them. People drive cars before signing up for a three-year lease. People take a close look at travel information before committing to an expensive vacation. Thus people should have sex with each other before committing to marriage.
4. Different cultures have different beliefs about morality. Thus there is no objective basis outside of cultural norms for any moral claim.
5. Wei-jin's math teacher says that God exists. Thus God probably exists.
6. Two teenagers were found writing graffiti on the school walls yesterday. Thus all teenagers are delinquents.
7. A reliable study showed that 90 percent of Bellevue College's students want better food in the school cafeteria. Latisha is a student at Bellevue College. It follows that Latisha probably wants better food at the cafeteria.
8. Hakim has eaten at Joe's Café every day for two weeks, and has liked the food each time. Hakim plans to go to Joe's Café tonight for dinner, and on the basis of his past experiences concludes that he will likely enjoy this meal, too.
9. Paul has eaten at Joe's Café once before for breakfast, and liked the food. On that basis, Paul concludes that he will love the food at Joe's Café tonight when he goes there for dinner.
10. Upon landing at the SeaTac Airport, plane passengers saw broken buildings, large cracks in the runway, fire engines running about, and paramedics assisting injured people. The passengers concluded that an earthquake just occurred.
11. A box contains 1000 U.S. coins. Two selected at random were one cent pennies. Thus the entire box probably contains nothing but pennies.
12. An official state parks sign at a beach says, "Attention: Beyond this point you may encounter nude sunbathers." Therefore the beach in front of you is probably sanctioned for clothing-optional use.
13. An elderly lady drove 50 miles out of her way to visit the officially sanctioned clothing-optional beach at the state park, and complained to the park ranger there that she was offended by the nudity she saw through her binoculars. Thus the ranger should arrest every nude sunbather at the beach for disorderly conduct.
14. A spokeswoman for the nude sunbathers at the officially sanctioned clothing-optional beach plans to explain politely to the elderly woman complainant that no one at the beach had broken any law. Therefore it is likely that this particular elderly woman will subsequently and happily join the nudists for a game of Frisbee on the beach.
15. Ranger Dan has listened to the elderly woman's strident complaint about beach nudity. Ranger Dan has also listened to over a dozen nudists shout their points of view regarding the elderly woman's complaint. Ranger Dan works under an incompetent site administrator who

demands that Dan resolve all beach user-conflict quickly and in such a way that avoids negative media attention. Thus Ranger Dan is probably feeling frustrated.

Answers:

- | | | |
|-----------|------------|------------|
| 1. Strong | 6. Weak | 11. Weak |
| 2. Strong | 7. Strong | 12. Strong |
| 3. Weak | 8. Strong | 13. Weak |
| 4. Weak | 9. Weak | 14. Weak |
| 5. Weak | 10. Strong | 15. Strong |

Looking for Missing Premises

In far too many cases, when people suspect they are presenting a weak or invalid argument, they leave a key premise out. Often, these key premises—if stated explicitly—would sound ridiculous, and make the argument sound equally foolish. Many times the so-called arguer offers a rhetorical question instead of a declarative premise, forgoing the opportunity to present boldly and honestly his or her claims clearly. For example, “There is no objective standard by which moral claims may be judged as true or false. For who’s to say what’s right or wrong?” The conclusion here is stated at the beginning clearly, but absolutely no reason is provided for anyone to believe it. The rhetorical question offered afterwards is usually expected to elicit a shrug and a tacit response of “Gee, I don’t know; there must *not* be any point of view with the authority to judge moral claims objectively).” But surely the mere fact that the person responding to the argument can’t think of a good answer doesn’t show that there *is* no good answer. Also, the question reveals a highly questionable assumption on the part of the arguer that the truth value of ethical judgments is determined by some *perspective*; but the truth or falsity of the claim that “Two plus two equals four” is not determined by a perspective (whether holding power or not, or socially constructed or not). It must be shown whether moral truths are analogous to such claims or relevantly different. To assume what one is trying to prove begs the question.

When an inductive argument seems patently weak, or a deductive argument seems patently invalid (and none of the clearly stated premises are false), the problem may be that a substantive premise is missing. We need to see if the explicitly stated premises by themselves provide good reason to believe the conclusion, and if they do not due to a missing premise, we should determine what the missing premise must be. We can then take a close look at that premise, and determine whether we have good reason to believe it. Those missing premises are often the Achilles’ heel of bad arguments.

Consider the following four arguments:

- * All dogs are mammals. Thus, all dogs are animals.
- * Either the Senator is a Republican or she’s a Democrat. Thus she’s a Democrat.
- * The norms of our culture include activity X. Thus, people of our culture are morally obliged to do activity X.
- * No one has come up with a universally accepted object basis for morality. Thus there is no objective basis for morality.

In each case, there is a non-trivial, substantive premise missing. The stated premise alone cannot give good reason to believe the conclusion. The arguer may believe that the missing premise is so obviously true as to be not worth stating explicitly, but the arguer would be thus intellectually confused. Whether the missing premise is true or not is often at the heart of the argument; and for at least the second and third examples above, the missing premise claims something that may very well be false.

In the first argument, the missing premise is easily found: “All mammals are animals.” This claim is both undeniably true and something nearly any informed adult will agree to, so the arguer is probably not trying to slide something by us. The same is likely the case with the second argument. The missing premise is “The Senator is not a Republican.” This kind of claim is easily verified or falsified, so again, the arguer is probably just speaking loosely, and gives us little need for serious critical challenge.

The missing premise of the third and fourth arguments, though, make important, substantive claims, and many thoughtful and informed people will disagree with them. For the third argument, the missing premise needs to be something like “All activities that are the cultural norm for our culture are morally obligatory for people in our culture.” But that’s a severely difficult claim to take seriously. Do we really want to say that because the present norm of our culture is (at least partially) racist and sexist that we thereby have a moral duty to be racist and sexist (to the present extent)? Of course not.

The missing premise of the fourth argument needs to be something akin to “If an objective basis for morality exists, then someone would have presented it to so that it would be accepted universally by now.” But this claim too is highly questionable. There is no good reason to believe that just because everyone has not accepted a particular point of view that there is no objectively correct point of view on the subject. Scientists do not yet agree on what causes gravitational pull, yet we don’t walk about saying that there’s nothing objectively true about the cause of gravitational attraction.

Once the woefully misguided missing premises of the third and fourth arguments are brought into the open, the weakness of the arguments becomes apparent. But people arguing for such conclusions often do not want the weakness of their inferences glowing like neon signs in the night air. It’s our task as critical thinkers to recognize when important premises are missing, and to take them into consideration in determining if an argument is successful.

****Practice Problems: Missing Premises**

What substantive premise is missing in each of the following arguments? Note: the missing premise may be stated in more than one way.

1. No cats are birds. Thus, Garfield is not a bird.
2. Sue says that Mars is larger than Venus. Thus, Mars is larger than Venus.
3. If Bob is a mouse, then Bob is a mammal. Thus Bob is not a mouse.
4. If Camila is a logician, then Camila is a philosopher. Hence, Camila is a philosopher.

5. The sign outside our school is authoritative and informative. Thus we can believe that this school is Bellevue College.
6. Zahra goes to class every day and studies regularly, and she gets pretty good grades. Thus, Alina probably gets pretty good grades, too.
7. A strong wind storm is coming to our town tomorrow morning. Thus the rowing regatta scheduled for tomorrow on our town's lake will probably be canceled.
8. Alejandro is not from Argentina. Thus he's from Peru.
9. If Giulia is from Rabat, then she is from Morocco. Thus, if Giulia is from Rabat, then she is from North Africa.
10. I did not receive what I asked for in my prayer to God. Thus God does not exist.
11. Pastor Bustle is a social conservative. Thus he is a Republican.
12. Senator Sunny Shine is a nudist. Thus she is liberal.
13. All social conservatives want to see nudists put in jail. Thus, Bustle wants to see nudists put in jail.
14. Sunny Shine is willing to break a law to promote publically her family's lifestyle. Thus, Shine is an anarchist and has no respect for social order.
15. Knowledge of Sunny Shine's backyard skinny-dipping will bother some people. Thus Sunny Shine should be put in jail.

Answers:

1. Garfield is a cat.
2. Sue is authoritative on Mars and Venus.
3. Bob is not a mammal.
4. Camila is a logician.
5. The sign outside our school says that this is Bellevue College.
6. Alina goes to class every day and studies regularly.
7. If a strong wind storm comes to our town tomorrow, then the scheduled rowing regatta will be canceled.
8. Alejandro is either from Argentina or Peru.
9. If Giulia is from Morocco, then she is from North Africa.
10. If I do not receive what I ask for in my prayer to God, then God does not exist.
11. All social conservatives are Republicans.
12. All nudists are liberals.
13. Bustle is a social conservative.
14. Anyone willing to break a law to promote publically his or her family's lifestyle is an anarchist and has no respect for social order.
15. If an activity bothers some people, those engaged in that activity should be put in jail.

Chapter 5: Deductive Soundness and Inductive Cogency

We now need to bring truth and the real world back into our discussion. Recall that a statement makes a claim that is either true or false (never both, and never neither). A statement is true if and only if its claim corresponds to, that is, describes the relevant portion of reality; it is “false” if the claim does not correspond to reality. That is a definition of truth. Let’s look at some examples. The following statements correspond to reality:

The Earth is (roughly) round.

The Moon has mountains.

The Sun is presently over one hundred miles away from the Earth.

The following statements obviously do not correspond to reality.

Richard Nixon was President of the United States on January 1, 2012.

The Earth as a whole is flat as a pancake.

The Sun is only 100 miles away from Mars.

The definition just presented, called the “correspondence theory of truth,” was first formulated in the West by the ancient Greek philosopher Aristotle (384-322 B.C.). Ancient and medieval philosophers from India and China also agreed with the correspondence theory with near unanimity, as do most contemporary philosophers around the world today. Moreover, it’s what most folks walking the streets today will believe, even if they do not presently know how to articulate the position. (Debate on this issue is important to the philosophic field of epistemology, however.) You may have noticed that the correspondence theory presupposes a distinction between the *claim* expressed by a statement, and an extra-linguistic reality existing beyond the statement, a reality that is “there” independently of language (and a statement is *true* if and only if its claim corresponds to that reality). Someone can *say*, for example, “Adult elephants are larger than adult mice,” but saying or believing so does not make it *true*; the claim expressed by this statement is true if and only if it correctly describes or corresponds to the relevant part of reality, an extra-linguistic part of reality, a reality beyond the sentence, namely, all *adult elephants* and *adult mice*.

Correspondence theory enters logical theory via the concept of soundness. A deductive argument is sound if and only if it is both *valid* and has *all true* premises. A deductive argument that is not sound is unsound. In other words, in order to qualify as sound, a deductive argument must satisfy two conditions:

1. It is valid.
2. All of its premises are true.

Or, “Sound = Valid + True premises.”

An inductive argument is cogent if and only if it is both *strong* and has *all true* premises. An inductive argument that is not cogent is uncogent. In other words, in order to qualify as cogent, an inductive argument must satisfy two conditions:

1. It is strong.
2. All of its premises are true.

Or, “Cogent = Strong + True premises.”

The following deductive argument is valid but not sound:

All squirrels are fish.
All fish are good swimmers.
So, all squirrels must be good swimmers.

The following deductive argument is sound:

All whales are mammals.
No mammals are reptiles.
So, no whales are reptiles.

The following inductive argument is strong but uncogent:

The air temperature has been over 300 degrees F in New York City for ten straight weeks with no change in sight.
Therefore, the air temperature will probably be over 300 degrees F in New York City tomorrow.

The following inductive argument is cogent:

The temperature at the surface of the Sun has been over 100 degrees F for ten straight weeks with no change in sight.
Therefore, it will probably be over 100 degrees F on the surface of the Sun tomorrow.

The soundness or cogency of an argument does not depend on my (or your) knowing if the premises of the argument are true. Soundness and cogency depends (in part) on whether the premises are actually true or not. And we may not know if premises are true or false in many cases, or we may disagree on such matters. This does not make the argument’s soundness or cogency “relative to our point of view”; it simply means that in some cases we will not be able to know or agree upon the ultimate assessment of the argument. Consider the following deductive argument:

There is a mountain on Pluto 3000 meters high. Thus, there is a mountain on Pluto over 2000 meters high.

As logicians, we can be confident that this argument is valid, for a 3000-meter-high mountain is definitely higher than a 2000-meter-high mountain. *If* the premise is true, then the conclusion is guaranteed. But is the argument sound? We do not know if the premise is true or not. We are presently ignorant of this matter given the limitations of today's technology in astronomy. This does not make the argument sound or unsound; it simply means that although we can tell (being students of logic) that the argument is valid, we cannot tell if it is sound or unsound. No problem. We're simply ignorant and don't know everything there is to know.

Now consider this additional argument:

Each person has a guardian angel.
The U.S. president is a person.
Thus, the U.S. president has a guardian angel.

Valid or invalid? Well, *if* the premises are true, then it's impossible for the conclusion to be false; so the argument is clearly valid. Basic knowledge of logic can tell us this much. But is the argument sound or unsound? Hm. Some people believe in guardian angels, and will think the premises are both true. Others do not believe in guardian angels, and will think the first premise false. If the angel believers are correct, then the argument is actually sound. *If* those not believing in guardian angels are correct, then the argument is actually unsound. But both groups cannot be correct at the same time. Guardian angels (whatever we mean by such heavenly creatures) either exist or not, and our belief about them won't make them pop into or out of existence. Belief does not imply reality here. For me to say, "What's true for me is true for me, and what's true for you is true for you," is just an intellectually lazy way of saying I believe one thing, and you believe something else (perhaps the opposite of what I believe). That's fine; people can have differing beliefs; but let's not confuse what someone believes with reality. It would be great if our beliefs matched up with reality on a regular basis, but as the intellectually alert among us will have recognized, we each have had false beliefs before (and likely even today).

So, what shall we make of this guardian angel argument? It's clearly valid, but it looks like the two groups talking about it will disagree on whether the argument is sound or unsound. They can't both be correct in this judgment, but they are presently unable to convince the other side of their position. The best they may be able to do here is to agree to disagree. The sharpest in each group, however, will refrain from thinking that the soundness of the argument is relative to the group's belief about angels. What's relative is their *belief* about the soundness of the argument, and that's a substantially different matter.

Given all this said about mountains on Pluto and guardian angels flitting about our shoulders we are, for the purposes of this course, going to focus our discussions primarily on things that nearly all of us will agree to exist (e.g., cats and U.S. presidents) or not to exist (e.g., vampires and square circles).

****Practice Problems: Deductive Soundness and Inductive Cogency**

For each of the following arguments, determine three things: (a) whether it is deductive or inductive, (b) whether it is valid or invalid (if deductive), or strong or weak (if inductive), and (c)

whether it is sound or unsound (if deductive), or cogent or uncogent (if inductive).

1. All rats are mammals, and no mammals are fish. Thus it is necessary that no rats are fish.
2. Paris is in France, and France is in Africa. Hence it must be the case that Paris is in Africa.
3. No human has ever swum across the Atlantic Ocean. The president of the USA is a human. Thus the president of the USA will likely not swim across the Atlantic Ocean.
4. Mexico City's human population is today well over 1000. Thus it is guaranteed that the human population today of Mexico City is over 500.
5. India is north of the Antarctic. It follows that the Antarctic is south of India.
6. Beijing—the capitol of China—is a large, famous, and interesting city. Thus Beijing probably receives at most a dozen tourists a year.
7. Highly respected physicists say that it is important to learn math in order to excel at advanced physics. Thus it is important to learn math to excel at advanced physics.
8. Different cultures have different beliefs about morality. Thus it is certain that there is nothing absolute or objective about morality.
9. Our moral beliefs are produced through environmental conditioning. Thus is highly likely that there is nothing absolute or objective about morality.
10. Thinkers have yet to agree on an absolute or objective basis for morality. Thus it is certain that there is no absolute or objective basis for morality.
11. It has never snowed in the mountains of Tibet. Thus it will not likely snow there this year.
12. The USA has never elected a woman as president of the country. Thus in the next election, the USA will likely elect a woman as president of the country.
13. In 1950, basketball star Michael Jordan was president of Argentina. All basketball players are athletes. Thus in 1950, Argentina had an athlete as president.
14. The capitol of Costa Rica is San Jose. The capitol of Panama is Panama City. Most of Costa Rica is north of Panama. Thus it is certain that San Jose is north of Panama City.
15. Ethiopia is north of Kenya, and Kenya is north of Botswana. Therefore it is guaranteed that Ethiopia is (at least in part) north of Botswana.

Answers:

1. Deductive, valid, sound
2. Deductive, valid, unsound
3. Inductive, strong, cogent
4. Deductive, valid, sound
5. Deductive, valid, sound
6. Inductive, weak, uncogent
7. Inductive, strong, cogent
8. Deductive, invalid, unsound
9. Inductive, weak, uncogent
10. Deductive, invalid, unsound
11. Inductive, strong, uncogent
12. Inductive, weak, uncogent
13. Deductive, valid, unsound
14. Deductive, invalid, unsound
15. Deductive, valid, sound

Chapter 6: The Counterexample Method

Suppose some people present a deductive argument and believe it's valid although their argument is actually *invalid*. Suppose they insist their argument is valid even when everybody is telling them their argument is logical junk. How can you show these blighted fools their error? One way is called the Counterexample Method. Recall that a deductive argument claims that if the premises are true then the conclusion must be true. A counterexample to a deductive argument is a description of a possible circumstance in which the premises of the argument are clearly true while the argument's conclusion is clearly false. When you present a counterexample to someone's deductive argument, you help him or her see one way in which the premises of the argument could be true while the conclusion is false. In other words, you show that the argument is invalid.

For instance, suppose someone proposes the following deductive argument and stubbornly insists it's valid:

Aya is Jane's biological mother.

Aya is married to Tom.

Therefore, it's guaranteed that Tom is Jane's biological father.

This is a deductive argument, since it claims that the conclusion must be true if the premises are true. But the following counterexample shows that this is an *invalid* deductive argument:

It is *possible* that Aya gave birth to Jane before ever meeting Tom. Perhaps Aya married Tom after Jane had already grown up, gone off to college, majored in dance, and got a job at Kinko's.

This shows that the argument's structure allows for the possibility of the premises being true with the conclusion being false. This is a counterexample to the argument. Note: The mere *possibility* that the premises are true and the conclusion is false shows that this deductive argument is invalid, for a deductive argument is invalid if there is any possibility at all, no matter how unlikely, that the premises could be true and the conclusion false.

Here is another invalid argument, although people commonly mistake it for a valid inference.

Some dogs are brown animals.

Thus, some dogs are not brown animals.

The structure of this argument is:

Some D are A

Some D are not A

If we replace the D with "bears," and replace the A with "mammals," we'd get the following counterexample containing a premise that is clearly true and a conclusion that is clearly false.

Some bears are mammals.
Some bears are not mammals.

Since the structure of the original argument allows for a true premise and a false conclusion, it is by definition invalid, as will be *any* argument of this structure.

Your friend's following invalid argument requires a little more effort to dislodge from his intellectually blinkered mind:

If it's snowing outside, then the picnic will be canceled. And our picnic is indeed canceled. Thus it must be snowing outside.

What we need to do here is to replace entire *statements* with other *statements* so that the premises are clearly true and the conclusion is clearly false. You could respond to your friend as follows:

"Look good buddy, your argument is illogical. It's just like arguing this way: 'If I'm a one-month-old baby boy, then I'm a human. But I'm not a one-month-old baby boy. Thus I'm not a human.' That's just nuts! The two premises are obviously true, while the conclusion is obviously false. Any argument with that pattern will be invalid."

With all this in mind, let's outline a simple procedure for showing invalid deductive argument to be indeed invalid. We'll focus first on deductive arguments made up of *categorical statements*. Such statements begin with "All," "No," or "Some," and contain two *terms* each. A term is a plural word or phrase that picks out a class of things. Oftentimes terms are made up of plural nouns or phrases like "dogs," "black dogs," or "black dogs that bark in the night." Words that describe things or actions—like adjectives (e.g., "black," "fast," "unscrupulous") or adverbs (e.g., "swiftly," "bravely")—do not point to or pick out a class of things, and thus are not terms.

Categorical statements come (technically speaking) in the following four patterns (with S and P used to abbreviate terms):

All S are P
No S are P
Some S are P
Some S are not P

The words "All," "No," and "Some" are called the statement's *quantifier*. The connectors "are" and "are not" are called the statement's *copula*. The term referred to as "S" is called the statement's *subject term*, while the term at the end of the statement and referred to here as "P" is called the *predicate term*. Vocabulary is a wonderful thing.

Note also that the word "some" in logic means "at least one." So it is true that some dogs are animals, because at least one dog is an animal. The fact that all dogs are animals does not contradict the claim that at least one of them is.

We'll call arguments made up of categorical statements and that have only one premise *immediate inferences*. Also, we'll call arguments made up of categorical statements and that have two premises *categorical syllogisms*. The counterexample method provides an easy, intuitive, and polemically useful procedure to show that invalid deductive arguments of these two types are indeed invalid.

Beforehand, for clarity's sake and by convention, we'll rewrite arguments containing two terms that (in the argument's context) clearly refer to the same thing, reducing the two terms to one. Consider the following argument:

We can conclude that some striped tigers are not angry lions, because some cats are striped tigers, and some felines are angry lions.

We need to notice that the two terms *cats* and *felines* surely refer to the same thing, so let's just pick one of the terms (it doesn't matter which) and use it consistently in both instances. We then get the following wording.

We can conclude that some striped tigers are not angry lions, because some *cats* are striped tigers, and some *cats* are angry lions.

Step 1: We can dispense with the premise and conclusion indicator words, and label each premise with *P*: and the conclusion with *C*:. We then replace each term consistently with a single upper-case letter to show the structure of the argument. If you have two terms that begin with the same letter, then select two different capital letters for the two different terms. Step 2 is an easy task, but it shows your understanding of the structure of the argument.

C: Some S are not A
P: Some C are S
P: Some C are A

Step 2: Now comes the fun, creative part. Replace each upper-case letter with a simple, easy-to-understand term of your own choosing so that the premises are *obviously* true and the conclusion is *obviously* false. Be sure not to use the same capital letter to stand for two different terms (e.g., don't use "A" to stand for both "animals" and "aardvarks"). Make it so obvious that your confused and uninformed friend who offered the invalid argument in the first place will agree readily that your premises are true and conclusion is false. Once you've done so, you'll have shown that the argument's structure allows for true premises and a false conclusion; by definition, that means the argument is invalid. And since your invalid counterexample argument has the same structure as your friend's original argument, the original argument *must* be invalid, too. There will be an infinite number of options open to you in selecting terms. Three ways of making the above argument's premises true and the conclusion false include:

C: Some dogs are not animals (obviously false!)
P: Some poodles are dogs (obviously true!)

P: Some poodles are animals (obviously true!)

C: Some trees are not plants

P: Some pines are trees

P: Some pines are plants

C: Some boys are not humans

P: Some 10-year-old children are boys

P: Some 10-year-old children are humans

Notice how the terms are simple and easy to understand. There is nothing controversial here that might motivate your intellectually-challenged friend to get sidetracked into a weird discussion about the meaning of words or the ontology of Nothingness. Avoid using terms that *might* confuse or allow for ambiguous interpretations. Keep it simple!

Other tips:

* When possible, use simple, single-word plural nouns (e.g., rocks, dogs, cats, tigers) that have one clear meaning.

* It's often easiest to begin with the conclusion, fill in the two "blanks" there first making the conclusion clearly false, plug those two terms into the equivalent blanks of the premises, and then think of a remaining term that will fit in the remaining two blanks making the two premises clearly true.

Here's another example, this time examining an invalid immediate inference:

All meat-eaters are loyal Americans. Thus all loyal Americans are carnivores.

We note that "meat-eaters" and "carnivores" are easily understood as referring to the same thing, so we'll select one term to use ("meat-eaters").

Step 1: We replace the two terms with single upper-case letters of our choosing, although the first or key letter from each term is a good choice.

P: All M are L

C: All L are M

Step 2: We replace the single capital letters in Step 1 with simple terms of our own choosing to make for a clearly true premise and a clearly false conclusion.

P: All dogs are animals

C: All animals are dogs

Here, "M" is replaced with "dogs," and "L" is replaced with "animals." The premise is obviously true, and the conclusion is obviously false. This counterexample is thus clearly invalid, so the

original argument is also invalid (and thus unsound, and thus worthy of being rejected as a bad inference).

The Counterexample Method may be used on any deductive argument, although sometimes it gets a little trickier. Consider the following argument that looks at first like a categorical syllogism.

Some dogs are poodles. Some dogs are white. Thus some dogs are white poodles.

Here, the words “dogs” and “poodles” are terms, but the word “white” is an adjective. Moreover, the argument does not quite have the structure of a categorical syllogism. The use of an adjective and the structure of the conclusion are making this deductive argument something a little different. Abbreviated, the argument looks like this:

P: Some D are P
P: Some D are W
C: Some D are WP

To use the Counterexample Method in this case, we need to replace D and P with simple terms, and replace the W with an adjective (again making the premises obviously true and the conclusion obviously false). One way to do this is:

P: Some cats are females
P: Some cats are male
C: Some cats are male females

or

P: Some men are nice people
P: Some men are mean
C: Some men are mean nice people

Consider the following more complex invalid deductive argument:

All baseball players who are right fielders are athletes who are sport fans. Hence, all baseball players are sport fans.

This one-premise argument is an immediate inference, but the premise is more complex than a standard-form categorical statement. Still, we can use the Counterexample Method to show that it is invalid.

P: All B who are R are A who are S
C: All B are S

Given the use of “who” in the argument, we should limit our choice of terms to those referring to people. One combination of a true premise and a false conclusion is:

P: All women who are mothers are females who are people who have given birth.
C: All women are people who have given birth.

Sometimes even a potentially easy problem can be meddlesome. Consider the following example of an invalid argument to the left and its abbreviation to the right:

P: No poodles are mice.	P: No P are M
P: All poodles are dogs.	P: All P are D
C: No mice are dogs.	C: No M are D

It might be easiest to begin by filling in the two conclusion blanks to make the conclusion false; here we can replace “M” with “animals,” and replace “D” with “dogs.” We’d then want to be consistent and replace the other “M” with “animals,” and replace the other “D” with “dogs.” We’d then have the following:

P: No P are animals
P: All P are dogs
C: No animals are dogs

The problem we’d face here is that whatever we use to replace the “P” in the first premise to make the premise true, will not provide a true claim for the second premise. If we replace the first “P” with “rocks,” the first premise would be true, but when we then replace the “P” in the second premise with “rocks” to be consistent, that second premise ends up being false. We’re gummed up because whatever is true of no animals (e.g., being a rock, being a ballpoint pen) would be false of dogs.

Does this mean the argument is valid? Not necessarily. What we need to do is go back to the beginning of Step 2 and plug in different terms for the conclusion. Soon enough, you’ll see how to juggle simple terms to get all the blanks to fill in properly. Oftentimes all we need to do is exchange the two terms we initially used in the conclusion in Step 2. If switching our choice of terms still makes for a false conclusion, the rest of the procedure usually works out smoothly. For example:

P: No cats are dogs (True!)
P: All cats are animals (True!)
C: No dogs are animals (False!)

The argument’s structure does indeed allow for true premises and a false conclusion; that is, it is possible given this structure to have true premises and a false conclusion. The argument is therefore invalid.

Sometimes if we hit the kind of “brick wall” above and we see that we can’t possibly fill in the final two blanks with a single term so that both premises are clearly true, what we need to do is go back and try a different pair of terms to create the initial false conclusion. We’ve seen that simply switching the two terms might work. Another option is to go from using two terms of unequal size (note that “animals” contains a much bigger set of things than “dogs”) to two terms of equal size (e.g., “dogs and “cats”); or vice versa. The bottom line is that doing Step 2 of the Counterexample Method on categorical syllogisms should be easy and take no more than 15 or 30 seconds to complete. If it takes much longer, it may be best to simply start off with a different choice of two terms when making the conclusion false.

Some deductive arguments contain no categorical statements. Consider the following invalid argument:

If Snoopy [the *Peanuts* cartoon character] is a dog, then Snoopy is mammal. Snoopy is a mammal. Thus Snoopy is a dog.

Here we are working in part with simple statements, with two of them combined in the first premise. In this kind of case, instead of replacing terms with a single capital letter, we replace entire simple statements with a capital letter. Here we’d get:

P: If D, then M

P: M

C: D

Here “D” stands for “Snoopy is a dog,” and “M” stands for “Snoopy is a mammal. Our next step is to replace these capital letters with simple statements of our choosing so that the premises are obviously true and the conclusion is obviously false. The following should do it:

P: If Lassie [the dog from 1950s television show] is a cat, then Lassie is an animal.

P: Lassie is an animal.

C: Lassie is a cat.

It’s certainly true that *if* Lassie is a cat, then Lassie *would* be an animal; that’s true of each student in this course, as well as the computer you are using right now. Also, the second premise is clearly true, and the conclusion is clearly false. The argument’s structure allows for true premises and a false conclusion, so it’s *invalid*.

One final note. Suppose that you work for 10, 15, or 60 minutes on an argument, and you can’t seem to fill in the blanks so that the conclusion is false and the premises are true. Does that show that the argument is *valid*? No. It *might* be the case that you’re simply not up to speed today, or that the argument is complex in some fashion. *Not* successfully filling in the blanks proves nothing; but if you *do* fill in the blanks so that the premises are true and the conclusion is false, then you’ve proved the argument to be invalid. The Counterexample Method is thus only useful for showing that invalid arguments are indeed invalid; it cannot show that valid arguments are valid. For that, we’ll need other techniques to be learned later.

****Practice Problems: The Counterexample Method**

Use the two-step Counterexample Method to show that the following arguments are invalid.

1. Some tigers are striped animals. Thus, all striped animals are tigers.
2. Some birds that fly through jungles are not green animals, because some green animals are not birds that fly through jungles.
3. All the men in my biology class are funny people. Thus, all the humorous people in my biology class are men.
4. All parrots are birds. All parrots are animals. Thus, all birds are beasts.
5. All pit bulls are mammals, since some dangerous dogs are pit bulls, and because some dangerous dogs are mammals.
6. No people learning logic are completely irrational people. Some logic students are mathematicians. It follows that some completely irrational people are not mathematicians.
7. No eagles are mammals. Thus, since no mammals are fish, no eagles are fish.
8. Some dogs are not pigs, because some pigs are animals and some animals are not dogs.
9. If it's raining outside, then the ground is wet. But it's *not* raining outside. Consequently, the ground is *not* wet.
10. All chess players with a high USCF rating are either chess masters or chess fans. Thus all chess players are chess fans.
11. If Fischer was superior to Spassky, then Fischer could have beaten Karpov. If Kasparov was the world's best chess player, then Fischer could have beaten Karpov. Thus, if Fischer was superior to Spassky, then Fischer could have beaten Karpov.
12. All black dogs are mammals. Thus all dogs are mammals.

Answers: (with proposed changes when needed, followed by the two steps of the method, with an illustration of one of many possible counterexamples)

1. P: Some tigers are striped animals. P: Some T are S
C: All striped animals are tigers. C: All S are T

Counterexample: P: Some dogs are animals.
C: All animals are dogs.

2. P: Some green animals are not birds that fly through jungles. P: Some G are not B
C: Some birds that fly through jungles are not green animals. C: Some B are not G

Counterexample: P: Some animals are not dogs.
C: Some dogs are not animals.

3. P: All the men in my biology class are funny people. P: All M are F
C: All the funny people in my biology class are men. C: All F are M

Counterexample: P: All cats are animals.
C: All animals are cats.

4. P: All parrots are birds. P: All P are B
 P: All parrots are animals. P: All P are A
 C: All birds are animals. C: All B are A

Counterexample: P: All poodles are animals.
 P: All poodles are dogs.
 C: All animals are dogs.

5. C: All pit bulls are mammals. C: All P are M
 P: Some dangerous dogs are pit bulls. P: Some D are P
 P: Some dangerous dogs are mammals. P: Some D are M

Counterexample: C: All lions are tigers.
 P: Some cats are lions.
 P: Some cats are tigers.

6. P: No logic students are completely irrational people. P: No L are C
 P: Some logicians are mathematicians. P: Some L are M
 C: Some completely irrational people are not mathematicians. C: Some C are not M

Counterexample: P: No dogs are cats.
 P: Some dogs are animals.
 C: Some cats are not animals.

7. P: No eagles are mammals. P: No E are M
 P: No mammals are fish. P: No M are F
 C: No eagles are fish. C: No E are F

Counterexample: P: No tigers are monkeys.
 P: No monkeys are cats.
 C: No tigers are cats.

8. C: Some dogs are not pigs. C: Some D are not P
 P: Some pigs are animals. P: Some P are A
 P: Some animals are not dogs. P: Some A are not D

Counterexample: C: Some cats are not mammals.
 P: Some mammals are bears.
 P: Some bears are not cats.

9. P: If it's raining outside, then the ground is wet.
 P: It's *not* raining outside.
 C: The ground is *not* wet.

P: If R, then G

P: If Lassie is a cat, then Lassie is a mammal.

Chapter 7: Fallacies

A *fallacy* is an argument that is logically bad but often psychologically persuasive. Advertisements trying to convince you to buy a new shade of lipstick are often fallacious: they offer no logically good reason to believe that you need or should buy that lipstick, yet such ads (if effective) can convince you that you simply must buy it nonetheless or face a life of social penury fit only for a troll.

Fallacies fall into two camps: formal and informal. A *formal fallacy* is a psychologically persuasive but logically bad argument whose problem reveals itself to the keen eye of logic students through its form, or structure. The logical problem with an *informal fallacy* lies in its content, that is, in what the premise is claiming, and not merely in the argument's structure. We will in this section of the text speak most thoroughly about informal fallacies. A few words on formal fallacies are warranted, though.

Formal Fallacies

There are a number of formal fallacies, but two are quite common and often trip up the sleepwalking thinker. Consider beforehand the logically good (i.e., valid) argument known by logicians as *Modus Ponens* (Latin: "the affirming mode"):

If A, then B
A
Thus, B

An example of a Modus Ponens argument about the famous American dog and star of 1950s television, Lassie, would be:

If Lassie is a dog, then Lassie is an animal.
Lassie is a dog.
Thus, Lassie is an animal.

This argument is valid because it is impossible for the premises to be true and (at the same time and with the same meaning) for the conclusion to be false. We can see this even if we had never heard of Lassie or knew that she was a dog. The premises here are guaranteeing that the conclusion is true, and only a valid argument can do that. Consider, however, the following invalid argument:

If Lassie is a dog, then Lassie is an animal.
Lassie is an animal.
Thus, Lassie is a dog.

This argument is invalid because it is possible for the premises to be true and the conclusion false. The premises (and conclusion) happen to be true, but the information in the two premises does not by itself guarantee the conclusion. It is possible that Lassie is an animal (the claim of

the second premise), and all dogs may very well be animals (implied by the first premise), but for all we know from the premises, Lassie may be a bird or a cat (with the premises holding true), so the conclusion fails to be guaranteed.

This pattern of invalid argument is so common that it's been given a name: *Affirming the Consequent*. The name comes from the vocabulary of "if, then" statements: the "if" part is called the *antecedent*; the "then" part is called the *consequent*.

In Affirming the Consequent, one premise is a conditional statement of the form "If..., then..." A second premise (provided beforehand or afterwards) affirms the consequent of the conditional. The arguer improperly concludes with the affirmation of the antecedent. All arguments of this pattern are invalid, and are said to be formally fallacious. We don't even need to know what the argument is about to see its fallaciousness. For example, all the following are guilty of Affirming the Consequent even though we don't know if the premises are actually true or false:

If H, then O	L	If K, then not-U	Not-R
O	If M, then L	Not-U	If not-G, then not-R
Hence, H	Therefore, M	Thus, K	Thus, not-G

A second common formal fallacy is called *Denying the Antecedent*. Its structure looks like this (with the two premises and conclusion arranged in any order):

If A, then B
 Not-A
 Thus, not-B

An example of this invalid line of reasoning is this:

If it's raining outside, then the ground is wet.
 It's not raining outside.
 Thus, the ground is not wet.

Surely the two premises do not guarantee the conclusion, as it's quite possible for it to have been raining heavily five minutes ago (or sprinklers are on, or a pack of territory-marking dogs had recently walked by) and the ground still be wet. The following are examples of Denying the Antecedent. You might compare them to the Affirming the Consequent examples above. In each case below, there is a conditional statement along with a shorter one that denies what the conditional has as its antecedent. The conclusion is then the denial of the consequent.

If H, then O	Not-M	If K, then not-U	Not-G
Not-H	If M, then L	Not-K	If G, then R
Hence, not-O	Therefore, not-L	Thus, not-not-U	Thus, not-R

What makes Denying the Antecedent tricky for many people is its resemblance to a common valid form of reasoning called *Modus Tollens* (Latin: "the denying mode"):

If A, then B
Not-B
Thus, not-A

For example,

If it's raining outside, then the ground is wet.
It's false that the ground is wet (i.e., it's dry as a bone).
Thus, it is false that it's raining outside.

Here, with Modus Tollens, it is impossible for the premises to be true and at the same time for the conclusion to be false. All arguments in this form will be valid, just like all arguments in the forms of Affirming the Consequent or Denying the Antecedent will be invalid.

Formal fallacies are found among invalid deductive arguments. It's usually not the structure of an inductive argument that makes it weak. With induction, the weakness relates to the content of the argument. Consider the following two inductive appeals to authority:

My four-year-old niece (who has never studied botany or native plants) says that the scientific name for coast redwood trees is *Sequoia sempervirens*. Thus, the scientific name for coast redwood trees is *Sequoia sempervirens*.

Each botany professor in Pacific Rim universities says that the scientific name for coast redwood trees is *Sequoia sempervirens*. Thus, the scientific name for coast redwood trees is *Sequoia sempervirens*.

The two arguments have the same structure:

X says Z.
Thus, Z is true.

Looking at the structure, however, does not let us know if either argument is strong or weak; we need to examine the content. Only then can we determine if the authorities we appeal to are sufficiently authoritative on the subject of redwood trees. Formal fallacies can be recognized as such without knowing any of this content; all we need to see is the argument's form. That's what makes them *formal* fallacies. To recognize an *informal* fallacy, we need to know the content of the argument, and know how it may or may not provide good reason to believe the conclusion. In the next section, we'll examine 20 common informal fallacies.

****Practice Problems: Formal Fallacies**

Determine whether the following arguments are examples of Modus Ponens, Modus Tollens, Affirming the Consequent, Denying the Antecedent, or something else (the name of which you don't need to know here).

1. If Jack likes fruit, Jack likes apples. Jack does indeed like fruit. Thus, Jack likes apples.
2. If Sally likes oranges, then Sally likes fruit. Sally likes fruit. Thus Sally likes oranges.
3. If Jugo is a logician, then Jugo understands fallacies. Jugo is not a logician. Hence, Jugo does not understand fallacies.
4. Either Pat is a baseball player or a football player. Pat is not a baseball player. Therefore, Pat is a football player.
5. If Laura starred in the movie *Logicians are Hot!*, then Laura is an actress. But Laura is not an actress. Thus, Laura did not star in the movie *Logicians are Hot!*
6. If Joipl is ghjkl, then Joipl is qwert. Joipl is ghjkl. Thus, Joipl is qwert.
7. If June is tall, then Tim is 30 years old. But Tim is 30 years old. Thus June is tall.
8. Bob Marley was not a member of the Beatles. If Bob Marley was a member of the Beatles, then Bob Marley knows Ringo Starr. Thus, Bob Marley does not know Ringo Starr.
9. Olga is French. If Olga was born and reared in France, then Olga is French. So, Olga was born and reared in France.
10. If Carrie is from Canada, then Mel is from Mexico. If Mel is from Mexico, then Patty is from Panama. Thus if Carrie is from Canada, then Patty is from Panama.
11. Kate is not a dancer. If Kate is a ballet star, then Kate is a dancer. Thus, Kate is not a ballet star.
12. If *, then \$. \$. Thus, *.
13. Not-*WWW*. If 3456, then *WWW*. Thus, not-3456.
14. If A, then B. If C, then D. A or C. Thus, B or D.
15. If Alice is a sun-lover, then Alice is a nudist. Thus, since Alice is not a sun-lover, Alice is not a nudist.

Answers:

1. Modus Ponens
2. Affirming the Consequent
3. Denying the Antecedent
4. Something else (this valid pattern is called Disjunctive Syllogism)
5. Modus Tollens
6. Modus Ponens
7. Affirming the Consequent
8. Denying the Antecedent
9. Affirming the Consequent
10. Something else (this valid pattern is called Hypothetical Syllogism)
11. Modus Tollens
12. Affirming the Consequent
13. Modus Tollens
14. Something else (this valid pattern is called Constructive Dilemma)
15. Denying the Antecedent

Informal Fallacies

Informal fallacies are inductive arguments that are commonly thought to be strong when they are actually weak. That is, they are weak inductive arguments that fool a lot of people. There are

dozens that have been analyzed and named over the centuries. Multiple websites list and discuss many (do a word search for “informal fallacies” and set aside multiple weeks to read them all). We’ll select a few that show up often, especially in discussions ranging from thoughtful philosophy to really, really inept talk radio from both sides of the dial.

Many informal fallacies have Latin names that are now known internationally. It should be no surprise that ancient Greek philosophers used Greek, and classical Indian philosophers used Sanskrit to name many of these same fallacies. When most English-speakers inside and outside philosophy use the Latin name, we’ll use it here, too. If most English-speakers refer to the fallacy by a common English name, we’ll use the English. This is just an introduction; a full study of informal fallacies can take weeks and gallons of good Northwest microbrew.

Students should be able to recognize an informal fallacy when it’s before them, and be able to explain why it’s a weak argument (i.e., why the premises do not make it likely that the conclusion is true). One potential complexity is that in many cases in the so-called “real world” a weak argument may be guilty of more than one informal fallacy. No problem! The argument is thus bad for more than one reason. For our purposes, in this text we’ll attempt to look at examples that most clearly illustrate one informal fallacy. Class discussions and your instructor can help sort out the arguments that seem to be guilty of more than one logical infraction.

With no particular order in mind, we’ll split 20 informal fallacies into four groups (for ease of study):

Appeal to Pity
Appeal to the People
Ad Hominem
 abusive
 tu quoque
 circumstantial

Accident
Straw Man

Red Herring
Appeal to Ignorance
Weak Authority
Genetic Fallacy
Hasty Generalization

False Cause
 post hoc ergo propter hoc
 non causa pro causa
 oversimplification

Slippery Slope
Weak Analogy
Begging the Question

False Dichotomy

Is-Ought Fallacy

Equivocation

Amphiboly

Composition

Division

Appeal to Pity

Appeal to Pity occurs when someone argues that his or her woeful, pitiable condition justifies acceptance of some conclusion, when that woeful, pitiable condition is irrelevant to the conclusion. All teachers know this to be a favorite fallacy of many students. For example,

Student to philosophy teacher: “I know that the essay was due yesterday and that it was supposed to be on India’s classical logic system [i.e., *Nyāya*], but I think you should accept this late paper I wrote on Ernest Hemingway’s *The Old Man and the Sea*. My mom and dad are fighting, my dog is sick, and because I got a D on my Algebra test I’m really depressed right now.”

All of these semi-tragic events in this student’s life may very well merit sympathy from the teacher and anyone else within earshot, but the circumstances are logically irrelevant to whether the teacher should make special arrangements to accept a late essay on a topic far removed from the assignment.

Other examples:

“Chief architect and project manager Bob Shine should not be held responsible for the engineering flaws causing the recent Floating Bridge collapse. He’s got a lot of problems right now. His wife Sunny rides nude in Seattle’s annual World Naked Bike Ride, his kids are all vegetarian activists, and he lost every game in a recent Bellevue Chess Club tournament. We can’t throw more misery his way!”

“The U.S. should allow my mother to acquire citizenship in this country. She was so poor in Mexico, and her illegal border crossing into the U.S. was very, very dangerous. *And* she now has four children to feed. Surely the U.S. should ignore immigration laws that applied to her.”

Of course, if the appeal to the pitiable situation *is* relevant to the conclusion, then we don’t have a fallacy. For instance,

“I’ve been out of work for eight weeks due to a work-related injury, and I’ve been looking steadily for a new job all the while. My wife and three kids are now hungry, we have no money left, and we have no family at hand willing to help us out. Thus we merit consideration for assistance from your well-funded Food for the Needy foundation.”

Logicians need to be a bit cold-hearted, but not all the time.

Appeal to the People

If I argue that you should believe X and use as my reason your (logically irrelevant) desire to be associated with a portion of society, I am probably guilty of the Appeal to the People fallacy. For instance, I might reason that wearing Nike T-Shirts will make you look like Michael Jordan or Tiger Woods, thus you should buy Nike T-shirts. Or I might argue that *everyone* who is cool at school wears Ray-Ban sunglasses, and you want to look cool, don't you? I can appeal to your desire to be cool, sexy, fit, young, rich, smart, or any other demographic of society, and if I make that appeal to demonstrate that you ought to believe X, then unless that desire is relevantly related to X, it's a fallacious appeal.

A broader instance of Appeal to the People is to argue that since all (or most) people believe, say, or do X, we should thereby agree that *we* should believe, say, or do X. But, just because the majority of Americans at one time thought slavery was morally permissible, does not in itself give any good reason to believe that slavery was—even back then—morally permissible. What all (or the majority) of the people believe, say, or do does not by itself show that we should follow suit. Independent reasoning is needed to get to that conclusion with any degree of logic.

Additional examples of Appeal to the People include:

“Tecate is the best selling beer in Mexico. Thus you should drink Tecate!”

Melodramatic teenage daughter to mother: “But moooom, *all* my friends are like dating biker gangs these days. I should be able to, also, you know!”

“Our present cultural norms dictate that murder is morally wrong. Thus murder is morally wrong for us.”

Ad Hominem

Known almost universally as *Argumentum ad Hominem*, Argument against the Person is one of the most common fallacies heard today. Instead of challenging a person's conclusion or argument, the wielder of an *Ad Hominem* (Latin: “against the man”) chooses to attack irrelevantly the other's character traits, lack of consistency, motives, or situation. Logicians often name three fairly distinct kinds of Ad Hominem arguments: abusive, *tu quoque*, and circumstantial.

Ad Hominem abusive occurs when the fallacious arguer ignores another's position, argument, or conclusion and makes an irrelevant personal attack on the other's character traits. For instance:

“Senator Knight says we should raise tax revenue for higher education. But Knight has been making sexual advances towards Bob Shine's wife, she's been known to kick her dog, and she smokes at least two packs of cigarettes a day. We can thus safely reject Knight's position on taxes and education.”

Timber company advocate before a public hearing on increasing wilderness roadless areas in a forest: “These young people calling for increasing the wilderness area are a bunch of long-haired, maggot-infested, environmental whackos. Thus no one should listen to their arguments on wilderness areas.”

Ad Hominem tu quoque (Latin: “you also”) occurs when the fallacious arguer points out the other person’s inconsistency in holding her position, and concludes that the inconsistency alone warrants everyone’s rejection of her position or argument. The problem here is that being inconsistent does not mean that your argued position is in any way mistaken or shaky. Inconsistent people can be correct, and they can reason well. They may be making some kind of mistake in not having two of their beliefs match up, or in having a lifestyle not match up with what they claim all others should be doing, but that alone says nothing about whether their stated positions are true or false, or argued for well or not.

Examples:

Patient to doctor: “You tell me that I should lose weight and exercise more. But you’re easily 20 pounds too heavy. Therefore I am justified in rejecting your claim that I am overweight and need to exercise more.”

Timber company advocate continuing his attack on pro-wilderness activists: “These people calling for an expansion of wilderness out of love of Nature are inconsistent. They wear leather shoes! And some of them eat meat! And they all drove cars guzzling fossil fuels to this meeting today! With such inconsistency, their position regarding this tract of land is surely without merit.”

“Conservative talk show host Rush Limbaugh rails against drug addicts and government assistance programs for them. But he was hooked on the pain reliever Vicodin! How inconsistent! We can thus reject everything Limbaugh has to say about drug addiction and tax-supported recovery measures.”

“Pastor Smith has routinely referred to homosexuality as a sin. But we just found out that he has for months been having gay sex at the local city park. His inconsistency is appalling! We can thus reject all of his claims against homosexuality.”

Ad Hominem circumstantial occurs when a fallacious arguer points to a vested interest another might have in people agreeing with his or her position, and concluding on that basis that we reject that person’s position or argument. Imagine that State Senator Sunny Shine wants the Justice Committee she chairs to approve a bill allowing people to sunbathe nude in their backyards. Senator Ima Dresser, an opponent to Shine and her position, stands up and says, “You just want that bill passed so you can sunbathe starkers in your own backyard. You have a personal interest in seeing this bill passed. On that basis, we should all vote against it.” In this case Dresser is guilty of *Ad Hominem* circumstantial because Shine’s position might be well-

argued and stand up to the light of reason, regardless of whether Shine stands to gain anything from it personally or not.

Additional examples of *Ad Hominem* circumstantial include:

Timber company advocate challenging yet again some pro-wilderness activists: “We need not pay attention to the arguments of these wilderness advocates; they just want the land preserved for their own selfish recreational use: backpacking, snowshoeing, and Lord knows what else!”

Anti-development activist writing to the editor of a newspaper: “We can reject developer Juan Murphy’s claim that the city will be benefitted by his new condominium development. He stands to make tens of thousands of dollars over the course of ten years on this project. It thus can’t possibly be good for our city!”

Accident

People are guilty of Accident when they appeal to a generally accepted rule and misapply it in a specific situation for which that the rule was never designed or intended. For instance, we agree that it is generally wrong to steal. But if I see a bank robbery taking place and I notice the get-away car idling on the street, I’d be arguing fallaciously if I reasoned as follows: “It’s wrong to steal, thus it’s wrong for me to take the car keys out of the robbers’ get-away car to prevent their escape.”

The name “Accident” refers to a non-essential (what philosophers refer to as an “accidental”) feature of a general rule misapplied fallaciously. It’s not the mere *taking* of the keys that’s wrong above; it would have been the *wrongful* taking, and in this case the taking is justified and not wrongful.

Other examples of Accident include:

It’s wrong to stick people with knives. Thus it’s wrong for that surgeon to stick a scalpel into her patient to remove his ruptured appendix.

We should not speed in cars on city streets. Thus I should not break the speed limit at 2:00 a.m. on this deserted street to get my bleeding friend to the Hospital’s Emergency Room.

The fallacy of Accident is often an important and debated issue in courts. If a billboard sign company wishes to place a large, garish billboard in your residential neighborhood, you and your neighbors will likely cry, “No way! It would be an eyesore!” The billboard company may go to court and argue that it has a constitutional right to free speech, and it wishes to exercise that right in your neighborhood. The court will have to decide if the billboard company is guilty of Accident; you, of course, believe it is; the company, of course, will argue it is not. Ideally a rational, informed judgment will prevail.

Straw Man

How many times have you been listening to talk radio (ideally, not too many), and the host misinterprets what his opponent says, or restates his opponent's argument so that it sounds like moronic mush...and then begins to pound away at the apparent stupidity of his opponent's position. If you are on your intellectual toes, you are shouting into the radio saying, "That's not what the other guy is claiming! You're misrepresenting his position!" Ever been there? You were listening to an example of the Straw Man (aka "Paper Tiger") fallacy.

A Straw Man fallacy occurs when person A holds a position (or offers an argument), and person B misinterprets that position, attacks that weaker misinterpreted version, and shows to all who will listen that *that* version is bad and should be rejected. But B has done nothing to show A's actual position is bad; all B has done is trash a weak version of A's position. If we—the listeners—are not careful, we'll be suckered into believing that B just refuted A's position.

A weak version of any position is easy to refute, as a straw man or paper tiger is easy to burn and be rid of. Let's not let logically dishonest people fool us. If someone wishes to show that someone else's argument or position is weak, let's make sure the first person is giving a fair and accurate presentation of the other's real position, and not attacking a ridiculous version that's easy to spoof.

Here are three examples of intellectually blinkered reasoning of the Straw Man variety:

An animal rights group argues that cosmetic companies should use fewer animals in their cosmetic tests. A cosmetic company advocate responds fallaciously: "These pro-animal people are wrong and should be ignored. Why? They would have advances in science stop in its tracks. They think all animals should be preserved, and that all scientists and product testing should never use animals. But if we do that, we'll never get cures for cancer or diabetes or other tragic diseases. That's just crazy!"

"Senator Billy Barker argues that we should consider federal background checks for anyone wishing to purchase automatic weapons. But denying gun ownership to American citizens is...well...*un-American*... and against the Constitution! We should thus oppose Barker's federal policy."

Pastor Bustle argues that our state is far too permissive in its acceptance of nudity. He challenges Senator Sunny Shine's proposal to allow homeowners to enjoy their backyards nude when they make a reasonable effort to make their yards private and out of public view. "Shine and her nudie friends," he argues, "would have us put up with seeing their immodest bodies on every street corner, in every restaurant, and along every grocery store produce aisle. This is decadence of the most offensive kind! We must give an emphatic 'No!' to Senator Shine's proposal."

****Practice Problems: Informal Fallacies**

Are the following fallacious lines of reasoning best understood as examples of Appeal to Pity, Appeal to the People, *Ad Hominem*, Accident, or Straw Man?

1. We should not run with knives in our hands. So even though your friend is entangled in a rope tied to a heavy rock pulling him over a cliff edge, you should not run to him with a knife to cut the rope.
2. Driver to traffic cop: “Man, has my day ever been bad. I spilled coffee in my keyboard at work, my girlfriend found out I’m married, and my stocks are continuing to take a nosedive. With all of this going on, you thereby shouldn’t give me a speeding ticket for going 50 mph in a 25 mph zone.
3. Teenager to teen friend: “My parents say I need to be home by 11:00 tonight. That’s so unfairly random! They don’t want me to have any fun or ever meet anyone. They want to control every aspect of my life! Since that’s clearly unjust, I need not take seriously their curfew.
4. Senator Sunny Shine argued for her bill allowing backyard nudity by saying, “Benjamin Franklin, John Quincy Adams, Walt Whitman, Norman Rockwell, and other famous American figures sunbathed or skinny-dipped nude regularly. Backyard nudity is American as apple pie. If you love your country, you should support my bill!”
5. Architect Bob Shine gave his friends his reasons for reluctantly supporting his wife’s backyard nudity bill. But others argued, “Oh, you’re married to her so it’s in your best interest to have her be happy. Thus we can reject your arguments for the bill’s endorsement.”
6. College student Stu tells his friend that she needs to study to pass her Sociology test. She responds, “Stu, I’ve *never* seen you study beyond reading the required text. You are hardly one to talk. I can thus discount your claim about the need to study for my Sociology test.”
7. “Wheaties: The breakfast of champions!”
8. Nervous student to himself: “It’s always good to study before an important math test. So, even though it’s late the night before the test, and I haven’t slept for 48 hours, I should force myself to study the rest of the night!”
9. Professor Barnes teaches Latin American History. But he never dresses well, needs a haircut, and has bad breath. I guess we can ignore what he has to say about Latin American history.
10. Professor James says we students should spend less time with our phones and video games and spend more time traveling, reading, and talking to a larger variety of people. James obviously hates technology and would have us go back to the Stone Age, rubbing sticks together to light fires and to send smoke signals. What nonsense. We can thus reject his claims about broadening our activities.

Answers:

- | | |
|---------------------------------------|----------------------------------|
| 1. Accident | 6. <i>Ad Hominem (tu quoque)</i> |
| 2. Appeal to Pity | 7. Appeal to the People |
| 3. Straw Man | 8. Accident |
| 4. Appeal to the People | 9. <i>Ad Hominem</i> (abusive) |
| 5. <i>Ad Hominem</i> (circumstantial) | 10. Straw Man |

Red Herring

How many times have you heard an interview between a reporter and a politician in which the reporter asks a potentially embarrassing question pointing out a fault with the politician, but the latter subtly changes the subject and concludes that everyone should think he’s the coolest thing since sliced bread? Far too often. We’re in the world of the Red Herring fallacy here. Red

Herring occurs when someone subtly changes the subject and draws a conclusion that unaware listeners take to be regarding the original issue. For instance:

A pulp mill is said to be dumping large amounts of pollutants into a local river, making it unsafe to swim or fish downstream. A reporter asks the mill manager about this, and the manager says, “Well, you know, our mill employs dozens of local men and women, and we’ve been donating to the high school football team every year. The city taxes we pay support the police and fire departments in our small town. I’d say that we’re an asset to this community!”

The mill may be doing all these grand things, and it may even—overall—be an asset to the community, but the spokesman is avoiding the issue of whether the mill is polluting the river and in that manner harming the community. He changes the subject slightly (to do so too bluntly would make his evasion of the issue too openly apparent; he thus cannot effectively change the subject to the strength of the Seattle Mariners’ outfield), talks about the mill’s strengths regarding community effect, and concludes that the mill should be applauded for the good work it does locally. Rarely do reporters have the nerve to call such people on their evasion of the question, but we listeners are often frustrated when politicians, business leaders, and other officials sidestep an issue, focus on their strengths, and conclude that everything is okay.

The phrase “Red Herring” refers to the old British practice of training hounds to chase foxes for horse-mounted fox hunts. Trainers would guide dogs to follow the scent of foxes, and not go off track looking for something else. The trainers would sometimes take a smelly herring, wipe it across the fox’s track, throw it into the bushes, and see if the dog would stick with the fox scent or get sidetracked and follow the tantalizing aroma of rotting fish. Going after the herring indicated a poor hunter, as today it indicates a poor thinker.

Note that the fallacious Red Herring arguer is not restating in a weak manner the original argument or position as he would if engaged in a Straw Man fallacy. Here, the arguer is changing the subject subtly, and offering a conclusion he hopes listeners will take to pertain to the original issue. Here’s another example of Red Herring doing exactly this:

An opposing senator argues: “Senator Shine would have us support backyard naked gardening. But what this is really about is the moral decay of our society. People are not willing to take responsibility for acting in an upright manner anymore. Just last week we heard of an entire high school football team getting together to rob a liquor store. A woman was assaulted on our streets last night! Kids are writing graffiti all over the walls of our town hall. We’ve thus got to say ‘No!’ to Senator Shine’s proposal.”

Appeal to Ignorance

The fallacy of Appeal to Ignorance occurs when someone argues that because we do not know that X is true, that gives us reason to believe that X is false, or, because we do not know that Z is false, that gives us reason to believe that Z is true. For instance:

No one has proved that astrology does not work. Therefore it *does* work!

A logical, convincing argument for God's existence has yet to be provided. Therefore, God does not exist.

No person or culture has ever come up with a clear objective, universal basis for morality. Thus there is no objective, universal basis for morality.

Pretty clearly, just because no one has proved one side of the issue, it does not follow that the other side is true. It could be that the issue is difficult and challenging, and that good arguments are still to come. Scientists *still* do not know *why* massive objects fall in normal gravitational fields. We can see that such objects *do* fall, and we can measure accurately the rate at which they fall, but as to why...? Still, that does not mean that there is no answer; it could be we just have not discovered it yet. Also, an answer may indeed have been found, but for some reason most of us have rejected it or forgotten it.

All that said, if we don't find an answer *and we would* have if there was one, then we can safely conclude something. For instance, if a team of expert mountain climbers spend three years scouring the top 500 feet of Mt. Rainier looking for a mature palm tree, and they don't find one, the team would be justified in saying that there is no such tree there. This is because if the tree was there, they'd have surely found it. This may not be the case with inquiries into the existence of God or an objective basis for morality. Such issues are far more complex than looking for a palm tree on a single mountain top.

Weak Authority

Appeals to authority may be strong or weak, depending on how authoritative the authority is on the issue in question. Consider the following argument:

All astronomers say that Jupiter and Saturn are the largest planets in our solar system. Thus Jupiter and Saturn are the largest planets in our solar system.

This is a strong argument because astronomers are authoritative on the size of our solar system's planets. They *might* be mistaken, but this is an inductive argument, and the arguer is not intending to prove deductively anything here. Consider, though, the following argument:

My four-year-old niece Jessica [who has never studied astronomy and does little but burp and watch cartoons] says that Jupiter is a large planet. Thus Jupiter is a large planet.

This is what we'll call an example of Weak Authority because Jessica—at least at this young stage of her life—is no authority on astronomy or the size of planets. She is correct in her claim, but it would be a weak inference on our part to believe this claim about Jupiter simply because she said it was so.

Authorities can be individual persons, committees, universities, books, encyclopedias, online sources, signs, plaques, or any other generally reliable source. Of course, thoughtful people

might disagree on the authoritativeness of some figures. One might believe a man really knows his stuff and is authoritative, while another might have good reason to doubt the man's knowledge in the area. Induction often requires some discussion or debate, so we won't let those possibilities bother us.

Also, some kinds of claims just don't lend themselves well to appeals to authority. For instance, moral claims or claims about God's existence are generally weak if merely made by an appeal to authority. "Presidents George Washington and Ronald Reagan said that God exists. Thus God exists." These two guys may have been smart, and they may have been correct about God's existence, but the mere fact that they *say* that God exists really can give us no reason to believe it. If either of them has a good argument for God's existence, that *argument* may give us good reason for becoming believers, but then we'd no longer be appealing to authority, but to that argument.

One more point. Sometimes someone might be an authority on one issue, but not authoritative on what he or she is talking about today. For example, if I appeal to Stephen Hawking (a famous and reputable astrophysicist) on how the U.S. should respond to the Israel/Palestine conflict, I'd likely be making a Weak Authority argument, because Hawking is not an authority on international politics.

Examples of clearly *strong* appeals to authority include:

The official sign outside our school says "Bellevue College." Thus this school must be Bellevue College.

Stanford University's philosophy website says that Plato was Greek. Thus Plato was probably Greek.

The title of your textbook purchased at the college bookstore is *Introduction to Biology*. Thus your textbook is likely about biology.

Zoology teachers have told me that mice are mammals. Hence, I am justified in believing that mice are mammals.

Clear examples of Weak Authority include:

The marking pen scrawl on the men's restroom wall says, "Buffus rules!" Therefore, Buffus must rule.

Famous actress and singer Barbara Streisand says that the nuclear power plant would be unsafe for our community. Therefore we should not allow such a plant to be built here.

Edgar Rice Burroughs' fictional novel, *Tarzan of the Apes*, says that Tarzan lives in Africa some time and in England other times. Thus if I went to Africa, I might be able to meet Tarzan!

My mother told me long ago that Jesus is the son of God. Thus Jesus is the son of God.

The majority of our culture believes and says that sexual assault is morally wrong. Thus, sexual assault is morally wrong.

Genetic Fallacy

How someone comes to acquire a belief is distinct from whether that belief is true or not. Surely a little boy can acquire a true belief in an odd, unreliable way. For instance, he might come to believe that a man walked on the Moon by reading of such an occurrence in a Sci-Fi comic book. If he were to come running in excitement to us saying, “A man walked on the Moon!” and we replied that we should reject his belief because he acquired it in a manner that is not usually trustworthy for belief-acquisition, we’d be guilty of a weak line of reasoning known as the Genetic Fallacy.

The Genetic Fallacy gets its name from the same root word we find in *genesis*, referring to the beginning. We commit this fallacy when we argue that because the origin of the belief—that is, how the belief came to be held—is questionable, the belief itself is questionable. But this does not follow. We can acquire true beliefs in bizarre ways, and we can acquire false beliefs in reliable ways (e.g., when little kids come to believe that Santa Clause exists because all the adults in their lives say so).

Examples of the Genetic Fallacy include:

Confused Freudian to believer: “You came to believe in God due to your desire for a father figure. Thus God does not exist.”

Confused behaviorist to believer: “You came to believe in God due to cultural conditioning. Thus God does not exist.”

Confused theist to atheist friend: “You believe God does not exist because you suffered so much in your life. Thus God does exist.”

Obviously the Genetic Fallacy shows up in less theological discussions, too:

Five-year-old Juan Garcia came to believe that two plus three equals five from his student friend in kindergarten. But kindergarten kids are hardly authorities on math. So it is false that two plus three equals five.

Five-year-old Juan Garcia came to believe that two plus three equals *six* from his student friend in kindergarten. But kindergarten kids are hardly authorities on math. So it is false that two plus three equals six.

Friend to friend: You believe that Mary loves you because Alice told you so. But Alice is a lunatic who is always gossiping. Alice is not trustworthy. Thus Mary does not love you.

Our moral beliefs (e.g., that stealing is wrong) were acquired subjectively through cultural conditioning. If we had different conditioning (e.g., different teachers, parents, pastors, friends), we'd have different moral beliefs. Thus there is nothing objectively true about moral beliefs.

Hasty Generalization

Hasty Generalization occurs when the arguer appeals to what's known about a portion of a group and then makes a weak inference to that claim being true of the whole group. Clear examples are weak polls. For example:

Professor Tran asked three students in her graduate class on American Literature if they like to read books. All three said they did. Tran said that she thereby could conclude that *all* the students at her university like to read books.

This generalization from three students to the entire school student population is weak for at least two reasons: the sample is woefully small, and the sample was not neutral (American Literature graduate students would obviously like to read books, and probably like it more than most others on campus).

Other examples of Hasty Generalization:

Confused professor to friend: "College students today are all slackers. I know this is true because last week two of them showed up for my class late."

Radio talk show host: "Driving to work I saw three young people writing graffiti on public walls. The youth today are a bunch of thugs."

Bob selected at random two cards from a large box containing hundreds of baseball cards. Both cards selected were of Seattle Mariners. Therefore every card in the box is probably of Seattle Mariners.

I ordered two books from Amazon.com, and both arrived in damaged condition. I can only conclude that Amazon.com always sends damaged materials.

****Practice Problems: Informal Fallacies**

Are the following fallacious lines of reasoning best understood as examples of Red Herring, Appeal to Ignorance, Weak Authority, Genetic Fallacy, or Hasty Generalization?

1. No arguments or evidence has proven alchemy to be true. Thus the claims of alchemy are bogus.
2. I heard a guest philosopher try to answer some questions on the radio yesterday. Boy, did he sound like an idiot. He kept mumbling, saying "ah...ah...ah," and had a hard time answering any question directly. All philosophers must be morons.

3. Alvin Plantinga is a respected American philosopher of the last three decades. He says the Mariners will win the next World Series. Thus I'm putting my money on the Mariners!
4. Senator Sunny Shine in response to a critic of her backyard nudity bill: "Senator Smith, you oppose my bill because—as you say—you don't want to force neighbors to see ugly bodies. But ugliness is a complicated aesthetic concept. Many art theorists have labored unsuccessfully to define 'beauty' and 'ugliness.' Philosopher Ludwig Wittgenstein thought the task hopeless. Thus people should support my bill."
5. A Roman Catholic priest, speaking publicly from his pulpit, asked five of his parishioners if they believed in God. All five said yes. The priest responded, "See, this country is full of believers!"
6. Most people come to believe that George Washington was the first U.S. president due to the cultural conditioning of the educational system. Thus there is no objective truth to whether or not George Washington was the first U.S. president.
7. Many dieticians think we should eat chemical-free foods. But chemicals are vitally important to our lives. Without them, we'd not have the plastics we all rely on today, nor would we have access to strong, light-weight materials for cars or planes. These dieticians are just foolish.
8. Raw vegans have come up with no good reason to believe that cooked food is harmful to us. Thus cooked food is perfectly safe for human consumption.
9. Michelle Obama (the U.S. president's wife) says American children should eat less fat. Because she says this, we can conclude that American children should eat less fat.
10. Friend to friend: "You heard that President Barack Obama is a Muslim on a talk radio show. But those radio show hosts are nuts. Thus Obama is not a Muslim."

Answers:

- | | |
|-------------------------|------------------------|
| 1. Appeal to Ignorance | 6. Genetic Fallacy |
| 2. Hasty Generalization | 7. Red Herring |
| 3. Weak Authority | 8. Appeal to Ignorance |
| 4. Red Herring | 9. Weak Authority |
| 5. Hasty Generalization | 10. Genetic Fallacy |

False Cause

A False Cause fallacy occurs when someone argues from cause to effect or effect to cause, but does so when there is no good reason to believe that such a causal relationship may be expected. For instance:

As Joe drives by a partially destroyed building and sees flames jutting out of its windows and rescue vehicles all around, he concludes that Martians must have just attacked.

Joe's inference is weak, as there is no good reason to conclude a Martian attack is a likely cause of the damage to the building. Far more sensible causes could be inferred.

Logicians sometimes like to distinguish at least three kinds of False Cause. One, receiving the fancy Latin enumeration of *non causa pro causa* ("not the cause for the cause") is the most

general, positing something as a cause of something else when the first thing is not actually the second thing's cause. Examples include:

All great historians for the Roman Empire read Latin. Thus, if Sara learns Latin, she'll be a great historian of the Roman Empire.

In any major city, when more ice cream is eaten, the crime rate goes up. We must therefore ban all eating of ice cream.

Wherever there are more churches, there is more crime. Thus we must get rid of churches.

Whenever I've gotten an A on a college test, I was wearing clothes. Clothing must thus heighten intellectual abilities.

A more specific form of False Cause is called *post hoc ergo propter hoc* ("after this therefore because of this"). Sometimes people refer to it as simply a *post hoc* fallacy. These examples of False Cause make the point that merely because action B occurred immediately after action A, it follows that A caused B. But this line of reasoning is surely weak, because it often happens that one thing follows another, when there is no direct causal connection between the two. The sun rises every day prior to setting, but it would be odd to say that the morning's sunrise causes the evening's sunset. Examples of False Cause *post hoc* fallacies include:

Senator Garcia spoke at our university today at noon. Immediately afterwards a tornado struck our city. For the love of God, we must never let Garcia speak here again!

I broke a mirror yesterday. That's why I got in a car accident today.

While riding on a train through the mountains I was eating my lunch. All was going well until I bit into my bologna sandwich. All of a sudden I could not see anything. Immediately after finishing the sandwich I could see again. Thus, eating bologna sandwiches causes blindness!

Sammy Slugger hit two home runs over the past two days, and right before those two at-bats he rubbed his lucky rabbit's foot. He should thus rub the heck out of that thing throughout every game!

A third distinct variety of False Cause is sometimes called False Cause oversimplification. Here, the causal connection appealed to obtains (that is, it accurately reflects how the world works), but it's not the whole story. Other causes ignored in the argument were at play. For instance, President Ronald Reagan's foreign policy was probably a cause—in part—of the fall of the Soviet Union's political system. But to say that Reagan caused the fall of the Soviet Union is to oversimplify the situation. The USSR's economic situation, internal political dissent, their foreign policy, and other factors contributed together to usher in the dramatic change. Other examples include:

Bob finally got an A in a college course. The textbook Bob used was written by Horace Kline. We can conclude that Kline's writing produces student success.

The relationship between big businesses and local communities is declining these days. Obviously the CEOs are not doing their jobs.

The Backyard Nudist Association has acquired large contributions from both Democrats and Republicans in our state. Thanks must therefore go to the BNA's chair, Senator Sunny Shine, as she must be a great fundraiser.

Slippery Slope

Slippery Slope arguments can be strong or weak, but they all share a pattern in common. They're actually a form of False Cause, but have their own unique characteristics. A Slippery Slope line of reasoning argues that A will cause B, B will then cause C, C will then cause D, and D will then cause E. But we don't want E! So we should not even allow ourselves to get started down the slope; we should thus reject A. This argument can be strong if we have good reason to affirm each causal link (e.g., between A and B). But if we have good reason to reject one or more of the causal links, then just because we allow A to take place, it does not mean that E will occur.

Here is a pretty good causal chain argument:

If straight-A student Beatrice stops studying for her physics class, she'll not ace her next Physics test. And if she does not ace her next Physics test, she'll not get an A in the class. And if she does not get an A in Physics, she'll no longer be a straight-A student. And if she is not a straight-A student, she'll not get the \$1000 her aunt promised her if she *did* finish with straight As. Beatrice wants that \$1000; so Beatrice should continue studying for her Physics test.

What do you think of the following causal chain argument? Is it a truly slippery slope or a fallacious Slippery Slope?

If we allow the government to register private ownership of automatic weapons, next they'll want to register all private ownership of hand guns. And if they register private ownership of all handguns, next they'll ban all private ownership of automatic weapons. And once they do that, they'll next ban private ownership of *all* guns. Canada will view this as a window of opportunity and attack the USA. We don't want that. (Think of the horror!) So we must not allow the government to register ownership of automatic weapons.

Is there a break in the logic links anywhere? If so, it's a fallacy; if not, be very, very afraid, eh. Here's an example of a clear Slippery Slope fallacy:

Teacher to school administrator: "We can't let students choose which classes they take, for they'll next want to choose how many they need to graduate. After that, they'll not be satisfied until they teach the classes themselves. Then they'll want to take charge of the buildings and sell

them to China. Chinese business groups would take over our country and we'd all have to be Communist. If you value democracy, we must say 'No!' to these students!"

Weak Analogy

Analogical arguments compare two things or two groups of things. Such arguments point out some relevant similarities between the two things, and conclude that they are so relevantly similar that what is true of one is probably true of the other. Arguments from analogy can be incredibly strong, and as we'll see later in this course, we bet our lives on them every day. Still, some arguments from analogy are seriously weak, and we can call these Weak Analogy fallacies.

One basic problem with many weak analogies is that there is a significant and relevant difference between the two things being compared. For instance:

Both Albert Einstein and Aristotle were males, very smart, highly respected, and wrote books. Einstein believed that nothing travels faster than the speed of light. Thus Aristotle probably believed this, too.

The problem here is that Aristotle, being a 4th century BC Greek philosopher, did not have the physics or math background to understand things like the theory of special relativity.

Other examples:

Ima Dresser and Sunny Shine are both women, married, Washington State senators, Democrats, and chairs of congressional committees. Ima Dresser thinks anyone convicted for a third time of skinny-dipping within view of anyone other than one's spouse should receive a \$10,000 fine and life in prison [Dresser was born in Montana where that's the current law]. Thus Sunny Shine likely agrees with Dresser on this response to such behavior.

Pastor Bustle of the Central Baptist Church believes in the existence of God, thinks associating with fellow believers is important to him, and votes regularly. The same can be said for Rabbi Maimonides. Maimonides celebrates Hanukah with his friends each year. Therefore, Bustle likely does, too.

I've gone out to dinner four times this week. I've eaten at the Carnivore Carnival BBQ Pit, the Hungry Man Burger Barn, the Tuesday night all-you-can-eat sausage feed at Mel's Meat House, and the Sizzlin' Steak Shack. I loved each meaty meal. Thus I'll probably also love the dinner tonight at a new raw vegan restaurant in Seattle called Thrive.

There are a number of ways analogies can go bad, and we'll look at them in more detail later.

For now, let's consider two more examples of Weak Analogy:

Tim dated Mary once and had a great time. Thus Tim expects to have a great time when he dates her again tonight. [This is not the strongest argument, because Tim has only one experience to

work from. If he had dated Mary multiple times before and enjoyed each date, he'd have a better argument.]

Sue took three classes from Professor King and liked them all. Sue has signed up for another King class next quarter, and concludes that on the basis of her past experience in King classes, this one will be the best experience of her entire life. [Here Sue's conclusion is far too specific. *Liking* her previous three classes with King gives her little reason to think the next one will be the highlight of her life.]

Begging the Question

Begging the Question is far too common even among sharp thinkers. It's one the brightest philosopher still can struggle to avoid. Begging the Question (we'll consider this the same as Circular Reasoning) occurs when someone assumes the truth of the conclusion when offering a premise for that conclusion. The premise is needed to support the conclusion (as is proper and expected), but also the conclusion is needed to support the premise (hence the "circle").

An obvious example of Begging the Question is this: "The Book is the word of God. Why should we believe this? I'll tell you! The Book says it's the word of God, and God would not lie." Clearly one cannot assume that God wrote (or inspired the writing of) The Book to show that God wrote The Book. Believers can argue for the divine authorship of a holy work without arguing in a circle, however. One might argue for a text this way: "The Book contains language so beautiful that no human could have written it. Thus The Book has a divine source." This may not be the greatest argument, but at least it's not assuming from the get-go that God wrote The Book.

Other examples include:

Aristotle was a more intelligent philosopher than Epicurus. We know this to be true because insightful philosophers say so. And we know them to be insightful philosophers because they are the kind of people who recognize that Aristotle was a more intelligent philosopher than Epicurus.

Euthanasia is morally wrong, because it's a sin.

Murder is wrong; thus abortion is wrong.

Opponent to Senator Shine's backyard nudity bill: "Backyard nudity is wrong because...well, we know it is."

All that exists is material in nature and determined by causal laws. Thus there is no free will.

Jim: "We know mass creates gravity because dense planets have more gravity." Margo: "How do we know which planets are more dense?" Jim: "They have more gravity."

Some people use the phrase “Begging the Question” in an additional sense. In a conversation someone might say something questionable that prompts a question. The listener might confusedly say, “That begs the question, ‘What about such-and-such?’” Saying something questionable that prompts a question or a demand for independent support is not the same as assuming information to get to that same information. If someone offers a premise for a conclusion, and that premise needs support itself, one can certainly demand the arguer supply that support. But this is different from the fallacy of Begging the Question.

False Dichotomy

A Disjunctive Syllogism is a two-premise argument of the form

P or Q
P is false.
Thus, Q

This is a deductive line of reasoning, and perfectly valid. However, if someone offers an “either-or” statement as one of the premises, and neither disjunct (the two statements to either side of the word “or”) is true, then ruling out one in the other premise does not actually give good reason to say the other is true. Since this problem is a matter of content—and of understanding that the disjunction is too limiting to be true—we are in the realm of informal fallacies.

Parents of teenage children are the world’s experts at False Dichotomy. These nearing maturity often come to their parents with the following kinds of “either-or” lines of illogic:

Daughter to mother; “Mooooom, either you let me date the platoon based at Fort Lewis, or I will surely die. And you don’t want me to die, right?”

Son to father: “Dad, either you buy me these \$200 running shoes or my life will be ruined!”

Parents will adroitly point out to their adoring children that a third option lies squarely in their future. Or how about the beleaguered guys facing an onslaught of Valentine’s Day ads?

“Either you buy your sweetie a really, really expensive piece of diamond jewelry, or she’ll think you loathe her.”

“Buy X, Y, and Z, or women won’t think you’re hot!” [This has the acrid aftertaste of Appeal to the People, too.]

Guys are not the only ones facing this nonsense:

“Either you have the figure of a super model, or no guy will ever like you. And surely you don’t want that. So buy the latest version of Super-Beauty Body-Shaping Cream!”

And let's not think too long and hard about what crazy claims talk show hosts and politicians too often give us:

"Either you vote Republican or government will take away all your guns. For the safety of your family and our country, vote Republican!"

"Either you vote Democrat or the government will take away your rights to privacy. For your own well-being, vote Democrat!"

****Practice Problems: Informal Fallacies**

Are the following fallacious lines of reasoning best understood as examples of False Cause, Slippery Slope, Weak Analogy, Begging the Question, or False Dichotomy?

1. Senator Billy Barker advocates allowing terminally ill patients to receive voluntary passive euthanasia. But if we allow that, patients will next be asking for voluntary active euthanasia, and then it will be non-voluntary passive euthanasia, then non-voluntary active euthanasia. My God, we could then see involuntary euthanasia, first of the mentally ill, then for those with colds, then for those who wear too much Spandex in public. We'd lose half of Los Angeles! We can't allow that, so we must disagree with Barker on this.
2. Surely you want to send the very best card to your grandmother for her birthday. And it's either Hallmark brand cards or...well...garbage. Thus you should send her a Hallmark card.
3. We try on shoes before we buy them. We give cars a test drive before committing to a purchase. Thus we should have sex with our boy- or girlfriend before we get married.
4. Free market capitalism is an ideal system, because having the liberty to buy and sell anything you want is better than any other economic approach.
5. I had a coffee right before my chemistry test yesterday, and I bombed the test. I'm never drinking coffee again!
6. Either you'll marry me this week or you don't love me! Surely you love me; so it's settled, you marry me this week.
7. Each time I arrived with flowers two hours late for my dates with Susan, she was upset with me. Susan must really dislike flowers.
8. Melanie and Trudy have the same parents and the same upbringing. Both go to Bellevue College, and both like computer science and sports. Melanie wants to go out with me. Thus probably Trudy does, too.
9. My daughter scored a goal in her soccer game yesterday. I can conclude that extra exercise I encouraged her to do paid off.
10. Reading racy novels leads to looking at porn. Looking at porn leads to viewing other people as mere objects. After a while, that will cause you to assault someone. But we don't want assaults in our society. So we should ban racy novels.

Answers:

- | | |
|-------------------------|---|
| 1. Slippery Slope | 6. False Dichotomy |
| 2. False Dichotomy | 7. False Cause (<i>non causa pro causa</i>) |
| 3. Weak Analogy | 8. Weak Analogy |
| 4. Begging the Question | 9. False Cause (oversimplification) |

Is-Ought Fallacy

The line of reasoning we will examine here probably merits as much discussion and debate as any of our other informal fallacies. There are some thoughtful philosophers who will want to defend the so-called move from “is” to “ought, or “ought” to “is.” Still, we’re going to call such an inference a fallacy. Surely it will be so in many cases. So what *is* the Is-Ought Fallacy?

An “Is” statement is *descriptive*; it describes how the world actually is. The statement may be false (as it fails to correspond to reality), but it still claims to be accurately describing the way things are. Examples of “is” or descriptive statements include:

The Empire State Building is in New York City.

George Carlin is not president of the USA.

$2 + 2 = 4$

A blue pen is colored.

Triangles have four sides.

The USA has 145 states.

Most people believe that $2+3=5$.

It is a cultural norm (or custom, more, folkway) in the USA not to murder people.

It is a cultural norm in Culture X for husbands to assault their wives.

An “Ought” statement is *prescriptive*; it prescribes how things ought to be (whether or not they happen to be that way now, and whether the prescriptive claim is true or false, or whether everyone agrees to it or not). They are thus claims about the moral nature of something or other. For instance:

We ought not to murder people.

Sexual assault is wrong.

Truth-telling is usually the right thing to do.

The USA ought not to have engaged in slavery.

We should not be sexist.

We should be sexist.

People should worship God on the Sabbath.

People should believe only what they are rationally justified in believing.

The Is-Ought Fallacy occurs when someone argues from a merely descriptive claim (or set of merely descriptive claims) to a prescriptive claim, or vice versa. The idea behind the problem is that just because something *is* (or is not) the case, does not mean that it *ought* to be (or not be) the case. Moreover, just because something *ought* to be (or not be) the case does not mean that it *is* (or is not) the case.

For instance, we ought not to murder, but that does not mean that murder never takes place. Also, we ought to tell the truth, but that does not mean that no one ever lies. On the flip side, just

because robberies take place does not mean that there should be (or should not be) such actions. Just because no one in our culture is doing X, does not mean that X is wrong (or right). And the norm for our culture being Y gives us no logical reason by itself to believe that we ought to do Y.

Here are some examples of what we here refer to as the Is-Ought Fallacy:

The lifestyle and public demands of religious and political leaders ought to be consistent [a prescriptive claim]. Thus the lifestyle and public demands of religious and political leaders are perfectly consistent [a descriptive claim].

In the early 1800s, slavery was perceived by the majority in the USA as morally permissible [a descriptive claim]. Thus slavery was not wrong in the USA in the early 1800s [a prescriptive claim].

Racist segregations policies were a norm for the southern USA in the 1960s [a descriptive claim]. Thus people like Martin Luther King, Jr. and Malcolm X were wrong to fight racist segregation policies in the southern USA in the 1960s [a prescriptive claim].

Female genital mutilation (a common practice in some cultures) goes against the cultural norms of the USA [a descriptive claim]. Thus female genital mutilation is wrong in the USA [a prescriptive claim].

Various rich, thoughtful theories of ethics will claim that the cosmos has an eternal order. Hindus have called this order Rta; the Chinese have referred to it as the Dao; Natural Law Theorists from the West often refer to it as Eternal Law. The cosmic order offers something like a blueprint for the universe, and if things participate in it according to its plan for them (i.e., as humans, animals, plants, rocks, or whatever they are), the universe will flow in an orderly, ideal manner. This all sounds rather descriptive to some critics, who then say that no prescriptive claim can logically follow merely from it. Thus, they say, it is an example of the Is-Ought Fallacy to argue (as Martin Luther King, Jr. seems to argue in his “Letter from Birmingham Jail”) that because within Eternal Law humans have the role of X, humans are thus morally obliged to do X. Hm. Let the philosophizing begin!

There is one last point to make regarding the Is-Ought Fallacy: it often is found in arguments guilty of other fallacies. Appeal to Pity, for instance, often argues from the descriptive claim that I am in a pitiable situation, and concludes prescriptively that someone ought to do such-and-such for me. Appeals to the People fallacies also often begin from a descriptive claim about the sexiness of X, or the athleticism of Y, and concludes prescriptively that we ought thereby to embrace X, or Y. It is common that bad arguments fit more than one fallacious pattern. That makes it challenging to come up with practice examples that have only one clear answer, but it’s worth our while to understand the distinction between these fallacies nonetheless.

Equivocation

To equivocate is to use a word in two different ways and with two different meanings, while allowing others to think you are meaning only one thing. The fallacy of Equivocation occurs when someone begins an argument meaning one thing by a word, but switches meanings (tacitly or explicitly) later in the argument, and the conclusion follows only because of that illicit switch in meanings. A simple illustration is this:

Rivers have banks. But I heard that Bob keeps his cash in a bank. Thus Bob must have some pretty soggy cash.

A dumb argument, yes. The word “bank” refers to the muddy sides of rivers in the first premise, but to financial institutions in the second. The conclusion is playing off the ridiculous double meaning of the word “bank” here. Look for a double meaning of a word impacting the conclusion in these additional examples.

My wife hit me over the head with a bat. Thus, someone should call Animal Control. [“Bat” can refer to an elongated tool used in baseball or to a flying mammal. The conclusion is working off the bizarre secondary meaning of the word.]

Nothing is better than having good friends and bodily health. And a stale bologna sandwich is better than nothing. So, a bologna sandwich is better than having good friends and bodily health.

Tennis star Serena Williams seems often to get an ace against her opponents. Thus she would do very well at draw poker.

A turtle is an animal. Thus, a fast turtle is a fast animal. [The word “fast” is being used in two different senses here: in terms of turtle speed and in terms of animal speed; they are not the same thing.]

Architect Bob Shine recalling his recent problems over the Floating Bridge collapse: “Boy, that bridge has taken its toll on me.” A befuddled friend concluded that Shine is mistaken: “But you don’t live anywhere near that bridge, so you never had to pay the road toll to cross it.”

Senator Ima Dresser to committee: “I wish to speak confidently and openly, to stand naked before you.” Immediate fallacious response from Senator Sunny Shine: “Finally, she agrees with me!”

Student to student friend: “Professor Storey’s logic lecture today was so exciting that we sat glued to our seats.” Intellectually challenged friend’s response: “You thus must have had difficulty standing up. How’d you get up off that sticky chair?”

Amphiboly

The fallacy known as Amphiboly is akin to Equivocation, but whereas Equivocation plays off an ambiguity of a single word or term, Amphiboly plays off a grammatical ambiguity. Some sentences can be phrased poorly so that two distinct meanings might be understood. If an arguer

draws an inference from a secondary, clearly ridiculous meaning or interpretation, that arguer is guilty of Amphiboly. For instance:

Senator Ima Dresser was so distraught over losing the Justice Committee vote to Senator Shine yesterday that she was seen crying and wiping her eyes on the State Capitol lawn. We can conclude that Dresser must have grass stains on her face.

Botany student to class-skipping fellow student: “Our professor spoke about trees in our classroom today.” Fellow student: “Wow! You must have moved outside, because I don’t recall any trees in our classroom.”

Court reporter to intellectually blinkered friend: “After sentencing the criminals and addressing the members of the jury, the judge sent them off to jail.” Friend: “Oh dear. I had no idea that people had to go to jail for serving as jurors.”

Groucho Marx: “One morning I shot an elephant in my pajamas. How he got into my pajamas I’ll never know.”

Suspect to police detective: “You guys suspect several people of robbing the store. Thus, I can’t be under suspicion, since I was alone the afternoon of the robbery.”

Sign on café window: “Eat here and you’ll never eat anywhere else again!” Rationally confused passerby: “Boy! They must serve some dangerously bad food there!”

Composition

This and the final informal fallacy are the flip side of each other. In Composition, the arguer contends that merely because something is true of each of the parts of a thing, that character trait is true of the whole, as well. But that does not always follow; it’s not a reliable inference. For instance:

Each brick in this wall is well made. Thus this wall as a whole is well made.

Each sentence in Sarah’s English essay is well formed. Thus her essay is well formed.

I like fresh tuna, cheese, chocolate, spaghetti sauce, black licorice, and hot bread. Thus I’d like a licorice/chocolate/tuna pizza.

I can lift every individual part of my car. Thus I can lift my entire car.

Each part of Rene Descartes’s metaphysics is internally consistent. Thus his metaphysics as a whole is internally consistent.

Each player on the NBA All-Star team is excellent at basketball. Thus the NBA All-Star team is excellent at basketball.

Division

If Composition is “Parts → Whole,” then Division is “Whole → Parts.” Division occurs when someone argues that merely because something is true of a thing as a whole, that trait is also true of each part of that whole. As with Composition, this is a weak line of reasoning. For example:

The ACME Company did quite well this past year. They made higher profits than usual, and the local community loves them. Thus each employee of ACME is doing well at his or her job.

The USA is not in favor of Americans visiting Cuba. Thus Maria Drake, an American tour agent for U.S. travelers, is not in favor of Americans visiting Cuba.

My Subaru did not cost very much! Thus I can expect if the headlight ever breaks, it will be inexpensive to replace it.

Water quenches thirst. Therefore, since hydrogen is part of what makes up water, it quenches thirst, too.

Stanford University is an excellent academic environment. Thus, Stanford freshman Bill Dense is an excellent student.

I love Mexican *mole negro* sauce. It contains among other things dried chilies, nuts, fruit, chicken broth, onions, garlic, and chocolate. Thus I'd love to eat a bowl of dried chilies.

****Practice Problems: Informal Fallacies**

Are the following fallacious lines of reasoning best understood as examples of the Is-Ought Fallacy, Equivocation, Amphiboly, Composition, or Division?

1. Senator Billy Barker is a handsome man. Thus Barker's left elbow is handsome.
2. Wife feeling physically uncomfortable while driving her car with her husband: “I think I have gas.” Unthinking husband: “Good! We can thus avoid stopping at the service station and we'll make it to the movie on time.”
3. Senator Sunny Shine went out into her backyard to garden in her tennis shoes. Those must therefore be extra large tennis shoes to hold a garden.
4. Senator Ima Dresser: “The majority of this fine city is opposed to naked backyard gardening. Thus such activity is morally wrong!”
5. Each of Sunny Shine's backyard garden crops taste good: her beets, beans, squash, kale, and spinach. Thus if she mixed them all together they'd taste good, too.
6. Pastor Bustle: “It's clear that adultery is wrong, which by itself convinces me that no one in this congregation is be engaging in such immorality.”
7. Bellevue citizens contributed \$100,000 to charity for the homeless last night. Rabbi Maimonides is a Bellevue citizen. Thus Maimonides contributed \$100,000 to the homeless charity last night.

8. Teenage daughter to friend: “My mom censors my phone calls by telling me not to take calls from that really cool biker gang.” Confused friend in reply: “That’s horrible! Your mom is thus breaking the law, as censorship is against the First Amendment.”
9. Each grain of sand in this sandcastle can withstand great pressure and maintain its shape. Thus, this sandcastle can withstand great pressure and retain its shape.
10. Older woman to friend: “As a girl, my grandfather told me stories about living in the Wild West.” Befuddled friend in response, “I can conclude that his accounts were not nearly as wild as his story about his sex change.”

Answers:

- | | |
|---------------------|---------------------|
| 1. Division | 6. Is-Ought Fallacy |
| 2. Equivocation | 7. Division |
| 3. Amphiboly | 8. Equivocation |
| 4. Is-Ought Fallacy | 9. Composition |
| 5. Composition | 10. Amphiboly |

****Practice Problems: Formal and Informal Fallacies**

For each fallacious line of reasoning below, determine which of the two formal fallacies or 20 informal fallacies it most clearly illustrates.

1. We should re-elect Senator Garcia. After living through a tornado, he’s feeling kind of bad recently. He needs encouragement.
2. Pastor Bustle has argued against the new proposal demanding that churches pay property tax. But we can ignore his arguments because he’s a pastor and passage of the bill would cost his church money.
3. The school’s PTA has argued that our students are not reading at acceptable levels due to our teachers using progressive classroom methods. But we can reject the PTA’s position. Students spend a lot of time playing video games and sending text messages on their phones. Technology seems to be taking over everyone’s attention these days. Every bill coming out of Congress seems to support one new-fangled gizmo or another. We really should get back to the good old days where people talked to each other each evening face-to-face.
4. Democracy is the best form of government. From this we can conclude that the rule of centralized authority is not ideal.
5. The claims of alchemy must be true, for scientists like Francis Bacon believed in it.
6. The *Seattle Times* told today of a two Albanians who rescued a drowning child from a raging river. Albanians must be a courageous people.
7. Mid-level management has requested a water cooler in their lounge. But if we give them that, they’ll next want full kitchen access. Then it will be their own gym, then an entire building devoted to them. We can’t afford to purchase a new building, so we must not give them that water cooler.
8. Kale is a leafy green that grows easily in Washington. Poison hemlock is a leaf green that grows easily in Washington. Kale is nutritious for humans. Thus poison hemlock is likely nutritious for humans, too.
9. No one has understood the complete nature of God. Thus no one can understand God’s complete nature.

10. The immorality of a society brings on political collapse. Thus, since the U.S. is acting immorally these days, our country will soon undergo collapse.
11. An elephant is an animal. Hence, a small elephant is a small animal.
12. Either our country gets on its knees and asks God for forgiveness, or our political and social systems will collapse. Surely we don't want collapse! Thus get on your knees and pray!
13. My friend says that I should vote Democrat in the next election. But she's a granola-eating nudist, and an environmental extremist. Thus her arguments for voting for Democrats are faulty.
14. We should do unto others as we would have them do unto us. Since, as a masochist, I like experiencing pain, I ought to inflict pain on others.
15. Senator Smith's assistant says she found a picture of Senator Shine in Smith's desk drawer. But Shine is a grown woman. How on Earth did Shine fit in that drawer?
16. "I saw an old John Wayne Western movie last night. The Wayne character rode a horse quite well. There apparently were quite a few people who could ride horses in the Wild West." Friend's response: "That's a ridiculous way to come to believe something. Movies are often fictional. Thus your belief is false."
17. The factory manager charges me with stealing tools from work. But I've seen him steal office supplies week after week. Thus his claims about me may be rejected.
18. Culture Y believes that sexual activity among unmarried people is morally okay. Thus such activity is morally okay in that culture.
19. Everyone cool at school is wearing ripped jeans. Thus, I too should wear ripped jeans.
20. Every academic department at this college works efficiently as individual units. Thus all the departments work efficiently together.
21. Believer to illogical atheist friend: "The Bible says that the Jews lived in captivity under Egyptian rule." Illogical friend's response: "But the Bible is a highly complex book, with a great deal of metaphor. It's not a reliable way to gain knowledge about history. If you believe the Jews lived in Egypt due to reading it in the Bible, then it must be false that the Jews lived in Egypt."
22. Senator Ima Dresser has stated that people should never be without clothes in public view. Apparently she can't stand the sight of the human body, and would throw into jail any doctors who examine their patients, and would label as sex offenders any actors who performed nude on stage. That's absurd! We can thus reject Dresser's position.
23. Fellow student Roger argued to the teacher that the class would learn the material better if we all were given essay assignments instead of research projects. But Roger is an English major who knows how to write well and easily. Essays would be a piece of cake for him! Thus Roger's arguments are bad.
24. If Senator Smith votes for Senator Barker's gun control bill, Barker votes for Smith's education tax bill. But Smith will not vote for Barker's gun control bill. So, Barker will not vote for Smith's education tax bill.
25. Budweiser is the king of beers! Thus, you should drink Bud!
26. I got food poisoning at Joe's Café last week. I'm eating at a restaurant tonight, so I better prepare for stomach cramps.
27. Tax dodger to IRS: "I know I declared multiple business expenses that never took place, but if I had to pay my full tax bill, I'd not have enough money left over to take my three kids and their grandmother to Disneyland for vacation. And the kids and Granny are so looking forward to that. They cry each night thinking they won't be able to go. Thus the IRS should waive all penalties for my recent tax fraud."

28. Dieticians tell us that if we eat too much fast food, we'll get sick. Let's *not* eat too much fast food, so we'll thereby never get sick!
29. Terminally ill patients should have the right to doctor-assisted euthanasia, because many of them cannot commit suicide on their own.
30. Either you join the U.S. Chess Federation, or you don't like to play chess. But you love to play chess, don't you? The choice is obvious.
31. Different cultures have different beliefs about the morality of gay marriage. Therefore, there is no objectively correct answer to whether or not cultures should allow gay marriage.
32. People are driving like crazy on the highway tonight. Thus there must be a full moon.
33. Logic tutor: "All three students who met with me today were confused about validity. Therefore, everyone at Bellevue College is confused about validity."
34. If Al likes apples, then Barry likes bananas. Thus, since Barry likes bananas, Al likes apples.
35. The Grand Wizard of the Ku Klux Klan says we ought to reject Senator Barker's affirmative action programs. On the basis of the Wizard's understanding on such matters, we should reject Barker's proposals.
36. Cows are similar to lions. Both have hair, have four legs, have a tail, and give birth to live young. Thus, because cows are vegetarians, lions are, too.
37. It seems like every time I step into my bathtub, the phone rings immediately afterwards. I'm kind of lonely tonight, so I think I'll take a bath.
38. Bob says that he's never mistaken and that we should always believe him. Well, I guess that's that. Since he's never mistaken, we can trust what he says here.
39. We should never lie. Thus when your friend asks what you think of her particularly ugly tattoo, you should state your true opinion openly and clearly.
40. Google is an efficient company. Thus Bert, one of Google's marketing managers, is an efficient Google employee.

Answers:

- | | |
|--|--|
| 1. Appeal to Pity (& Is-Ought F.) | 21. Genetic Fallacy |
| 2. <i>Ad Hominem</i> (circumstantial) | 22. Straw Man |
| 3. Red Herring | 23. <i>Ad Hominem</i> (circumstantial) |
| 4. Begging the Question | 24. Denying the Antecedent |
| 5. Weak Authority | 25. Appeal to the People (& Is-Ought F.) |
| 6. Hasty Generalization | 26. Weak Analogy |
| 7. Slippery Slope | 27. Appeal to Pity (& Is-Ought F.) |
| 8. Weak Analogy | 28. Denying the Antecedent |
| 9. Appeal to Ignorance | 29. Begging the Question (& Is-Ought F.) |
| 10. False Cause (<i>non causa pro causa</i>) | 30. False Dichotomy |
| 11. Equivocation | 31. Is-Ought Fallacy |
| 12. False Dichotomy | 32. False Cause (<i>non causa pro causa</i>) |
| 13. <i>Ad Hominem</i> (abusive) | 33. Hasty Generalization |
| 14. Accident | 34. Affirming the Consequent |
| 15. Amphiboly | 35. Weak Authority |
| 16. Genetic Fallacy | 36. Weak Analogy |
| 17. <i>Ad Hominem</i> (<i>tu quoque</i>) | 37. False Cause (post hoc ergo propter hoc) |
| 18. Is-Ought Fallacy | 38. Begging the Question |

19. Appeal to the People (& Is-Ought F.) 39. Accident
20. Composition 40. Division

Chapter 8: Arguments from Analogy

We use and hear arguments from analogy every day. They can be so trustworthy that we'll bet our lives on them. Or, they can be horribly, unredeemably bad.

Arguments from analogy compare two things, or two groups of things. After noting that the two things are relevantly similar, we conclude that what is known to be true of one is probably true of the other. For instance, let's imagine that I have three modern jazz CDs featuring Miles Davis: *Cookin' with the Miles Davis Quintet*, *Relaxin' with the Miles Davis Quintet*, and *Workin' with the Miles Davis Quintet*. I see that there's another Miles Davis CD available: *Steamin' with the Miles Davis Quintet*. The three I presently own all feature Miles Davis as lead musician playing trumpet. The fourth CD also features Miles Davis as lead musician playing trumpet. I like the music on the three CDs I have, so I conclude that I'll probably like the music on *Steamin' with the Miles Davis Quintet*, too.

Every time you buy a CD or download music from your favorite musician, you are arguing from analogy. Every time you go to your favorite restaurant, or buy your favorite candy bar, or buy a book by your favorite author, or date your favorite girl- or boy-friend, you are arguing from analogy. Are you guaranteed that tonight's date will go as well as the others? No, but this is inductive reasoning, and deductive certainty will never be quite within reach. Still, you pony up some money, put your ego on the line, and go on that date expecting to have about as good a time as you did previously.

In fact, every time you get in your car and drive to the store, you bet your life on an argument from analogy. Think about it: you could be struck by a falling meteorite (or a drunk driver) and perish. Yet you get in your car and serenely drive into town. Why? Well, you've driven to the store a thousand times before and have never died in the process. This trip is relevantly similar (i.e., analogous) to the hundreds of others, so you conclude that you will probably not perish on this trip, too. *Quod erat demonstrandum!*

Arguments from analogy will have a distinct structure:

Entity A has characteristics W, X, Y, and Z.

Entity B has characteristics W, X, and Y.

Thus, B probably has Z, too.

Entity A is the *primary analogate* (aka the *primary analogue*). Entity B is the *secondary analogate* (aka the *secondary analogue*). It's about the primary analogate that we know the most; it's to the secondary analogate that the conclusion is directed. In our Miles Davis example above, the three CDs I already own are the primary analogates. The CD I'm thinking of buying is the secondary analogate. W, X, and Y are the *similarities* known to be shared by the primary and secondary analogates. For instance, the group of three CDs I have are each similar to the one I'm thinking of buying, as both the primary and secondary analogates contain modern jazz music, the lead musician is Miles Davis, and Davis is playing trumpet. Z, in our example, is my liking the

music on the first three CDs. I'm thereby inferring that I'll probably like the music on the secondary analogate.

The argument is clearly inductive, as my liking the first three CDs in no way can guarantee that I'll like the fourth. Davis may play poorly on this fourth CD, or he may end up playing a radically different style of jazz, or no jazz at all. He may play nothing but monochromatic Buddhist chants on a harmonica! Still, given the information we have, it looks like a pretty strong argument. I really should buy that fourth CD.

In assessing arguments from analogy, five principles should be considered. For any given argument, it may be the case that it looks strong given consideration of one or two principles, but shamefully weak considering one or more others. Assessing arguments from analogy is more a matter of informed consideration than in cranking out a black-and-white answer. There is also room for debate and for one party in the assessment process to know more than the others, and thus to have more relevant knowledge at hand to make a more informed judgment of the argument's merits. Knowledge is often power in assessing inductive arguments.

Five principles meriting close consideration in assessing arguments from analogy include:

1. Number and relevance of *similarities*
2. Number of *primary analogates*
3. *Diversity* among primary analogates
4. Number, degree, and nature of *differences*
5. *Specificity* of conclusion to premises

Number and Relevance of Similarities

All else being equal (*ceteris paribus*), the more relevant similarities there are between the primary and secondary analogates, the stronger the argument is. The similarities need to be relevant to the conclusion, though. Let's take the Miles Davis argument above as our base example. We can add a couple *irrelevant* similarities and we end up inferring the conclusion with no more confidence. For instance:

I have three modern jazz CDs featuring Miles Davis: *Cookin' with the Miles Davis Quintet*, *Relaxin' with the Miles Davis Quintet*, and *Workin' with the Miles Davis Quintet*. I see that there's another Miles Davis CD available: *Steamin' with the Miles Davis Quintet*. The three I presently own all feature Miles Davis as lead musician playing trumpet. They also have a picture of a sailboat on the CD cover and the sailboat is painted red. The fourth CD also features Miles Davis as lead musician playing trumpet, and it has a picture of a red sailboat on its cover, too. I like the music on the three CDs I have, so I conclude that I'll probably like the music on *Steamin' with the Miles Davis Quintet*, too.

This revised argument is no stronger than the original one, because the added similarities (pictures of a sailboat that is red) are not relevant to my enjoyment of the music. Consider, however, if we left out the appeal to the red sailboat pictures and noted that John Coltrane played

saxophone and Red Gardner played piano on my three CDs, and that all three records were made on the same two weekends in the late 1950s, and all that is true also of *Steamin' with the Miles Davis Quintet*. Now the argument is much stronger, as I have additional *relevant* reasons for thinking that my original three CDs are very much like the fourth one I'm thinking of buying. Any musician's backup band will have a profound effect on the quality of music he or she produces, and Coltrane and Gardner are my favorite saxophone and piano players respectively. That the fourth record (now a CD) was recorded at the same time as the three I presently have tells me that Davis will likely be playing the same style of jazz on *Steamin' with the Miles Davis Quintet*.

Number of Primary Analogates

All else being equal, the more primary analogates there are, the stronger our argument from analogy. If I've eaten at Joe's Café ten times and I've always liked the food, I have pretty good reason to think I'll like the food if I go to Joe's for dinner tonight. If I've eaten at Joe's 20 times in the past and always liked the food, my argument is stronger. If I've eaten there 100 times and always liked the food, I'd say that it's almost (but not quite) guaranteed that I'll like the food there tonight.

We can change our original arguments to say I presently have ten (not merely three) CDs by Miles Davis, and I like them all. Thus I'll like the next Miles Davis CD I buy. If that's the only change to the original argument (i.e., all else is equal), I now have stronger reason to buy that new CD. If I presently have over 40 Miles Davis CDs, and I like them all, then I'd have even *stronger* reason to believe I'd like that additional CD.

Another potential concern related to this first principle is that the primary analogates might not *all* point to the conclusion. Assume, for example, that I have 40 Miles Davis CDs, and like 30 of them. This still gives me good reason believe that I'll like the next Miles Davis CD I buy, as I like most of those I presently own, but my new argument is not as strong as when I had 40 Davis CDs and I liked each of them. So, the higher the percentage of primary analogates favorably pointing to the conclusion, the stronger the argument will be.

Diversity among Primary Analogates

All else being equal (which seems to be a catchphrase here), the greater the diversity among the primary analogates, the stronger the argument is likely to be. If the primary analogates consist of only one kind of thing, and they are all pretty much the same, then that can tell you something about another thing (i.e., the secondary analogate) as long as it's relevantly similar to the first group. But if the first group has a mix of character traits, that can strengthen the argument. For instance, consider once again our original Miles Davis CD argument. Let's make one change: we'll diversify the primary analogates (in ways that admittedly do not accurately represent Davis's recordings from this period). They'll continue to share the same characteristics with *Steamin' with the Miles Davis Quintet*, but we now also learn that one CD has Davis playing electric trumpet while he's playing acoustic trumpet in the other two. In one CD he's playing bebop, in the second he's playing a cool style, and in the third he's fusing jazz with rock. In one

he has a full orchestra playing an opera with the band, in another he incorporates hip hop musicians, and in the third he includes classical music from Spain. And I like each CD! I can now conclude with greater confidence than before that I'll probably like a fourth Miles Davis CD, since it appears that I like his music no matter what he does.

Number, Degree, and Nature of Differences

Most of us have heard the criticism, "Oh, that's just comparing apples to oranges." The critic is here claiming that someone else—who has just offered an argument from analogy—is comparing two relevantly different things, claiming that they are similar, and that since something is true of one thing, it is thereby likely true of the other. *But*, our critic notes, the two analogates are quite different in relevant ways. What's true of one is *not* thereby likely to be true of the other.

For instance, aspirin is a drug, and so is heroin. They are similar in that both cost money, and both are available in the USA. Thus—we might weakly argue—since aspirin is safe for most people to take, so too is heroin. That's a ridiculous inference because aspirin and heroin are relevantly different. One is a mild pain reliever that has no serious side effects for most people, while heroin can cause serious harm if taken in too large a quantity and it is highly addictive. The arguer here is indeed comparing "apples to oranges."

Often—all else being equal—the more relevant differences between the primary and secondary analogates, the weaker the argument. Consider again our original Miles Davis argument. Let's imagine, though, that we find out that for the CD I am thinking of buying (*Steamin' with the Miles Davis Quintet*) Davis decided to feature the didgeridoo and a set of gongs. The first three CDs I already have use no such instruments, and are limited to the trumpet, saxophone, bass, piano, and drums one often finds in a mid-1950s modern jazz quintet. The change in musical instruments with *Steamin' with the Miles Davis Quintet* thus makes it relevantly different from the first three, and I no longer am as confident in my liking it as much as I like the ones I already have. I may like the fourth CD *more*, but that's up for question. The emergence of that *question* is what makes the conclusion less likely, and thus the argument weaker.

Of course, some differences are more relevant and critical to the conclusion than others, so the degree of difference can play a role in determining how strong or weak an argument from analogy is. If the new information learned above was simply that the band had played at night during the recording of the first three CDs I own, but they played during mid-day for the recording of *Steamin' with the Miles Davis Quintet*, that difference is far less relevant than the change in musical instruments, so it would weaken the argument less. Still, the time of day jazz musicians play *may* impact their style or readiness to play well, so it might be relevant, and thus my conclusion is not quite as strong as it was in the original argument.

Sometimes a difference can strengthen an argument from analogy. Imagine that after presenting the original Davis argument, I find out that for the first three CDs the band did not care too much about the quality of the music (perhaps because they were fighting with the producer), but put all their effort into the fourth CD. This is likely a relevant difference, and in this case, it may give me stronger reason to believe the conclusion (i.e., that I'll like the music on the fourth CD).

If you are hearing an argument from analogy, and you get the sense that it's weak, finding relevant differences (or "disanalogies") is often the most straightforward way of showing the argument to be weak. It's also the way that the befuddled person offering the weak argument will most readily understand. You will be showing that she is guilty of comparing "apples to oranges," which is one way of being guilty of the Weak Analogy fallacy.

Specificity of Conclusion to Premises

It would be nice to have a more clearly informative name for this fifth principle, but "Specificity of Conclusion to Premises" may be the best we can do. The idea is this. Sometimes a set of premises will justify our concluding something similar about the secondary analogate, but we might—if we're not careful—overstate the conclusion. For instance, in our original Miles Davis argument from analogy, I *liked* my three Davis CDs and conclude that I'll probably *like* the fourth Davis CD. If, instead, we had concluded that I would *love* the fourth CD, the argument would be weaker. Merely liking three CDs does not give much reason to believe that I will love another. If we change the conclusion instead to "I will therefore find *Steamin' with the Miles Davis Quintet* to be my favorite CD of all time," the argument is weaker still, as the conclusion is getting harder and harder to hit; it's become too specific given what the premises say.

We could change the original argument and make it stronger by adjusting the conclusion so that it is "easier to hit," that is, to make it less specific. We could conclude, for instance, that "I will thus not be made ill by listening to the fourth CD." Given that I *like* the first three, it's highly improbable that I will be sickened by a fourth similar CD.

We could also adjust the premises and achieve similar results. If we change the premises of the original argument to say that I *love* these three CDs, and conclude as originally that I will probably *like* the fourth one, the argument is stronger than the original. If, on the other hand, the original premises were changed to say that the three CDs I now have do not make me sick, and leave the conclusion as originally stated, the argument is weaker, as the conclusion is now made more specific in relation to the premises.

Argument Assessment

Let's now consider an argument from analogy, and see where its strengths and weaknesses (if any) lie. Consider the following argument:

Martin Luther King, Jr. and Mahatma Gandhi were activists struggling for social change. So too was Che Guevara. King and Gandhi encouraged non-violence. Thus, Guevara probably encouraged non-violence, too.

Note that King and Gandhi are the primary analogates, and Guevara is the secondary analogate. Let's consider each of the five principles and see what we wish to say about this argument.

(1) The only similarities appealed to here are being an activist and struggling for social change. We may happen to know that all three men were active in the 20th century, but that similarity is not appealed to here, and if that were relevant, we might help the arguer by providing this added premise. But we'll not consider it here. Noting that the King/Gandhi pair and Guevara are all men is probably an irrelevant similarity. Given that there are many activist ways of trying to achieve social change, that King and Gandhi advocated non-violence does not give us much reason to believe that Guevara did, too. We'd need to see some more relevant similarities to feel confident about this conclusion. The argument is looking fairly weak so far.

(2) There are only two primary analogates (King and Gandhi). That's not many, although that number might do if the argument meets the other demands of an argument from analogy well. The low number certainly is not helping the argument. It's still looking weak!

(3) Given that we have only two primary analogates, it's hard to have much diversity among them. Still, King is American; Gandhi is Indian. King fought against American racist segregation policies, while Gandhi fought against British control of India. King was active in the 1960s, Gandhi in the 1920s-1940s. All else being equal, the amount of diversity is not bad, but this argument still looks weak. It's not weak *due* to lack of diversity, which would be the case—to an extent—if we appealed to Fred Shuttlesworth (a compatriot of King's in the non-violent fight against segregation) instead of Gandhi.

(4) There are many irrelevant differences that are not worth bringing up: King was American, Gandhi was Indian, and Guevara was born an Argentinean; the King/Gandhi pair are best known for their struggle in the USA and India respectively, while Guevara is best known for his struggle in Cuba. A *relevant* difference is that King and Gandhi were advancing social change under the influence of the Christian and Jain ethic of non-violence, while Guevara was advancing social change under Marxist philosophy. Since Marxism makes far greater allowances for the use of violence in moving societies toward historical change, we have little reason to believe that because King and Gandhi advocated non-violence, that Guevara did, too. This argument is looking really weak now!

(5) The conclusion is worded as a thoughtful person might expect. King and Gandhi are said to advocate non-violence, and so too is Guevara. That's better than concluding that Guevara *demand*ed non-violence. And it's worse than concluding that Guevara didn't mind non-violence. The claim of the conclusion in our original King/Gandhi/Guevara argument is appropriately specific given the premises. All else being equal, this argument would look okay on that score.

So, what do we make of this argument? It's clearly weak, as the similarities are few, the number of primary analogates are few and not very diverse, and there is at least one major relevant difference between the King/Gandhi pair and Guevara. Che Guevara may have been an advocate for peace and non-violent means (he was not), but this argument does little to support that position.

****Practice Problems: Assessing Arguments from Analogy**

Consider the two arguments from analogy below. Then examine the following individual changes to the argument. For each sequential change, determine if it makes the argument stronger (S), weaker (W), or leaves it unchanged (U). Then determine which of the five principles (as numbered above) you are most clearly considering in making that first determination (1-5). For each change, assume that the changes preceding it still apply (unless the latest change changes that earlier change).

I. Tiago has taken three classes at Bellevue College, and liked each one of them. He just enrolled in Social Philosophy at Bellevue College, and expects that he will like it, too.

1. The three classes Tiago has taken are Accounting, Calculus, and Statistics.
2. The conclusion is changed to “he will love it.”
3. All four classes are taught by Professor Kim.
4. The previous three classes were held in the evening when Tiago is at his best, while the fourth class will be held at 7:30 am.
5. The first three classes had an average of 35 students in them, while the fourth class will have 32 students in it.
6. We change the premises to say that the first three classes were the highpoint of Tiago’s life, up to then.
7. Tiago has taken *five* classes at Bellevue College, the other two being Algebra and Symbolic Logic. All five classes were high points in Tiago’s life.
8. Those two additional classes were actually English 101 and Introduction to Philosophy.
9. Tiago functions poorly in the evening, and excels in the morning.
10. Tiago has a crush on Maria and has been trying to impress her with his academic skills. She was in all five of his previous classes, and she’s enrolled in his next class.
11. Maria has just told Tiago that she thinks he’s a weirdo and has gotten a court injunction to keep him from speaking to her.
12. Tiago loves to read and write essays, and has been taking the math-related classes at Bellevue College so far only because his father demanded that he become a mathematician. Since his father has recently run off to Bora-Bora with Maria, Tiago is now free to take what classes he likes.
13. In Social Philosophy, Professor Kim will require students to read a newspaper editorial presenting a debate between Senators Sunny Shine and Ima Dresser.
14. Pastor Bustle plans to work with Rabbi Maimonides to protest Professor Kim’s use of the Shine-Dresser editorial in a publicly-funded college. They plan to alert the media regarding their protest on campus.
15. Professor Kim has been playing tennis for the past ten years, and he plans to continue playing tennis for the indefinite future.

Answers:

1. W 4 (There’s a huge difference between the math-related classes he’s taken and the Social Philosophy class he recently enrolled in.)
2. W 5 (If he only *liked* the previous classes, there’s no good reason given for him to *love* a fourth class.)

3. S 1 (All else being equal, the teacher adds a relevant similarity and makes the argument stronger than it was.)
4. W 4 (The time of day is often quite relevant to whether a student will enjoy a class or not.)
5. U 4 (It's a difference, but it's so minor as to be irrelevant.)
6. S 5 (The premises now make the conclusion less specific—and easier to “hit.”)
7. S 2 (We now have five primary analogates instead of three; the argument is thus now a little stronger.)
8. S 3 (We are now starting to get some diversity in the kind of classes Tiago took. They are no longer just symbolic or math-oriented.)
9. S 5 (If Tiago did well in the evening, even though he's a morning person, he should do even better in the next early-morning Social Philosophy class.)
10. S 1 (Well, if Tiago has blood in his veins, this may very well be an additional relevant similarity.)
11. W 4 (*¡Que lastima!* This seems to be a relevant difference that may likely put a cramp in Tiago's educational motivations or his ability to enjoy the next class. We certainly have less reason to be confident in the conclusion than we did after the last change.)
12. S 4 (The difference that before seemed to weaken the inference now appears to strengthen it. Tiago is finally getting to take another kind of class he's always liked.)
13. U 4 (Unless all the other classes read a similar article—which is highly unlikely—this is a difference, but it does not appear to be relevant enough to noticeably change the likelihood of the conclusion being true or false.)
14. W 4 (Again, we've got a difference, but this time it's unclear if this will make Tiago's next class more or less enjoyable for him. Since we don't know if he'll welcome or not the ruckus caused by the protest, we are thus less sure about the conclusion. That new uncertainty on our part lets us know the argument is now a little weaker.)
15. U 1 (It's an additional similarity, but it's probably irrelevant to the conclusion.)

II. Mary is thinking of buying a car from Hal's Used Cars. Her friend Tom bought a car from Hal's, and it ran well for over two years. Mary concludes that if she buys a car from Hal's Used Cars, that her car will also run well for at least two years.

1. Mary has three other friends who purchased cars from Hal's Used Cars, and their cars have run well for over two years.
2. One of the friends bought a sedan; one bought a station wagon; the third bought an SUV; while Tom bought a hatchback.
3. Mary's four friends are avid amateur car mechanics, while Mary knows little about cars.
4. Mary adjusts her conclusion to say merely that her new car will likely not break down within a month of purchase.
5. Mary changes her conclusion again to say that her car will last her lifetime and will run better than any of her friends' cars.
6. Mary notices that her four friends bought their cars from Hal's on sunny days, and she plans on buying hers from Hal's on a sunny day, also.
7. Mary notices that all the cars bought from Hal's Used Cars by her four friends had been worked on by Mike, the experienced chief mechanic at Hal's. Mary's car will be worked on by Mike, too.

8. Mary finds out that her four friends regularly change their car oil, and she is determined to do so, too.
9. Mary's four friends live in Los Angeles, California, whereas she lives in Hollywood, California.
10. Unlike her friends, Mary plans to use her car in extensive off-road travel touring Baja California, Mexico.
11. Mary is an exotic dancer, while her friends are all computer programmers.
12. Mary plans to change the oil in her car every 2000 miles, while her friends did so only once per 50,000 miles.

Answers:

- | | | |
|-------|-------|--------|
| 1. S2 | 5. W5 | 9. U1 |
| 2. S3 | 6. U1 | 10. W4 |
| 3. W4 | 7. S1 | 11. U4 |
| 4. S5 | 8. S1 | 12. S4 |

Arguments from Analogy and Moral Reasoning

Given the current fad of moral relativism, it sometimes comes as a surprise to students and to many of their college instructors that we argue for moral claims all the time, and think we're pretty good at it. When someone does us wrong, we immediately jump at the chance to explain to others why they should believe we've been grievously wronged and that the scallywag who wronged us should suffer the woes of hellish perdition. For instance, if your normally beneficent logic teacher was to give an unanticipated 15-page, in-class essay as your upcoming logic test, and required for the essay that it be written in Sanskrit as a proof for Heisenberg's Uncertainty Principle, you'd likely struggle and get an F. Upon receiving that grade and hearing from your teacher that you should buck up and recognize that "Life throws you a curve every now and then," you'd probably walk over to the college Dean of Instruction and argue as follows: "My logic teacher said our next test was going to be on arguments from analogy, but she assigned us a long essay on sub-atomic physics instead, and demanded that we write it in Sanskrit! This is not fair, and tests ought to be fair. She ought to have given us a normal first-year test on arguments from analogy, and you thus ought to compel her to give us such a test instead of this Sanskrit essay."

Some such argument will probably prevail over the usually-rational deans of your institution. Why? Because it's a pretty strong argument, and your deans are usually rational. Some moral arguments are weak (or invalid), of course, but others can be quite strong (or valid). The point is, morality need not be considered "simply a matter of opinion" with no grounding in the world's state of affairs; and few people consistently believe it to be so, especially when handed an unfair F on a test. One common way to argue about ethical claims is with arguments from analogy, and understanding how such arguments work helps us divide much of moral reasoning into that meriting acceptance and that warranting dismissal.

For instance, consider the debate surrounding euthanasia. A loved one is undeniably in the last stages of a terminal illness, and is in un-relievable pain. This person has expressed a clear wish

to die, and asks for assistance in doing so. May the doctor or friend at hand take active steps to end the life of this person? It's a tragic situation, but unfortunately far too common. We rightly believe that it is at least usually wrong to kill a fellow human, but to let a someone suffer horribly for no apparent good seems cruel (and perhaps an example of the informal fallacy of Accident). Argument from analogy is often used to assist in the complex conversation attempting to resolve this quandary.

We might hear a sad but thoughtful proponent of euthanasia argue as follows: "If you were placed in charge of your neighbor's beloved pet dog while the owner was on vacation, and the dog is hit by a car, severely injured beyond all medical hope of assistance, and the dog is writhing in pain, surely you'd have the dog put to sleep as gently and quickly as possible. That's what the owner would want. If we may do that for a dog, surely euthanasia may be an option for humans, too."

An equally sad and thoughtful friend might respond: "But humans are not dogs; we have rationality, a soul, and we are put here on Earth by God to care for one another...not to decide who will live or die. No, though we may euthanize dogs, we may not take the life of terminally ill humans, even when their pain cannot be relieved and they *wish* to die."

Both persons here are arguing from analogy. The first person sees enough similarities between dogs and people (they each can be terminally ill, live with pain that cannot be relieved, and can have people nearby who can gently but purposefully end their lives or step back and "passively" withhold aid and allow them to die). Given that most people in the situation of watching after a neighbor's dog would opt to euthanize the pet, the arguer believes that we'll draw the conclusion that euthanasia can be a morally viable option for humans, too. The number of primary analogates is vast, as the number of potential people watching after neighbor pets is huge. Moreover, the case is clearly not limited to dogs: neighbors' cats, rats, guinea pigs, and a diverse array of other animals serve as beloved pets. The conclusion, moreover, is no more or less specific than we'd expect, especially if the arguer simply concludes that euthanasia is an option that may be considered. ("Thus euthanasia must be performed in *every* case involving human suffering and terminal illness" would be far too specific given what the premises say.)

The friend responds with a challenge to the first person's argument from analogy. He holds that there is a relevant difference between dogs and humans. He points to humans having rationality, a soul, and a particular mandate from God as evidence for there being such a relevant difference. Thus, he concludes, comparing dogs to humans is like comparing apples to oranges; they are relevantly different things, and we cannot argue that because it's morally permissible (and perhaps obligatory) to euthanize pets in certain tragic circumstances, it follows that it is morally permissible to euthanize humans in such circumstances.

Which line of reasoning seems more compelling? We can imagine the first person responding in turn, "Yes, there is a relevant difference between dogs and humans, but that makes my analogy even stronger. If it is permissible to take the life of an animal that a human loves greatly—and in no small part *because* that human loves his pet so much—how much greater our responsibility to euthanize a fellow human (only under analogous or stricter circumstances, of course) who is

loved by God even more than the pet is loved by the neighbor? If the neighbor returned home to hear about his dog's car accident, and found his dog in great pain over the last two days, the neighbor would ask, 'Why didn't you put my dog to sleep!?' If you wish to get theological here, what would your God think if He came to you after two days of your allowing one of his beloved to suffer terribly? Would you say, 'I did so because I did not wish to "play God"?' If God loves humans more than neighbors love their pets, I'd think God would be pretty upset with you."

The argument could go on, but perhaps we can see how arguments from analogy and the principles that guide them can play a role in moral reasoning. Consider also the issue of the use of animals in testing for health risks of new cosmetic products. ACME Company wants to sell a new shade of red lipstick (which the world *knows* we desperately need). ACME doesn't want to be sued by humans who use it and have their lips fall off, so they test it on 10,000 rats first. After smearing the lipstick on the rats' exposed skin, only an insignificantly small number appeared to get sick from it. ACME concludes that the lipstick is likely safe for human use.

ACME's line of reasoning will work only if rats respond to the lipstick in a similar (i.e., analogous) way as would humans. If they did not, any results from testing on rats would be irrelevant to whether or not the product is safe for humans. ACME must think rats and humans are relevantly similar. "But wait," the pro-animal activist might say, "if it's morally wrong to test this lipstick on humans due to the possible harm it might do them, *and* rats and humans are relevantly similar, it should be morally wrong to test the lipstick on rats, too. Since either the rats are similar to humans or they're not." Sometimes logic can make your head hurt. If you were a logician listening in on this conversation, how would you help clarify the use of argument from analogy here?

Chapter 9: Categorical Patterns

The ancient Greek philosopher Aristotle (384-322 BC) is the first we know of to place in writing a detailed analysis of what we call categorical logic. Logicians may have been doing similar work in India a century or two earlier, but we have no written record to verify the date.

Categorical logic is the study of arguments made up of categorical statements. Such arguments include categorical syllogisms (with two premises), like this one:

All dogs are mammals.

Some dogs are pets.

Therefore, certainly at least some mammals are pets.

They also include immediate inferences (with one premise) like the following:

Some fish are sharks.

Thus, some sharks are fish.

In the first argument the terms *dogs*, *mammals*, and *pets* refer to categories of things, while the words *all* and *some* specify the quantity of things in those categories. The first premise of this argument indicates that all the members of the dog category belong to the category of mammals; the second premise says that some members of the dog category belong to the pet category, and so on. Similar analysis applies to the second argument.

As an aside, we could be a little bit more precise here. A sentence is a grammatically correct string of words, like “Juan is from Spain.” This *sentence* has 15 letters in it, and begins with a capital *J*. The *statement* is what is claimed; in this case it’s claimed that a particular guy named Juan is from the country of Spain. Sentences are thus often used to convey the meaning behind a statement. Statements are—by definition—either true or false, that is, they have a *truth value* of True or False. We may disagree on the truth value of a statement, or we may be unable to determine with any confidence what the truth value of a given statement is, but—given a specified meaning, context, and perspective—it will be either true or false, and never both and never neither. Questions, commands, suggestions, or exclamations are examples of sentences that are not statements. Although statements are true or false, it would make little sense to say that a *statement* has 15 letters in it. All that said, we will often use the words *statement* and *sentence* interchangeably in a somewhat loose fashion.

Categorical Statement Forms

A *categorical statement* is any claim that all or some of one specified category of things belongs to (or do not belong to) a second category of things. Of course, some statements are true, and some are false, but either way they declare that something is the case. For example, “All cats are mammals” is a true categorical statement since it says that all the members of the category of cats belong to the category of mammals. “Some black mice are not rodents” is a false categorical statement because it erroneously claims that at least one (in Logic, “some” means one or more)

black mouse is not a rodent. So, “All cats are mammals” has a truth value of True, while “Some black mice are not rodents” has a truth value of False.

There are four basic forms of categorical statements. Examples of each include:

All dogs are animals.

No dogs are animals.

Some dogs are animals.

Some dogs are not animals.

Each statement consists of four parts:

quantifier—subject term—copula—predicate term

The quantifier always comes first, followed by a subject term made up of a word or phrase picking out a class of things. Next comes the copula. The predicate term invariably comes last. A categorical statement contains nothing else. It’s a nice, clean, consistent pattern.

The *quantifier* (*All*, *No*, or *Some*) tells us the quantity or number of things the statement is talking about (e.g., dogs). The words *are* and *are not* is each called a *copula* because it joins, or “copulates,” the subject term with the predicate term. In this case the first statement’s quantifier (*All*) teams up with the copula (*are*) to assert that all the members of the subject category belong to the predicate category, which is to say, that every dog is included within the animal category. An *A* statement thus always claims that every member of a certain class is part of another (predicate) class. (Logicians *do* seem to have a way of making the simple complex, don’t they?)

The subject term of a statement indicates what the statement is about. A *term* is a word or phrase (usually a plural noun) that picks out a group of things. A *category* or *class* is a group of things that have a specified characteristic in common. For example, the category of dogs consists of all things that have in common the characteristic of being a dog. The *subject term* in all four examples above is “dogs” and refers to a category or class of things: dogs. The *predicate term* (“animals”) also denotes a category or class—in this case, animals. If the subject term tells us what the statement is about (e.g., dogs), the predicate term tells us what we are to know about them (e.g., that they are or are not animals). A term may consist of more than one word. For instance, “black cats” is one term, as is “dogs that bark all night long and leave ghastly presents on my doorstep each morning.”

So, for the categorical statement, “Some large dogs are golden retrievers that bark incessantly,” the quantifier is “Some,” the subject term is “large dogs,” the copula is “are,” and the predicate term is “golden retrievers that bark incessantly.”

****Practice Problems: Parts of Categorical Statements**

For each of the following categorical statements, state its (a) quantifier, (b) subject term, (c) copula, (d) predicate term, and (e) truth value.

1. Some dogs are poodles.
2. No peacocks are bright fish.
3. All black bears are polar bears.
4. Some reptiles are not lizards.
5. No dogs that do not bark much are animals that are feline.
6. Some animals that are fast are not cats that are not purple.
7. All fully green parrots are birds that are not white.
8. Some green beans are not vegetables.
9. Some logicians are male professors.
10. No college students are people who are biology majors.

Answers:

1. (a) Some, (b) dogs, (c) are, (d) poodles, (e) True
2. (a) No, (b) peacocks, (c) are, (d) bright fish, (e) True
3. (a) All, (b) black bears, (c) are, (d) polar bears, (e) False
4. (a) Some, (b) reptiles, (c) are not, (d) lizards, (e) True
5. (a) No, (b) dogs that do no bark much, (c) are, (d) animals that are feline, (e) True
6. (a) Some, (b) animals that are fast, (c) are not, (d) cats that are not purple, (e) True
7. (a) All, (b) fully green parrots, (c) are, (d) birds that are not white, (e) True
8. (a) Some, (b) green beans, (c) are not, (d) vegetables, (e) False
9. (a) Some, (b) logicians, (c) are, (d) male professors, (e) True
10. (a) No, (b) college students, (c) are, (d) people who are biology majors, (e) False

Standard Form Categorical Statements

Aristotle believed that we could take any declarative sentence and translate it into a standard form categorical statement. He was not quite correct in this broad claim, but he was right to think that it's true in many cases. We will say here that a categorical statement is in standard form if it matches precisely one of the four categorical patterns below (where *S* is the subject term and *P* is the predicate term):

All *S* are *P*
 No *S* are *P*
 Some *S* are *P*
 Some *S* are not *P*

Here are examples of non-standard form statements changed into equivalent categorical statements:

Non-standard: Dogs are animals. [There is no quantifier.]
 Standard: All dogs are animals.

Non-standard: Some cats are black. [“Black” is not a term; it’s an adjective.]
 Standard: Some cats are black things. [“Black things” is a term picking out a group of things.]

Non-standard: A few fish are large animals.

Standard: Some fish are large animals. [“Some” is a standard quantifier; “A few” is not.]

Non-standard: All birds are not reptiles. [“All S are *not* P” is not a standard form.]

Standard: No birds are reptiles.

Non-standard: A salmon is a fish. [There is no quantifier.]

Standard: All salmon are fish.

Non-standard: Mark is tall.

Standard: All persons identical to Mark are tall persons.

Non-standard: Only males are boys.

Standard: All boys are males.

Non-standard: If something is a dog, then it’s a mammal.

Standard: All dogs are mammals.

Non-standard: If something is a parrot, then it’s not a fish.

Standard: No parrots are fish.

Non-standard: Bob is nowhere to be found.

Standard: No places are places in which Bob is found.

The ancient Romans considered declarative statements that did not fit the patterns of standard form categorical statements (we’ll study some of these statements in the next chapter). For instance, “All cats are either animals or rocks,” “Bob ate pasta or a hamburger,” “If Sara is a mother, then she is a wife,” “It is false that pizza is a basic food group,” and “The beer tastes good if and only if it’s cold” do not fit into categorical patterns in any clean, straightforward way. Still, many ordinary declarative sentences can be turned into categorical statements, and once we do that, the arguments that they make up are more easily assessed as valid or invalid.

****Practice Problems: Translating into Standard Categorical Form**

Rewrite the following English claims into the standard categorical form of an A, E, I, or O statement.

1. Every cat is an animal.
2. All birds are not fish.
3. A few mice are rodents.
4. Most ducks are birds.
5. All horses are fast.
6. Some rich realtors are Republicans.
7. Parrots are birds.
8. Only mammals are pigs.
9. None but animals are llamas.

10. Julio is a logic teacher.
11. Beijing is in China.
12. Most fish swim.
13. I do not love Molly.
14. Some animals live in caves.
15. Bob always wears a hat.
16. Whoever studies will do well on the test.
17. A dog is not a cat.
18. If something is a goose, then it's a bird.
19. Love is everywhere.
20. The only people who voted for Tran are women.

Answers:

1. All cats are animals. All C are A.
2. No birds are fish. No B are F.
3. Some mice are rodents. Some M are R.
4. Some ducks are birds. Some D are B.
5. All horses are fast things. All H are F.
6. Some wealthy realtors are Republicans. Some W are R.
7. All parrots are birds. All P are B.
8. All pigs are mammals. All P are M.
9. All llamas are animals. All L are A.
10. All people identical to Julio are logic teachers. All J are L.
11. All things identical to Beijing are things in China. All B are C.
12. Some fish are swimmers. Some F are S.
13. No people identical to me are people who love Molly. No M are L.
14. Some animals are things that live in caves. Some A are L.
15. All people identical to Bob are people who wear a hat. All B are W.
16. All people who study are people who will do well on the test. All S are W.
17. No dogs are cats. No D are C.
18. All geese are birds. All G are B.
19. All places are places with love. All P are L.
20. All people who voted for Tran are women. All P are W.

Quantity and Quality

Two further properties of logical forms must be defined to make talking about and using them later easier. Every categorical statement has a *quantity* and a *quality*. The quantity is *universal* if the statement makes a claim about *every* member of the subject-term category, and the quantity is *particular* if the statement makes a claim about *some* members of the category referred to by the subject term. In standard form categorical statements, the universal quantity is expressed by the quantifiers *all* or *no*, while the particular quantity is expressed by the quantifier *some*.

Thus a sentence such as “All cows are mammals” is a universal statement, since it makes a claim about every member of the subject category. “No salmon are fish” is also universal, because it too makes a claim about everything by its subject term, “salmon.” In contrast, statements such as

“Some dogs are poodles” and “Some dogs are not cats” are particular since they talk about only some (i.e., at least one) of the subject category.

Each statement also has one of two *qualities*: *affirmative* or *negative*. A statement is affirmative if and only if it claims that such-and-such *is* true of all or some of the subject class. “All dogs are animals” and “Some dogs are animals” are both affirmative statements. A statement is negative if and only if it claims that such-and-such is not true of all or some of the subject class. “No birds are parrots” and “Some tigers are not horses” are negative statements. Note that *affirmative* is not the same as *positive*. *Affirmative* refers to statements, while *positive* refers to numbers.

It has been customary since Europe’s Medieval Period to name the two affirmative forms A and I (from the first two vowels of the Latin word *affirmo*, meaning “I affirm”), and to label the two negative forms E and O (from the two vowels in the Latin word *nego*, meaning “I deny”).

Name	Quantity / Quality	Standard Form
A	Universal / Affirmative	All S are P
E	Universal / Negative	No S are P
I	Particular / Affirmative	Some S are P
O	Particular / Negative	Some S are not P

****Practice Problems: Characteristics of Categorical Statements**

I. For each of the categorical statements below, determine its name (or type; i.e., A, E, I, or O), quantity, and quality.

1. No salmon are eels.
2. Some antelope are not prairie denizens.
3. All mountain goats are acrobatic animals.
4. Some parakeets that live in cages are birds that do not sing.

Answers:

1. E, universal, negative
2. O, particular, negative
3. A, universal, affirmative
4. I, particular, affirmative

II. Given a statement with the one characteristic provided below, what can be said about that statement’s other two characteristics? The statement is...

1. an E statement.
2. a negative statement.
3. an I statement.
4. an affirmative statement.
5. a universal statement.
6. a particular statement.

7. an A statement.
8. an O statement.

Answers:

1. The statement would be universal and negative.
2. The statement would be E and universal, or O and particular.
3. The statement would be particular and affirmative.
4. The statement would be A and universal, or I and particular.
5. The statement would be A and affirmative, or E and negative.
6. The statement would be I and affirmative, or O and negative.
7. The statement would be universal and affirmative.
8. The statement would be particular and negative.

Existential and Hypothetical Interpretations

Before we look at categorical argument patterns closely, there's an issue we must understand. Universal statements like "All trout are fish" and "No birds are pigs" are ambiguous. It took over 2000 years for logicians to figure this out clearly, but the English logician and mathematician George Boole (1815-1864) pointed out two different meanings behind these A and E statements. (There is no such ambiguity in particular I and O statements, so they are no problem.)

Imagine two friends sitting down at a local watering hole sharing a pitcher of Elysian Brewing Company's Dragonstooth Oatmeal Stout. One sighs and says, "All the dogs in my neighborhood are barkers." What should the other friend think the first is saying? It's obvious, right? Any normal person having a normal conversation would think the weary guy is claiming that (i) dogs exist in his neighborhood, and (ii) they're all barkers. That's why his friend should take pity on him and buy the next round of stout.

Later in the evening, the first guy begins reminiscing about college classes he took, and recalls a course on mythology. "You know," he says, "all unicorns are white creatures." The friend, who also went to college and is both informed and intellectually adept, agrees. But wait! Does this friend think that his buddy is saying that (i) unicorns exist, and (ii) they're all white creatures?? Of course not. Any of us would understand the reminiscing friend as claiming only *one* thing: If there were any unicorns (and he's not saying there are), then they would all be white creatures. We can all agree to that, just as we all agree that all vampires are bloodsuckers (i.e., *if* there were any vampires, then they would all be bloodsuckers).

This second interpretation of A (and E) statements does not assume the things referred to in the subject term exist. In the first case about barking dogs, the interpretation *did* make that *existential* assumption. In everyday conversation, native English speakers bounce from one interpretation to the other without a problem, as we somehow understand people's intended meaning. Logicians have simply made this distinction clear and explained how it can impact an argument.

The first commonsense interpretation goes by various names: the *existential* or *traditional* or *Aristotelian* interpretation. The second interpretation that makes no assumption about the existence of things is often called the *hypothetical* or *modern* or *Boolean* interpretation. For consistency, we'll here use the first option of each.

As we think about the logical relations among categorical statements, we should consider whether the subject terms refer to existing things. Again, particular statements (of the form "Some S are P" or "Some S are not P") have existential import, as they claim to be about existing things. For such statements to be true, their subject terms must refer to existing things. However, for universal statements (i.e., of the form "All S are P" or "No S are P"), we need to consider whether the person making the claim believes that the things referred to by the subject term exist. If we assume that the subject terms of statements refer to existing things, we are giving the statement an *existential* interpretation. If we do not assume that the subject terms refer to existing things, we are giving the statement a *hypothetical* interpretation.

In everyday life we sometimes state a universal affirmative or universal negative statement without presupposing that the subject term refers to existing entities. For instance, a logic teacher may write in her syllabus, "All students who receive an A on each of their tests will receive an A for the course," or stated in more standard form for a categorical statement, "All students who receive an A on each of their tests are students who will receive an A for the course." She means it, but her statement does not presuppose that there will be any students who receive an A on each of their tests, as she understands that the best student for that class may receive all As except for one B+ (which still might result in an A for the course). Another teacher may say, "No late papers are eligible for extra credit." He means it, but his statement does not assume there are, or will be, late papers. So sometimes in everyday communication we make A or E statements when we do not assume that the subject terms refers to things that exist.

That said, in normal, everyday conversation, when we consider A or E statements, we are assuming (consciously or not) that the subject terms refer to things that exist. *Most* of the time when we utter A or E statements, they are about existing things like dogs or pints of ale. This is largely why for so many centuries only the existential viewpoint was considered. And that is why it makes such immediate and intuitive sense to argue that if all dogs are animals, that it must then be the case that some dogs (i.e., at least one dog) are animals.

Fortunately making the existential/hypothetical distinction is relatively easy: If a universal categorical statement's subject term refers to something that either does not exist or that we do not wish to assume to exist, we use the hypothetical interpretation. If a categorical statement's subject term refers to something the arguer believes to exist, then we are warranted in assuming the existential interpretation. So, if an argument is about dogs and pints of ale, then unless we have reason to believe otherwise, we should assume that the arguer knows such things exist and is intending her A or E statements to be understood as having existential import. If an argument is about unicorns, meat-eating vegetarians, square circles, or hypothetical straight-A students, then we should assume that the arguer does not intend to be understood as assuming the things referred to by the premises' subject terms exist.

A set of apparent problems may arise, but are easily dealt with. Sometimes it is challenging to know with any degree of certainty what an arguer intends by an A or E statement. Does she present it intending an existential or hypothetical interpretation? If you are talking face to face with a person, you can ask her: “Are you assuming that the subject terms of your premises refer to existing things?” If she’s willing to answer, you know how to interpret her claims. Also, some benighted people actually believe in unicorns and vampires. Fine; interpret their A or E claims about unicorns and vampires using the existential viewpoint. The argument may turn out to be valid, but the premise in question is false given this interpretation (although your confused friend may think it true). If you do not have the opportunity to determine what the arguer believes regarding the existence of the things referred to by the subject terms of his premises, you are left relying on an understanding of the context. For instance, if an argument is about objects traveling at or near the speed of light, you’d need to know something about massive objects in motion accelerating toward such speeds to know how best to interpret an argument referring to such things. Finally, if you are completely unable to determine if an arguer believes in the existence of the things referred to by the subject terms of the premises, you can always assess the argument using the existential interpretation, and then assess it again using the hypothetical interpretation. If the argument comes out valid only one way, then in charity assume the arguer intended that meaning. If the argument is valid both ways, then its logic is good regardless of the interpretation taken. If it comes out invalid both ways, then the argument is a piece of junk and the arguer should hold his or her head in shame (*and* buy the next round of stout).

****Practice Problems: Existential vs. Hypothetical Interpretations**

For each claim below, are you justified in giving it an existential or a hypothetical interpretation? What would the statement then be saying?

1. All tigers are cats.
2. No vampires are nudist sunbathers.
3. Some animals are amphibians.
4. All presently existing brontosaurus are big animals.
5. No frogs are fish.
6. Some circular rectangles are not geometric figures.

Answers:

1. Existential because tigers exist; (i) tigers exist, and (ii) all of them are cats.
2. Hypothetical because we know that vampires do not exist; if vampires existed, then none of them would be nudist sunbathers.
3. Trick question! This is an I statement, and particular statements are not ambiguous; the clear and unambiguous meaning is: (i) animals exist, and (ii) some of them are amphibians.
4. Hypothetical because no brontosaurus presently exists; if brontosaurus existed presently, then they would all be big animals.
5. Existential because frogs exist; (i) frogs exist, and (ii) none of them are fish.
6. Trick question again! Particular statements are unambiguous, and one does not need to make an existential/hypothetical distinction for them. The meaning of this false statement is clear: (i) circular rectangles exist, and (ii) some are not geometric figures.

Categorical Inference Patterns

There are a handful of basic categorical patterns of inference that are worth knowing. If we are familiar with these few patterns, we can avoid making common invalid inferences, and we can detect easily when someone else is foisting them upon us. In so doing, we'll be more adept at creating and assessing some basic immediate inferences. For now, and until we say we should do otherwise, let's consider only arguments and statements about things that exist. We'll thus be taking the existential interpretation of universal statements (i.e., A and E statements) for now. Later, when we wish to make claims about unicorns or meat-eating vegetarians, we'll need to assume the hypothetical interpretation. But let's put that off for the time being. We will here examine the following categorical patterns:

Contradiction
Contrary
Subcontrary
Subalternation
Obversion
Conversion
Contraposition

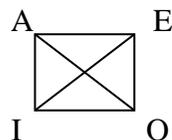
Contradiction

Two statements are said to be contradictory, or are a *contradiction*, if and only if whenever one is true the other is false, and whenever one is false, the other is true. Regarding categorical statements, an A statement will contradict an O version of itself, and vice versa; an E statement will contradict an I version of itself, and vice versa. Let's look at some examples.

All dogs are animals (True) – Some dogs are not animals (False)
All cats are fish (False) – Some cats are not fish (True)
No mice are birds (True) – Some mice are birds (False)
No lions are cats (False) – Some lions are cats (True)

We can be confident that our knowledge of the truth value of any statement can let us know the truth value of its contradictory statement. Thus, if we know that an I statement is false, we can justifiably infer that its contradictory E version is true (because I and E versions of a claim contradict each other, so if the first one is false, the second one must be true).

Aristotle presented these patterns in terms of a "Square of Opposition," a pictorial way of envisioning how Contradiction and other inference patterns work. Imagine a square with an X inside, and each corner representing a categorical statement:



A: All S are P
E: No S are P
I: Some S are P
O: Some S are not P

Each line represents a categorical logic pattern. Contradiction is represented by the X inside the square. A contradicts O, and E contradicts I. Other relations between the square's corners are *not* contradictory, as it's both the case that one being true does not guarantee the other being false *and* one being false does not guarantee that the other is true. For instance, A and I do not contradict each other, as an A statement might be true without the I version being false (e.g., "All dogs are animals" and "Some dogs are animals" are both true); neither do A and E contradict each other, as both might be false (e.g., "All dogs are poodles" and "No dogs are poodles").

Quick note: We will want to say that some statements are false. For the sake of simplicity, let's abbreviate "It is false that all dogs are birds" or "It is not the case that all dogs are birds" as "F: All D are B." There are symbolic ways of abbreviating this (e.g., $\sim(x)(Dx \supset Bx)$), but that gets covered in really cool Symbolic Logic classes (like PHIL& 120 at Bellevue College or the University of Washington). Using *F*: will do the job for us in our non-symbolic Critical Thinking class. We can also use \therefore immediately prior to a conclusion to abbreviate a conclusion indicator word like "thus" or "therefore."

Continuing... Given the trustworthiness of the Contradiction principle, we can recognize many immediate inferences as valid or invalid. For instance, let's consider the following four abbreviated arguments:

(i) All A are B \therefore F: Some A are not B

This is a valid inference. The argument moves from an A statement (said to be true) to its O version (said to be false). A and O are contradictories, so if the A version is true, then we should be able to conclude that the O version is false.

(ii) Some G are W \therefore No G are W

This argument is invalid. The premise is an I statement, and the conclusion is an E version of the same statement. They are thus contradictories. So, if the I statement is supposed to be true, then the E version should be false, but the conclusion claims the E version is true. That's a mistake according to Contradiction. The argument is thus guilty of the formal fallacy known as *Illicit Contradiction*, and it's thereby invalid.

(iii) F: Some K are not P \therefore All K are P

Valid! The inference moves from a false O statement to a true A version of that statement. O and A have a contradictory relationship, so if the first is false, the second needs to be true, which is what the argument claims.

(iv) F: No Q are S \therefore F: Some Q are S

Invalid! The premise is an E statement that is being denied, and the conclusion is the I version of the premise. But, according to Contradiction, E and I versions of the same statement must have

contradictory truth values; so if the E statement making up the premise is false, the concluding I version should be true. But the argument claims the I version is false. It's a screwed up use of Contradiction (i.e., Illicit Contradiction).

One additional point needs to be driven home. For two statements to be contradictory, they must have the same subject and predicate terms. The following two inferences look a bit like contradictions, since we are moving in the first case from an A statement to an O statement, and in the second case from an I statement to an E statement. But the conclusion for each is not the contradictory version of the argument's premise. Note that the terms have been switched. These inferences are thus not direct contradictions. The first argument happens to be invalid, and the second happens to be valid, but it's not due to failing or succeeding respectively in matching the Contradiction pattern.

All A are B ∴ F: Some B are not A
 F: Some U are W ∴ No W are U

****Practice Problems: Contradiction**

Determine for each immediate inference below whether it's a valid or invalid appeal to Contradiction.

1. Some A are not B ∴ F: All A are B
2. F: All N are J ∴ F: Some N are not J
3. No Y are E ∴ F: Some Y are E
4. F: No L are D ∴ Some L are D
5. F: No M are C ∴ F: Some M are C
6. All H are T ∴ F: Some H are not T
7. All X are B ∴ Some X are not B
8. F: Some U are not I ∴ All U are I
9. Some K are Z ∴ F: No K are Z
10. F: Some A are N ∴ No A are N
11. Some S are Y ∴ No S are Y
12. F: Some P are not I ∴ F: All P are I

Answers:

- | | | |
|------------|------------|-------------|
| 1. valid | 5. invalid | 9. valid |
| 2. invalid | 6. valid | 10. valid |
| 3. valid | 7. invalid | 11. invalid |
| 4. valid | 8. valid | 12. invalid |

Contrary

We can now move through the remaining patterns a little more quickly, as we'll use each in analogous ways. The relation between A and E versions of a statement is called *Contrary*. The principle behind Contrary is that if you have an A and an E version of a statement, *at least one of*

them must be false; they cannot both be true. Consider any A statement that you know to be true, and its E version will have to be false. For any E statement known to be true, its A version will be false. For example:

All dogs are animals. Thus, it is false that no dogs are animals.
No mice are fish. Thus, it is false that all mice are fish.

Contrary does *not* say that at least one of an A and E pair must be true, as it is indeed possible (although not always the case) that both may be false. For instance, “All dogs are poodles” and “No dogs are poodles” are both false. However, if we know that an A or E statement is true, then we can infer deductively that its contrary is false. So, the following inferences will be valid:

All A are G \therefore F: No A are G
No H are I \therefore F: All H are I

The following inferences relate two contraries, but they misuse use the principle, or pattern, of Contrary:

F: All J are E \therefore F: No J are E
No U are T \therefore All U are T

Contrary works between A and E versions of a statement, and only tells us that if one is true, the other is false. If we begin knowing that one is false, the other might be true or it might be false; all we know is that at least one of them must be false, but if we know the first is false from the beginning, the second statement’s truth value is undetermined (i.e., we do not have enough information to guarantee its truth value). If we misuse Contrary, we are guilty of the formal fallacy of *Illicit Contrary*.

Note that the following two immediate inferences are *not* examples of Contrary, as the conclusion is not an E (or A) version of the A (or E) premise. For two statements to be contrary to each other, each must have the same subject and predicate terms.

All A are B \therefore F: No B are A
No J are B \therefore F: All B are J

****Practice Problems: Contrary**

Determine for each immediate inference below whether it’s a valid or invalid appeal to Contrary.

1. No J are E \therefore All J are E
2. All K are W \therefore F: No K are W
3. F: All I are R \therefore No I are R
4. All O are D \therefore No O are D
5. F: No M are N \therefore F: All M are N
6. No S are I \therefore F: All S are I

7. F: All I are W ∴ F: No I are W
 8. F: No Y are D ∴ All Y are D

Answers:

- | | |
|------------|------------|
| 1. invalid | 5. invalid |
| 2. valid | 6. valid |
| 3. invalid | 7. invalid |
| 4. invalid | 8. invalid |

Subcontrary

The *Subcontrary* pattern is found at the bottom of Aristotle’s Square of Opposition, on the line between I and O statements. Note the prefix “sub,” and think “*submarine*,” a ship that moves underwater and below its surface. It’s underneath Contrary. For Subcontrary, *at least one of the two statements must be true*; they cannot both be false. For example, if “Some dogs are fish” is false, then the O version of that statement must be true: “Some dogs are not fish.” Or consider “Some cats are animals” and “Some cats are not animals.” The first is true, and the second is false. There is no way both could be false.

So, if we know an I or O statement is false, we can infer that its subcontrary (the O or I version of that statement respectively) is true. If we begin by knowing that the I or O statement is true, we cannot infer the truth value of its subcontrary, as it’s undetermined; we already know one of the pair is true, so the Subcontrary pattern is fulfilled: at least one is true. It would be a fallacious guess on our part to infer that the other is true, too. Such a misuse of Subcontrary is called *Illicit Subcontrary*.

Here are two more examples of valid inferences appealing to Subcontrary:

- F: Some H are P ∴ Some H are not P
 F: Some J are not U ∴ Some J are U

Let’s also look at two inferences that look a little bit like Subcontrary, but do not actually fit that pattern because the premise and conclusion do not have the same subject and predicate terms (they’ve been switched).

- F: Some H are not E ∴ Some E are H
 F: Some A are B ∴ F: Some B are not A

****Practice Problems: Subcontrary**

Determine for each immediate inference below whether it’s a valid or invalid appeal to Subcontrary.

- Some Y are T ∴ Some Y are not T
- Some K are J ∴ F: Some K are not J

3. Some L are not E ∴ Some L are E
4. Some G are not R ∴ Some G are R
5. F: Some Y are not I ∴ Some Y are I
6. F: Some D are not Q ∴ F: Some D are Q
7. F: Some A are C ∴ Some A are not C
8. F: Some I are W ∴ F: Some I are not W

Answers:

- | | |
|------------|------------|
| 1. invalid | 5. valid |
| 2. invalid | 6. invalid |
| 3. invalid | 7. valid |
| 4. invalid | 8. invalid |

Subalternation

The two vertical lines of the Square of Opposition represent the pattern we'll call *Subalternation*. A and I, as well as E and O, are subalterns of each other. This pattern is a bit more complicated than the others in the Square of Opposition, but it's intuitive. Subalternation says that if we know a universal statement to be true, then we may confidently infer that its particular subaltern below on the Square is true, too. So, if we know that "All dogs are animals" is true, then we can infer that "Some dogs are animals" is true, also. Moreover, if we know that "No dogs are cats" is true, then we can infer that "Some dogs are not cats" is true, too.

It does not work the other way, though. If we know the universal statement is false, that does not guarantee anything about its subaltern. Consider the following two examples:

No animals are cats [which is false]. Thus, some animals are not cats [which is true].
 No dogs are mammals [which is false]. Thus, some dogs are not mammals [which is false].

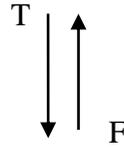
The structure of each inference is the same, but the falsity of the premise does not guarantee the truth value of the conclusion. It's a misuse of Subalternation, and it's called—get ready for this—*Illicit Subalternation*.

Subalternation has a second function. If we know an I or O statement is false, then we can confidently infer that its subaltern above (A and E respectively) is false, too. For if it's false that, say, *some* dogs are birds, it's certainly false that *all* dogs are birds. And, if it's false that *some* cats are not animals, then it's surely false that *no* cats are animals. It's kind of like the opposite of the first function or use of Subalternation.

Memory aid: A perfectly idiotic memory device for Subalternation is this, "Truth reigns down from Heaven, while falsity rises up from the pit of Hell!" If you have any theological leanings at all, this may resonate with you. Truth comes from Heaven above and drips down on us mortals. Lies and falsehood rise up from the devilish mire of Hades. Falsehood never comes down from Heaven, nor does truth ever go up from the Satanic Abyss.

Here are some examples of valid Subalternation inferences:

All J are T ∴ Some J are T
No R are K ∴ Some R are not K
F: Some I are not H ∴ F: No I are H
F: Some U are E ∴ F: All U are E



And here are some invalid examples of Illicit Subalternation.:

F: All O are T ∴ Some O are T
No R are K ∴ F: Some R are not K
Some P are not H ∴ No P are H
F: Some W are P ∴ All W are P

One again—as with the other patterns so far—let’s also look at two inferences that look a little bit like this rule (Subalternation), but which do not actually fit that pattern because the premise and conclusion do not have the same subject and predicate terms (again, they’ve been switched).

All J are I ∴ Some I are not J
F: Some N are not U ∴ F: No U are N

****Practice Problems: Subalternation**

Determine for each immediate inference below whether it’s a valid or invalid appeal to Subalternation.

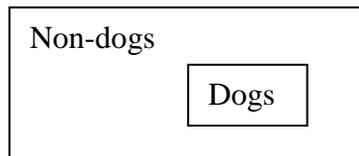
1. All J are R ∴ Some J are R
2. F: All I are O ∴ Some I are O
3. F: All K are W ∴ F: Some K are W
4. All P are H ∴ F: Some P are H
5. No K are Q ∴ Some K are not Q
6. F: No L are V ∴ Some L are not V
7. F: Some Y are L ∴ F: All Y are L
8. F: Some S are not O ∴ F: No S are O
9. Some I are E ∴ All I are E
10. F: Some N are C ∴ All N are C
11. Some Z are not O ∴ No Z are O
12. F: Some J are not H ∴ No J are H

Answers:

- | | | |
|------------|------------|-------------|
| 1. valid | 5. valid | 9. invalid |
| 2. invalid | 6. invalid | 10. invalid |
| 3. invalid | 7. valid | 11. invalid |
| 4. invalid | 8. valid | 12. invalid |

Obversion

The final three categorical patterns we'll examine fall outside the Square of Opposition. They "sit alongside it" as a little group unto themselves. For the first and third, we'll need to introduce a new concept: a *term complement*. A term complement is a term that complements another term. That is, the things the term refers to together with the things the term complement refers to make up all things existing in the universe. It's actually quite simple. Consider the diagram below. It's a picture of everything that exists. The inner box consists of all the dogs in the universe. Everything else in the outer box is everything that's not fully a dog (e.g., cats, trees, the U.S. Constitution, my desire for a cheeseburger, the CEO of Ford Motor Company, and angels (should they exist)).



D and non-D are term complements of each other, since every existing thing in the universe is either a dog or something other than a dog. So too are A and non-A, non-V and V, and non-H and H. In ordinary English, we probably would not say "non-P" or "non-W." We might, though, say something like "All dogs are things other than fish," which we can abbreviate as "All D are non-F."

Now we can describe *Obversion*. This categorical logic pattern says that any categorical statement (A, E, I, or O) will be logically equivalent to its obvert. Obversion involves two moves: we take the original statement and (i) change its quality and (ii) exchange its predicate term for its term complement. We'll look at one example slowly. Consider "All S are P." It's an A statement, and all A statements are universal and affirmative. To make the first change in Obversion, we change the quality from affirmative to negative, leaving the quantity alone: "No S are P." Now the statement is universal and negative. The second change is to swap out the predicate term for its term complement: "No S are non-P." This and the original statement are equivalent; their truth values remain the same as they are logically saying the same thing.

Additional examples to think about:

Original statement

Some dogs are poodles.

F: No mice are rodents.

All lions are non-dogs.

F: All horses are fish.

Some salmon are not mammals.

F: Some tigers are not non-birds.

Obvert of the original statement

Some dogs are not non-poodles.

F: All mice are non-rodents.

No lions are dogs.

F: No horses are non-fish.

Some salmon are non-mammals.

F: Some tigers are birds.

Obversion maintains the original truth value, so if the original statement is true, the obvert will be true, too. Also, if the original statement is false, the obvert will be false, as well. The following are examples of mistakes using Obversion, and they are guilty of—you guessed it—*Illicit Obversion*. These following examples are thus invalid inferences:

All M are non-U ∴ F: No M are U
 No K are M ∴ F: All K are non-M
 F: Some J are T ∴ Some J are not non-T
 F: Some H are not non-I ∴ Some H are I

****Practice Problems: Obversion**

Determine for each immediate inference below whether it's a valid or invalid appeal to Obversion.

1. Some K are not I ∴ Some K are non-I
2. No D are N ∴ All D are non-N
3. F: All O are non-Y ∴ F: No O are Y
4. F: Some Q are P ∴ Some Q are not non-P
5. F: No U are E ∴ All U are non-E
6. All K are D ∴ F: No K are non-D
7. F: Some W are non-U ∴ F: Some W are not U
8. F: Some S are not non-R ∴ F: Some S are R

Answers:

- | | |
|------------|------------|
| 1. valid | 5. invalid |
| 2. valid | 6. invalid |
| 3. valid | 7. valid |
| 4. invalid | 8. valid |

Conversion

Conversion simply trades the subject for the predicate term. The pattern produces a logical equivalence, however, only when performed on E and I statements. Here are four examples of Conversion working well in valid inferences:

- * No dogs are cats. Thus, no cats are dogs.
- * It is false that no cats are animals. Thus, it is false that no animals are cats.
- * Some mice are mammals. Thus, some mammals are mice.
- * It is not the case that some parrots are fish. Thus, it is false that some fish are parrots.

Conversion is not reliable on A and O statements. Just because “All dogs are mammals” is true, its convert, “All mammals are dogs” is not the case. And, just because “Some animals are not fish” is true, it does not follow that “Some fish are not animals” is true, too. If we make a Conversion inference on an A or O statement, or if we fail to “keep” the original truth value, we

are guilty of—wait for it—*Illicit Conversion*. Here are some examples of invalid uses of conversion. Socially polished students will avoid making these errors in public.

All S are Y \therefore All Y are S [Conversion does not work on A statements.]

Some L are E \therefore F: Some E are L [Conversion keeps the original truth value.]

F: No G are E \therefore No E are G [Again, Conversion maintains the same truth value.]

Some K are not Q \therefore Some Q are not K [Conversion does not work on O statements.]

Here are some valid uses of Conversion:

No A are B \therefore No B are A

Some J are W \therefore Some W are J

F: No L are K \therefore F: No K are L

F: Some I are D \therefore F: Some D are I

****Practice Problems: Conversion**

Determine for each immediate inference below whether it's a valid or invalid appeal to Conversion.

1. Some K are T \therefore Some T are K
2. F: All I are Y \therefore All Y are I
3. All O are P \therefore All P are O
4. No M are N \therefore No N are M
5. F: Some G are W \therefore F: Some W are G
6. F: No Q are W \therefore No W are Q
7. Some E are not I \therefore Some I are not E
8. F: Some O are not B \therefore F: Some B are not O

Answers:

- | | |
|------------|------------|
| 1. valid | 5. valid |
| 2. invalid | 6. invalid |
| 3. invalid | 7. invalid |
| 4. valid | 8. invalid |

Contraposition

Our last categorical pattern is *Contraposition*. Like Obversion and Conversion, if Contraposition is performed correctly, the truth value of the contrapositive is the same as the original statement, that is, the two statements will be logically equivalent. Contraposition (like Obversion) requires two steps. The order is unimportant, but two changes take place moving from an original statement to its contrapositive: (i) exchange the two terms with each other (as you do in Conversion), then (ii) exchange each term with its term complement.

Consider “All dogs are animals,” or “All D are A.” If we do the first step, we get “All A are D.” The second step swaps out each newly traded term with its own term complement: “All non-A are non-D.” It may be a headache to think about, but in English is says, “All things other than animals [e.g., plants, rocks, ballpoint pens] are things other than dogs [e.g., cats, fish, plants, rocks, ballpoint pens]. Or put another way, “Everything that is not an animal can’t be a dog,” because if you’re not an animal, you can’t be a dog.

Contraposition “works” only on A and O statements; inferences using Contraposition are not reliable on E and I statements. For instance, consider the following invalid uses of Contraposition:

No dogs are cats. Thus, no non-cats are non-dogs.
Some animals are non-dogs. Thus, some dogs are non-animals.

In both cases, the premise is true and the conclusion is false, showing the inferences to be invalid. Such an ill-advised inference may be called *Illicit Contraposition*. With A and O statements, however Contraposition will guarantee that the contrapositive (i.e., the second statement) *is* equivalent; and thus if we are making an inference with them using Contraposition, if the premise is true, the conclusion will be guaranteed to be true, too.

Here are some examples of valid uses of Contraposition:

All non-A are H ∴ All non-H are A
F: All J are B ∴ F: All non-B are non-J
Some K are non-I ∴ Some I are non-K
F: Some non-O are non-G ∴ F: Some G are O
All U are M ∴ All non-M are non-U

Memory aid: Obversion, Conversion, and Contraposition are a bit different from the rules associated directly with the Square of Opposition. With these three, one works on all statements (Obversion), one works only on E and I (Conversion), and one works only on A and O (Contraposition). How might students remember which is which? Maybe this will help:

Obversion is the odd one in that it works on all four statements. [“Obversion” and “odd” begin with “o.”] Conversion and Contraposition are cranky, as they work on only two types of statements. [Note that cranky, Conversion, and Contraposition all begin with “c.”] Conversion works only on E and I. [Note the middle vowels of “Conversion.”] Contraposition works only on A and O. [Note the middle vowels of “Contraposition.”]

****Practice Problems: Contraposition**

Determine for each immediate inference below whether it’s a valid or invalid appeal to Contraposition.

1. Some Y are non-P ∴ Some P are non-Y
2. F: All J are E ∴ F: All non-E are non-J

3. Some K are not W \therefore F: Some non-W are not non-K
4. No U are V \therefore No non-V are non-U
5. F: Some S are not non-L \therefore F: Some L are not non-S
6. All non-K are non-C \therefore All C are K
7. All Q are M \therefore F: All non-M are non-Q
8. Some O are not non-N \therefore Some N are not non-O

Answers:

- | | |
|------------|------------|
| 1. invalid | 5. valid |
| 2. valid | 6. valid |
| 3. invalid | 7. invalid |
| 4. invalid | 8. valid |

Putting it All Together

Now that we've learned seven categorical patterns, we can now more easily and quickly assess many deductive immediate inferences as valid or invalid. If we can recognize one of these patterns being used, and we can determine that the use is correct or improper, then we can know the argument to be valid or invalid (respectively). For instance, consider the following argument: "All dogs are animals. Thus, it is false that no dogs are animals." This argument containing an A and a negated E statement has the following structure:

All D are A \therefore F: No D are A

We see here a move from an affirmed A statement to a negated E statement, which is a use of Contrary. We now ask ourselves, is this a proper or improper use of Contrary? Well, Contrary says at least one of a pair of A and E statements must be false, and the premise tells us that the A statement is true. So, we can be confident that the E version of that statement must be false. And that's exactly what the conclusion tells us, so the argument is a correct use of Contrary, and thus valid.

Now consider the following argument: "Some things other than dogs are tigers. Thus, some things other than tigers are dogs." The argument may be abbreviated as follows:

Some non-D are T \therefore Some non-T are D

This argument fits the pattern of Contraposition, but it's a mistaken use of that pattern, as Contraposition only works on A and O statements. The premise here is an I statement, so the argument misuses Contraposition, it's guilty of Illicit Contraposition, and it's thus invalid.

Here's one more example:

F: Some K are not I \therefore F: Some K are non-I

This inference fits the pattern known as Obversion, and everything looks okay. Obversion works on O statements (e.g., this premise), and the false truth value of the premise is appropriately carried over to the conclusion.

****Practice Problems: Assessing Immediate Inferences**

Consider the following immediate inferences. Each is an example of a categorical pattern presented above. Name the pattern and determine if the inference is valid or invalid.

1. All H are N ∴ Some H are N
2. F: No G are H ∴ F: No H are G
3. F: Some A are U ∴ No A are U
4. F: All J are E ∴ F: No J are E
5. F: Some K are T ∴ Some K are not T
6. All G are non-Y ∴ F: No G are Y
7. Some S are non-P ∴ Some P are non-S
8. No M are non-Y ∴ All M are Y
9. F: Some T are not I ∴ All T are I
10. F: No L are J ∴ F: Some L are not J
11. All D are E ∴ All E are D
12. All O are P ∴ F: No O are non-P
13. No H are E ∴ F: All H are E
14. All K are non-S ∴ All S are non-K
15. All A are B ∴ Some A are not B
16. No U are Q ∴ Some U are not Q
17. Some K are not Y ∴ Some K are Y
18. Some I are M ∴ Some M are I
19. F: Some N are not M ∴ F: Some non-M are not non-N
20. All Z are W ∴ F: No Z are W

Answers:

- | | |
|-----------------------------|----------------------------|
| 1. Subalternation, valid | 11. Conversion, invalid |
| 2. Conversion, valid | 12. Obversion, invalid |
| 3. Contradiction, valid | 13. Contrary, valid |
| 4. Contrary, invalid | 14. Contraposition, valid |
| 5. Subcontrary, valid | 15. Contradiction, invalid |
| 6. Obversion, invalid | 16. Subalternation, valid |
| 7. Contraposition, invalid | 17. Subcontrary, invalid |
| 8. Obversion, valid | 18. Conversion, valid |
| 9. Contradiction, valid | 19. Contraposition, valid |
| 10. Subalternation, invalid | 20. Contrary, valid |

The Hypothetical Interpretation

We can now revisit the issue of the ambiguity in A and E categorical statements. As you recall, we often mean two completely different things when we utter claims of the form “All S are P” or “No S are P.” So far, we’ve been focusing on the existential interpretation in which we assume that the person making the claim believes that the subject term refers to existing things. But every now and then we want to talk about vampires, unicorns, or round squares. At these moments, we (who are not delusional or seriously confused) are not wishing to say that such things actually exist, but that *if* they did, they’d all be such-and-such. For instance, if the normally informed among us say, “All vampires are bloodsuckers,” what we’d mean is that *if* there were any vampires (and we’re not saying that there are), then they all *would* be bloodsuckers.

We need to care about this distinction because if we are not careful with it, we can quickly end up “proving” that vampires exist. Consider the two arguments below:

All dogs are animals. Thus, some dogs are animals.

All vampires are bloodsuckers. Thus, some vampires are bloodsuckers.

Both are examples of Subalternation. Doesn’t the first argument seem valid? Surely, if *all* dogs are animals, then at least *some* of them are. This is indeed a valid inference, but notice that the particular conclusion (an I statement) has *existential import*; it’s claiming that dogs exist and that at least one of them is an animal. That conclusion follows from the premise because the premise is claiming that (i) dogs exist, and (ii) they are all animals. But now look at the vampire argument. It has the same form, and the particular conclusion is claiming that vampires exist and at least one of them is a bloodsucker. Vampires exist!?! No! What went wrong? If we mean the same thing with the vampire premise as we meant with the dog premise, we can get in trouble; we can end up “proving” that vampires exist, but that’s nuts.

A solution is to use the existential interpretation when we have good reason to believe the person offering the A or E statement believes in the existence of the things referred to by the subject term, and to use the hypothetical interpretation when we understand him or her to believe the subject term refers to things that do *not* exist. We do this all the time in ordinary English conversation, and it’s no harder to do here in the context of categorical logic.

Note what happens when we give the premise of the vampire argument above a hypothetical interpretation (which is what we’d do normally in ordinary conversations). We’d understand the arguer to be opening his inference with “If there were any vampires, then they’d all be bloodsuckers (and we can readily agree to that, even though we don’t believe in vampires). If he then went on to conclude that therefore there are some vampires that are bloodsuckers, we’d say, “Hold on buddy! That’s an invalid inference! Just because vampires *would* be bloodsuckers if they existed does not mean that there actually *are* any vampires walking the Earth today. You *really* owe me another round of beer for that howler of an argument.”

Okay. We now know what to do with arguments. If a premise is an A or E statement, and the subject term refers to things that exist, we’ll give it the existential interpretation. If the subject term refers to things that the arguer pretty clearly does not assume to exist, we’ll give that A or E

statement the hypothetical interpretation. This will keep us from taking invalid arguments and making them appear to be valid. Making the unjustified interpretation and thereby making an erroneous assessment of an inference is known as the *Existential Fallacy*.

Given that A and E statements say less under the hypothetical interpretation than they do under the existential interpretation, fewer categorical logic patterns pertain to them. We've already seen how Subalternation works with arguments about dogs, but that it can misfire with arguments about vampires. There are thus fewer inference patterns that will be valid if we are arguing about vampires or leprechauns than if we are arguing about dogs or cats. The following lists detail which patterns are trustworthy for each of the two interpretations:

Existential Interpretation

Contradiction
Contrary
Subcontrary
Subalternation
Obversion
Conversion
Contraposition

Hypothetical Interpretation

Contradiction
Obversion
Conversion
Contraposition

If an inference about werewolves uses Contrary, it will automatically be an invalid argument (guilty of the Existential Fallacy). If an inference about horses uses Contrary, we need to see if Contrary is being used correctly, and then determine whether the argument is valid or invalid. Here are some examples:

No sharks are birds. Thus, some sharks are not birds.

The premise's subject term refers to sharks, which exist, so we use the existential interpretation. Given that interpretation, Subalternation gives us exactly this conclusion. The argument is thus valid.

No mermaids are fishy-smelling swimmers. It follows that it is false that all mermaids are fishy-smelling swimmers.

The subject term of the premise is "mermaids," and refers to beauties that sadly do not exist. We are thus justified in giving this E statement the hypothetical interpretation. But Contrary is not reliable under such conditions, and we cannot safely draw the conclusion. This argument commits the Existential Fallacy, and is thus invalid.

Some vampires are garlic haters. Therefore, some garlic haters are vampires.

The premise is an I statement, so there is no ambiguity or concern over existential/hypothetical interpretations. The premise straightforwardly says that there are vampires in existence and that at least one is a garlic hater. Given the inferential move is Conversion, and Conversion works

given no matter what the interpretation, the argument is valid. The argument is, of course, unsound, as the premise is false (vampires do *not* exist).

It is false that all round squares are unenclosed geometric figures. Hence, it is not the case that all things other than unenclosed geometric figures are things other than round squares.

The subject term of the premise here is “round squares,” which refers to things that do not exist. We are thus justified in using the hypothetical interpretation to understand this negated A premise. The inference pattern here is Contraposition, and that pattern provides a logical equivalent for A and O statements, whether or not they are about things that exist. We can thereby say this is a proper use of Contraposition, and that the argument is valid.

Note that we have examined only seven categorical inference patterns. There are other patterns that can form valid or invalid immediate inferences. We have learned these seven because they are fairly common. Note that the following immediate inferences do *not* follow any of the patterns we have studied here:

No jackrabbits are wildebeests. Thus, all wildebeests are jackrabbits.
All dogs are animals. Thus, some animals are dogs.

The first argument is invalid, while the second is valid. Neither, however, follows any one of our seven patterns. Other techniques may be used to prove such arguments valid or invalid (e.g., we can use the Counterexample Method to show the first to be invalid, if it wasn't obvious enough already). For our present purposes, though, we simply want to get better at recognizing some common patterns—good and bad—of fairly simple immediate inferences.

****Practice Problems: Assessing Immediate Inferences**

What pattern is used in each immediate inference below? Are the following immediate inferences valid or invalid? Use the existential or hypothetical interpretation as appropriate.

1. All tigers are cats. Thus, no tigers are things other than cats.
2. No 50-foot-tall humans are little people. Thus, no little people are 50-foot-tall humans.
3. All zombie strippers are sexy dead people. Thus, it is false that some zombie strippers are not sexy dead people.
4. Some black dogs are animals that bark. Thus, it is false that some black dogs are not animals that bark.
5. No rabbits are elk. Thus, it is not the case that all rabbits are elk.
6. Some nudist vampires are pasty-skinned night dwellers. Thus, all nudist vampires are pasty-skinned night dwellers.
7. It is false that no balding werewolves are sad creatures. Thus, some balding werewolves are not sad creatures.
8. It is false that all breakfast cereals are nutritious meals. Thus, it is false that all things other than nutritious meals are things other than breakfast cereals.
9. It is false that some logic teachers are weirdoes. Thus, some logic teachers are not weirdoes.

10. Some morally guiltless murderers are happy robbers. Thus, it is false that some happy robbers are morally guiltless murderers.
11. All jackalopes [a cross between a jackrabbit and an antelope, jokingly said to be found in Wyoming] are mythic creatures. Thus, some jackalopes are mythic creatures.
12. All sandy beaches are aquatic locales. Thus, it is false that some sandy beaches are not aquatic locales.
13. It is false that some pets are amoebas. Thus, some pets are not things other than amoebas.
14. Some makers of everlasting TV sets are not things other than popular manufacturers. Thus, it is false that some popular manufacturers are not things other than makers of everlasting TV sets.
15. All dogs are animals. Thus, it is false that no dogs are animals.
16. All unicorns are fierce beasts tamable only by female virgins. Thus, some unicorns are not fierce beasts tamable only by female virgins.
17. It is false that some dinosaurs presently living in Bellevue are large animals. Thus, some dinosaurs presently living in Bellevue are large animals.
18. All Beatle songs are great tunes. Thus, all great tunes are Beatle songs.
19. It is false that no native turkeys are things in the USA. Thus, all native turkeys are things in the USA.
20. All logic texts are joys to read. Thus, it is false that some logic texts are not joys to read.

Answers:

- | | |
|----------------------------|-----------------------------|
| 1. Obversion, valid | 11. Subalternation, invalid |
| 2. Conversion, valid | 12. Contradiction, valid |
| 3. Contradiction, valid | 13. Obversion, invalid |
| 4. Subcontrary, invalid | 14. Contraposition, invalid |
| 5. Contrary, valid | 15. Contrary, valid |
| 6. Subalternation, invalid | 16. Contradiction, invalid |
| 7. Subalternation, invalid | 17. Subcontrary, invalid |
| 8. Contraposition, valid | 18. Conversion, invalid |
| 9. Subcontrary, valid | 19. Contrary, invalid |
| 10. Conversion, invalid | 20. Contradiction, valid |

Categorical Derivations

Let's take use of categorical patterns one step further. We can appeal to them to prove an even wider variety of arguments to be valid. This will be a use of *natural deduction*. This process begins with a premise, and by appealing to the rules (or patterns) of logic, shows that the conclusion must follow from it. That is, natural deduction shows that the conclusion can be derived from the premise. The conclusion is thus the goal we aim for, and we have at our disposal to get that goal the tools of seven logic rules and the premise.

The idea behind the process is simple. We take the premise, and use categorical logic rules to transform it into the conclusion. The conclusion is thus our goal; it's what we're aiming for. An example will help. Consider the following argument:

All dogs are animals. Thus, it is false that some things other than animals are not things other than dogs.

Abbreviating the argument will make it much easier to work with. Let's place the conclusion after the premise, separating it off with a slash.

All D are A / F: Some non-A are not non-D

What we've done so far in our work in categorical logic is to look for a single categorical rule being used (e.g., Obversion, Contrary), and determine if that single rule is being used correctly or illicitly. Derivations of valid arguments can be more complex, though, as it may take a series of rules to move from the premise to the conclusion. It takes a creative mind and a healthy familiarity with all seven rules to figure out what "route" might be needed to go from the premise to the conclusion. With practice, this becomes somewhat easy.

For the argument above, we might note that the premise is an A statement. We can do Contradiction on it to get a negated O version: *F: Some D are not A*. We'll write that down as the second line, and justify that move by appealing to Contradiction.

All D are A / F: Some non-A are not non-D
F: Some D are not A Contradiction

We've not derived the conclusion yet, so we look to see what we might next do to change the second line into the conclusion—or at least something that will move us toward the conclusion. Upon reflection, we see that we can do Contraposition on it to get exactly what we want. We write *F: Some non-A are not non-D* as the third line, justifying the move by appealing to Contraposition.

All D are A / F: Some non-A are not non-D
F: Some D are not A Contradiction
F: Some non-A are not non-D Contraposition

Now that we've derived the conclusion, we're done! We've shown that if the premise is true, then the conclusion must be true, too. The argument is thus valid!

Natural deduction shows valid arguments to be valid, but it cannot show invalid arguments to be invalid (that takes alternative techniques). So, what we'll deal with here are valid arguments, and we'll use natural deduction to prove they're valid. Here's another example.

Some horses are mammals. Thus, it is false that no mammals are horses.

Abbreviated and set up as a derivation, this will look as follows:

Some H are M / F: No M are H

There are a number of changes we need to make to the premise in order to make it look exactly like the conclusion. In no specified order, we need to switch the subject and predicate terms, change the quantity from particular to universal, change the quality from affirmative to negative, and change the truth value from true to false. That's a lot of changes, but oftentimes using one rule will make more than one change. Let's start, though, by using Conversion on the premise; we then get *Some M are H*. That at least switches the terms to match more closely the conclusion.

Some H are M / F: No M are H
 Some M are H Conversion

We can make all the other needed changes now by using Contradiction. This gives us the conclusion, and we're done!

Some H are M / F: No M are H
 Some M are H Conversion
 F: No M are H Contradiction

Here are three more examples. Try to see how the use of each rule gradually changes the premise into the conclusion, thus showing that the premise guarantees that the conclusion is true. We'll assume for these three examples that the subject term of each premises refers to existing things. That will allow us to take the existential interpretation, and thus be able to use all seven categorical logic rules. If a premise was about unicorns or vampires (i.e., things that we do not wish to assume exist), we'd be limited to the use of Contradiction, Obversion, Conversion, and Contraposition.

No P are Q / F: No P are non-Q
 All P are non-Q Obversion
 F: Some P are not non-Q Contradiction
 F: No P are non-Q Subalternation

F: Some M are N / All N are non-M
 No M are N Contradiction
 No N are M Conversion
 All N are non-M Obversion

F: Some non-A are not H / Some non-H are A
 F: Some non-H are not A Contraposition
 All non-H are A Contradiction
 Some non-H are A Subalternation

Again, if the premise's subject term refers to things we do not wish to assume to exist, then we are limited to using Contradiction, Obversion, Conversion, and Contraposition. To use another rule on a statement about vampires or mermaids would make us guilty of the Existential Fallacy. Arguments about vampires might still be valid. For example:

All vampires are bloodsuckers. Thus, it is false that some vampires are things other than bloodsuckers.

A derivation of this valid argument can look like this:

All V are B / F: Some V are non-B
F: Some V are not B Contradiction
F: Some V are non-B Obversion

We can't use this natural deduction technique to show an invalid argument to be invalid, because we could not be sure that our inability to derive the conclusion from the premise is due to the invalidity of the argument or to our being somewhat obtuse that day. The fact that you cannot figure out how to get to the conclusion from the premise using natural deduction does not demonstrate anything in particular. The task may be impossible (in which case the argument is invalid), *or* you may simply have not eaten enough Wheaties for breakfast to stimulate your brain adequately. If you *do* succeed in deriving the conclusion from the premise, though, that indeed shows the argument to be valid.

****Practice Problems: Categorical Logic and Natural Deduction**

Use natural deduction and any of the seven categorical logic rules to derive the conclusion of the following valid arguments. Begin by abbreviating the argument and setting it up in derivation format.

1. No bears are fish. Thus, some things other than fish are not things other than bears.
2. It is false that all lions are cats. Thus, some lions are things other than cats.
3. All leprechauns are Irish people. Hence, it is false that some things other than Irish people are not things other than leprechauns.
4. It is false that some animals are bananas. Therefore, it is false that all things other than animals are things other than bananas.
5. It is false that some anarchists are lovers of law. It follows that some anarchists are things other than lovers of law.
6. All lesbians are people. Thus, it is false that all people are things other than lesbians.
7. No Algerians are Bolivians. Thus, it is false that no Algerians are things other than Bolivians.
8. No mermaids are skinny-dippers. Thus, it is false that some skinny-dippers are not things other than mermaids.
9. All radicals are enthusiastic people. Hence, some things other than enthusiastic people are things other than radicals.
10. Some Republicans are conservatives. Thus, some conservatives are not things other than Republicans.

Answers:

1. No B are F / Some non-F are not non-B
Some B are not F Subalternation
Some non-F are not non-B Contraposition

2. F: All L are C / Some L are non-C
 F: No L are non-C Obversion
 Some L are non-C Contradiction
3. All L are I / F: Some non-I are not non-L
 F: Some L are not I Contradiction
 F: Some non-I are not non-L Contraposition
4. F: Some A are B// F: All non-A are non-B
 F: Some B are A Conversion
 F: All B are A Subalternation
 F: All non-A are non-B Contraposition
5. F: Some A are L / Some A are non-L
 Some A are not L Subcontrary
 Some A are non-L Obversion
6. All L are P / F: All P are non-L
 F: No L are P Contrary
 F: No P are L Conversion
 F: All P are non-L Obversion
7. No A are B / F: No A are non-B
 Some A are not B Subalternation
 Some A are non-B Obversion
 F: No A are non-B Contradiction
8. No M are S / F: Some S are not non-M
 No S are M Conversion
 F: Some S are M Contradiction
 F: Some S are not non-M Obversion
9. All R are E / Some non-E are non-R
 All non-E are non-R Contraposition
 Some non-E are non-R Subalternation
10. Some R are C / Some C are not non-R
 Some R are not non-C Obversion
 Some C are not non-R Contraposition

Chapter 10: Propositional Patterns

The patterns of categorical logic can handle many arguments, and being familiar with these patterns allows us to determine if many arguments are valid or invalid at a glance. Ordinary conversation, however, often includes argumentation that in turn includes statements that are a bit more complex than A, E, I, or O statements. As we saw early in the previous chapter, many statements can be translated into a standard form categorical pattern, but there remain many others that cannot. So, we need to move forward to the next level of deductive reasoning: the patterns of *propositional logic*.

Whereas categorical statements are made up of terms referring to classes of things (e.g., dogs, black cats, unicorns that run through the streets of Bellevue), propositional logic is made up of *propositions*, or declarative sentences (i.e., statements; that's why it's sometimes called *sentential logic*). Since propositional logic is made up of the kinds of sentences we utter all the time, many people find it easier to work with, more natural, and more intuitive than categorical logic. It also forms the basis of more advanced deductive logic systems used around the world today. An introduction to those systems is presented in Symbolic Logic classes (like PHIL& 120 at Bellevue College and the University of Washington). We'll be getting a *non-symbolic* introduction to propositional logic here.

Basic Statement Patterns

Let's consider five different kinds of statements, or propositions. Each is used commonly in conversation and in arguments. Here are examples of each:

- (i) Yogi is a bear. [a simple statement]
- (ii) It is false that Yogi is a bear. [a negation]
- (iii) Yogi is a bear and Boo-Boo is a mammal. [a conjunction]
- (iv) Yogi is a bear or Boo-Boo is a mammal. [a disjunction]
- (v) If Yogi is a bear, then Boo-Boo is a mammal. [a conditional]

The first we'll call a *simple statement*. It does one and only one thing: it declares that something is the case. Here, it's declaring that the classic American cartoon character, Yogi, is a bear. Simple statements may be true or false, we may not know if they're true or false, or we might disagree on whether they're true or false—but they *are* true or false, which is what makes these sentences *statements/propositions* (instead of questions, commands, exclamations, or other non-declarative sentences).

The remaining four kinds of statements we'll call *compound statements*, since they are made of more than one part. The second kind above is made of "It is false that" and a simple statement. The third is made of two simple statements and the word "and." The fourth consists of two simple statements and the word "or." And the last is made of two simple statements united with "if...then...."

More specifically, we'll call the second kind of statement above a *negation*. A negation merely says that a statement is false. It disagrees with whatever the negating words are "pointing" to. Anyone growing up watching television cartoons in the U.S. will know that Yogi was a bear in Jellystone Park. So we'll know that "Yogi is a bear" has a truth value of True. "It is false that Yogi is a bear," or "It is not the case that Yogi is a bear," or "Yogi is not a bear" denies that Yogi is a bear. We'd disagree with that negation, since we agree with the simple statement denied by the negating phrases.

This is far easier than any of this sounds. Talking about it makes it appear somewhat convoluted. It's not. Here are some illustrations:

- * Oregon is north of California. [We agree! This simple statement is true.]
- * It is false that Oregon is north of California. [We disagree! This negation is false.]
- * Washington is east of Idaho. [We disagree! This simple statement is false.]
- * It is not the case that Washington is east of Idaho. [We agree! This negation is true.]

The third basic statement pattern above is called a *conjunction* because it *conjoins* two statements. To conjoin two or more statements is to say that they are all true. We agree that Yogi is a bear, and we know that Bob-Boo is his short bear friend and longsuffering mammalian sidekick. We thus agree that Boo-Boo is a mammal. Since we agree with both simple statements, we'll agree with a conjunction (an "and" statement) of the two: "Yogi is a bear *and* Boo-Boo is a mammal."

Would you agree or disagree with the following conjunctions? (Note that in English, there are many logically equivalent ways of saying *and*.)

- * Mexico is south of Canada *and* the Seattle Mariners are a baseball team.
- * Peru is north of Canada, *yet* the Seattle Mariners are a baseball team.
- * Costa Rica is north of Canada, *however* the Seattle Mariners are a football team.
- * Panama is south of Canada, *moreover* the Seattle Mariners are a hockey team.
- * Columbia is south of Iceland, *nonetheless* the USA is south of Cuba.
- * Honduras is north of Canada, *but* Belize is south of Argentina.

Conjunctions are true if and only if the two statements conjoined are true. The sides of a conjunction are called the *left conjunct* (or left-hand conjunct) and the *right conjunct* (or right-hand conjunct). So, a conjunction is true just in case both its conjuncts are true.

For the six conjunctions above, only the first is true, as only with it are both conjuncts true. For the others, one or both of the conjuncts is false. This is exactly what we mean when we use the basic word "and" in ordinary conversation, so there really is no weird complication here. As logicians, we're just trying to be precise.

The third basic kind of statement above is called a *disjunction*. It's an "or" statement claiming that one or the other of two claims is true. There are two uses of "or" in English: the *inclusive* "or" and the *exclusive* "or." If the disjunction is "Yogi is a bear or Boo-Boo is a mammal" and

we mean it in the inclusive sense, then we mean either (a) Yogi is a bear, or (b) Boo-Boo is a mammal, or (c) both statements are true. If we mean it in the exclusive sense, then we mean either (a) Yogi is a bear, or (b) Boo-Boo is a mammal, but not both. We exclude the option of having them both be true at the same time. Think of a friend who offers a ride in his car to school. He approaches you and your sister and says, “Either you or your sister can ride with me tomorrow.” If he has one seat available, he’s intending you to understand the offer as an exclusive “or,” that is, that one or the other—but not both of you—can ride with him tomorrow. If he has two seats available, then he likely means the offer as an inclusive “or,” that is, that either or both of you can get a ride. For the purposes of this critical thinking class and most introductory symbolic logic classes, “or” is understood as inclusive unless the context makes plain that it’s intended to be exclusive.

Disjunctions will then be false only when both *disjuncts* (right and left, or right-hand and left-hand) are false. If either side of the “or” is true, then the disjunction as a whole will be true. Again, this matches exactly the way we’ve been talking since early childhood, so there’s nothing bizarre going on here. Which disjunctions below would you agree to and say are true?

- * France is in Europe or Chile is in South America.
- * England is in Asia or Argentina is in South America.
- * Canada is in North America or Thailand is in Europe.
- * Australia is in the Atlantic Ocean and Poland is in Central America.

The only disjunction above whose simple statements are both false is the last. Thus, it’s the only one that as a whole is false; the other three statements as wholes are true. Again, a disjunction is true if and only if one or more of its disjuncts is true.

The fifth and final kind of statement we’ll consider is called by more than one name: *implication*, *hypothetical*, and *conditional*. We’ll opt for the latter name here, and call “If . . . , then . . .” statements *conditionals*. Conditionals have two parts: the “if” part is called the *antecedent*; the “then” part is called the *consequent*. The antecedent (the prefix *ante-* means “before”) is what conceptually comes prior to the consequent (the word is related to “consequently”: following). For example, in “If Bellevue is in Washington State, then Bellevue is in the United States,” “Bellevue is in Washington State” is the conditional’s antecedent, while “Bellevue is in the United States” is the statement’s consequent.

Below are some examples of conditional statements:

- * If Bob is a baseball player, then Bob plays sports.
- * If Sarah is a physician, then she is a doctor.
- * If José likes all seafood, then José likes fish.

There are quite a few ways of saying the same conditional claim in English, which can make conditionals the most awkward kinds of statements to work with. English is great for poetry, but miserable for logic. Note that each of the following examples of conditionals means the same thing:

- * If it's raining, then the ground is wet.
- * The ground is wet, if it's raining.
- * Provided that it's raining, the ground is wet.
- * Given that it's raining, the ground is wet.
- * The ground is wet, provided that it's raining.
- * The ground is wet, given that it's raining.
- * It's raining only if the ground is wet. [This one may take some thought, as the "only" throws many native English speakers.]

Conditionals are clearly the most complicated of the basic kinds of propositional statements. Not only are there numerous ways of getting the idea across in English, there is also more than one reasonable way of deciding if a conditional is true or false. The latter problem arises because "If..., then..." can mean different things in different contexts. If I say, "If I drop this bowling ball on my foot, then my foot would hurt," I'm using "If..., then..." in a causal sense. Dropping bowling balls on my feet causes my feet to hurt. Knowing that, most of us would say the conditional is true. However, if I say, "If I have three coins in my hand, then I have an odd number of coins in my hand," it's pretty clear I'm saying something different. Having three coins does not cause me to have an odd number of coins; it's a definitional issue and not one of mere cause and effect.

There's a third use of "If..., then..." logicians call a *material implication*. In this sense, the conditional statement comes out to be false only when the antecedent is true and the consequent is false. That may not seem intuitive, and that's likely because we use "If..., then..." in so many ways without even thinking about it. But consider the following scenario. I walk up to you one morning and make a promise: "If it's windy today, then I'll fly my kite." It's now at the end of the day; decide in which situations my promise ends up being false.

- (a) It was windy today, and I flew my kite. [No lie there! That's exactly what I promised to do.]
- (b) It was windy today, but I did not fly my kite. [Here's the clear falsehood; I did not do what I promised to do.]
- (c) It was not windy today, and I did fly kite. [No lie here! I promised to fly a kite if it *was* windy, but I might still fly my specially-designed indoor kites (there are some, but they're expensive). I've certainly not mislead you in any way here.]
- (d) It was not windy, and I did not fly any kite. [No lie here, either.]

The only place where I clearly mislead you was in the second case, the one with a true antecedent and false consequent.

Memory device: We can call the situation in which a conditional is false the "T-F situation," or "TUF." TUF (short for "tough") is hard and difficult. TUF is bad. Bad is false. If a conditional is

TUF, then it's false; if it's *not* TUF, then it's true. The following conditionals are thus true given the material implication sense of "If..., then...":

- * If George Washington was the first U.S. president, then a man walked on the Moon.
- * If all mothers are female, then all fathers are male.
- * If $2+2=5$, then squares have four sides.
- * If Ronald Reagan was a U.S. president, then the Mariners play baseball in Seattle.
- * If President Ronald Reagan was an astronaut, then he was a ballet star.

To see that each of these four statements is true, we simply note that none of them has a true antecedent *and* false consequent. They all avoid the TUF pattern. Now some of them may be false given a different interpretation of conditional statements, but for the purposes of this chapter, we are going to limit ourselves to considering only the material implication interpretation. Here are some false conditional statements:

- * If George Washington was the first U.S. president, then President Ronald Reagan played outfield for the Seattle Mariners.
- * If $2+2=4$, then $4+1=6$.
- * If Canada is north of Mexico, then the USA is north of Panama.
- * Libya is in the South Pacific, if Egypt is in Africa.

Complex Statements

Obviously, some compound statements are more complex than these basic patterns. We might, for instance, have a disjunction made up of two disjuncts, each of which is a compound statement. For example:

Either Bob is bald and Tom is tall, or it is false that Susan is short.

The comma helps us out, as its placement near "or" lets us know that this statement as a whole is a disjunction. The left disjunct ("Bob is bald and Tom is tall") is a conjunction, while the right disjunct ("it is false that Susan is short") is a negation. We can abbreviate this statement as "Either B and T, or not-S." To do so we took each simple statement and replaced it with a distinct upper-case letter. We also abbreviated the negating clause to a simpler "not-." Consider another example:

It is false that if June is jolly then Harry is happy.

The "It is false that" is denying something; in this case it's denying the conditional statement "if June is jolly then Harry is happy." The statement as a whole is thus a negation; it's a negated conditional. Abbreviated for simplicity's sake, this might look like "Not if J then H." Let's look at another:

Nathan is not in the Navy and Alice is not in the Army.

We can abbreviate this as “Not-N and not-A.” It’s a conjunction made up of two negations. The left conjunct is a negation negating the simple statement N (“Nathan is in the Navy”). The right conjunct is a negation negating the simple statement A (“Alice is in the Army”). Here’s another example:

If Alberto is not an artist, then either Daniel is a doctor or it is not the case that Phyllis is a physicist.

This can be abbreviated as “If A, then either D or not-P.” Again, the single comma helps, as it sits near “then,” which indicates a conditional. The statement as a whole is thus a conditional, with a simple statement (A) as its antecedent, and with a disjunction (“either D or not-P”) as its consequent. That disjunction, in turn, has a simple statement (D) as its left disjunct and a negation (not-P) as its right disjunct. That negation, in turn, is negating a simple statement (P). Whew!

What we are doing at this point is getting used to the structure of some simple and compound statements. Once we let that settle into our genetic structure, we’ll more easily be able to recognize the validity of some fairly complex deductive arguments.

****Practice Problems: Recognizing Propositional Patterns**

Determine if each statement below—taken as a whole—is a simple statement, negation, conjunction, disjunction, or conditional. Then determine the statement’s truth value (each proper name refers to well known places; don’t assume the statement refers to little-known places with names similar to better-known locations).

1. Either Nicaragua is in Central America or Germany is in Europe.
2. Both France and Paraguay are in Asia.
3. If Bolivia is in South America, then Tahiti is in the South Pacific.
4. Greenland is in the Middle East or Ukraine is in Central America.
5. It is false that Denmark is in Africa.
6. Algeria is in North America, but Guatemala is in Central America.
7. It is false that Brazil is in South America.
8. China is in Asia.
9. India is north of South Africa or it is false that the USA is south of Canada.
10. It is false that both Holland and Spain are in Central America.
11. It is not the case that either Congo or Angola is in Africa.
12. Russia is larger than Spain, if Japan is smaller than Mexico.
13. If it is false that Spain is near Portugal, then Uruguay is in South America.
14. Either Ecuador and Namibia are in Asia, or Iran is in the Middle East.
15. Both El Salvador and Venezuela are in the Western Hemisphere, or Turkey is in the Western Hemisphere.

Answers:

1. Disjunction, true
2. Conjunction, false

3. Conditional, true
4. Disjunction, false
5. Negation, true
6. Conjunction, false
7. Negation, false
8. Simple statement, true
9. Disjunction, true
10. Negation (i.e., a negated conjunction), true
11. Negation (i.e., a negated disjunction), false
12. Conditional, true
13. Conditional, true
14. Disjunction, true
15. Disjunction, true

Symbolization

Although we are not covering symbolic logic in Critical Thinking, it may be of interest to some students how logicians can make our lives easier by abbreviating propositional statements even further. Since we are not covering this in Critical Thinking, we'll look at translating English into contemporary symbolic logic very quickly. Feel free to pass over this section, as it will not be required for this course. We'll translate here simple statements, negations, conjunctions, disjunctions, and conditionals with symbols, or *operators*, commonly used by logicians today.

<u>English statement</u>	<u>Translation into symbolic logic</u>	<u>Operator</u>
Cats rule.	C	
It is false that cats rule.	$\sim C$	tilde
Cats rule and dogs drool.	$C \cdot D$	dot
Cats rule or dogs drool.	$C \vee D$	wedge
If cats rule, then dogs drool.	$C \supset D$	horseshoe

Translation into symbolic logic makes writing and working with propositional statements much easier and quicker. English can be messy, but symbolic logic has a clarity and precision that many people enjoy. Consider the following more complex examples, where we use parentheses (and brackets and then braces, if needed) to disambiguate the statement's meaning. If there is a need for more than one operator (as below), the *main operator* is the symbol that tells us what kind of statement the statement is as a whole. Thus the main operator of a negation will be the tilde, for a conjunction it's the dot, for a disjunction it's the wedge, and for conditionals it's the horseshoe. To be more precise, the main operator has the largest *range* (or *scope*), covering more of the statement than any other of the statement's operators. First, let's look at some examples of symbolic translations:

1. Al and Bob like apples, or Charlie does. $(A \cdot B) \vee C$
2. Al or Bob like apples, and Charlie does. $(A \vee B) \cdot C$
3. It is false that if Sue likes strawberries then Betty likes blueberries. $\sim(S \supset B)$
4. If it is false that Sue like strawberries, then Betty likes blueberries. $\sim S \supset B$

5. It is not the case that either Juan or Vic likes oranges. $\sim(J \vee V)$
6. Not both June and Kelly like prunes. $\sim(J \bullet K)$
7. If Gene is not grand, then Wally is not wonderful. $\sim G \supset \sim W$
8. If both Lou and Don like pears, then either Sally or Tracy likes apricots. $(L \bullet D) \supset (S \vee T)$
9. Either Abdul likes artichokes, or if Sasha likes salmon then Tran does not like tuna. $A \vee (S \supset \sim T)$
10. If Naomi does not like nan, then Petra likes pita and Stu doesn't like sourdough. $\sim N \supset (P \bullet \sim S)$

The range consists of that part of the statement that the operator is acting upon (including itself). Consider the following statement: $\sim(N \vee B) \supset A$

The range of the wedge is underlined here: $\sim(\underline{N \vee B}) \supset A$

The range of the tilde is underlined here: $\underline{\sim(N \vee B)} \supset A$

And here we underline the range of the horseshoe: $\sim(N \vee B) \underline{\supset A}$

The horseshoe's range includes itself, plus its full antecedent (i.e., $\sim(N \vee B)$) and consequent (i.e., A). Since the horseshoe has the largest range, it's the main operator of the full statement, and that makes the statement as a whole a conditional. Once more in life, size matters.

****Practice Problems: Main Operators**

Consider the ten translations above, and determine the main operator for each.

Answers:

- | | |
|--------------|---------------|
| 1. wedge | 6. tilde |
| 2. dot | 7. horseshoe |
| 3. tilde | 8. horseshoe |
| 4. horseshoe | 9. wedge |
| 5. tilde | 10. horseshoe |

****Practice Problems: Translating into Symbolic Logic**

Translate the following English statements into the language of propositional symbolic logic. Use the upper-case letters provided for simple statements.

1. Carl is from Canada and Bob is from Bolivia. (CB)
2. Paulo is not from Panama or Angie is from Argentina. (PA)
3. If Frank is from France, then Gerald is German. (FG)
4. Bart is British, and either Sandy is Spanish or Paula is Portuguese. (BSP)
5. Arnold is from Angola and Mandy is not from Mali, if Patty is from Paraguay. (AMP)
6. It is false that either Carlisle is Cuban or Tran is Tahitian, and Parisa is Peruvian. (CTP)
7. Isaac is from Israel, if it's the case that both Huy is Hungarian and Adaisheva is not from Albania. (IHA)
8. Tuan is Thai or it is false that Murtaza is Mongolian, or Simeon is not from Sudan. (TMS)

9. It is false that both Terrie is not from Togo and Gamani is from Ghana, or Ben is from Benin. (TGB)
 10. Julia is Jordanian, if Mark is Malaysian and Inez is not from India. (JMI)

Answers:

- | | |
|---------------------------------|-------------------------------------|
| 1. $C \cdot B$ | 6. $\sim(C \vee T) \cdot P$ |
| 2. $\sim P \vee A$ | 7. $\sim(H \cdot \sim A) \supset I$ |
| 3. $F \supset G$ | 8. $(T \vee \sim M) \vee \sim S$ |
| 4. $B \cdot (S \vee P)$ | 9. $\sim(\sim T \cdot G) \vee B$ |
| 5. $P \supset (A \cdot \sim M)$ | 10. $(M \cdot \sim I) \supset J$ |

Valid Argument Patterns

Now that we are familiar with the patterns of some basic propositional statements, we can begin to examine propositional *argument* patterns. There are a small number that are so common, they have names. Most are wonderfully intuitive and easy to see, but it takes a moment to slow down and develop some precision with them. Our goal is to become familiar enough with these patterns that we can recognize when an argument is valid. We'll limit ourselves to becoming familiar with eight common patterns of valid inference.

Modus Ponens

A simple one is called *Modus Ponens*, which is Latin for “the Affirming Mode.” It has the following pattern (the three dots aligned in a pyramid is a common way of indicating the conclusion of an argument):

If A, then B
 A
 \therefore B

The order of the two premises does not matter, as the following is Modus Ponens, too:

A
 If A, then B
 \therefore B

It is not really possible to prove that this is a valid inference, as it's so simple and basic. But it confuses no one. If A is true, then B has to be true; and A *is* true. Thus, B is guaranteed to be true. It does not matter what statements A and B stand for; *if* the premises are true, then the conclusion is certain. Of course, one or more of the premises might be false, but that would make the valid argument unsound. All we are doing here, though, is recognizing valid arguments to be indeed valid.

Here are some examples of Modus Ponens in full English:

If Ichiro played for the Seattle Mariners, then Ichiro played baseball.
 Ichiro played for the Seattle Mariners.
 Thus, Ichiro played baseball.

Barack Obama is the U.S. president.
 If Barack Obama is the U.S. president, then Barack Obama is a politician.
 Therefore, Barack Obama is a politician.

Modus Ponens is a pattern, so technically it can be illustrated using all sorts of things besides statements or upper-case letters. For example:

If #, then \$	If Ψ , then Φ	If ☺, then ☀	◇	$A \supset B$
#	Ψ	☺	If ◇, then □	A
∴ \$	∴ Φ	∴ ☀	∴ □	∴ B

It's just a pattern.

Modus Tollens

Another common pattern is called *Modus Tollens* (Latin: “the denying mode”), and it looks somewhat similar to Modus Ponens:

If A, then B
 Not-B
 ∴ Not-A

For Modus Tollens, we have a conditional (*If A, then B*), but then state that its consequent (*B*) is false. From those two claims (the order in which they appear is irrelevant), we may confidently conclude that the antecedent (*A*) is false. Here are some English instances of Modus Tollens:

If it's raining, then the ground is wet.
 The ground is not wet.
 Thus, it's not raining.

If Hong Kong is in Oregon, then Hong Kong is in the USA.
 But Hong Kong is *not* in the USA.
 Hence, Hong Kong is not in Oregon.

It is false that Yogi [the cartoon bear] is a fish.
 If Yogi is a salmon, then Yogi is a fish.
 Thus, it is not the case that Yogi is a salmon.

If President Barack Obama pitches for the Seattle Mariners, then he is a baseball player.
 But President Obama is not a baseball player.
 Therefore, he does not pitch for the Seattle Mariners.

Technically speaking, the following three valid inferences are *not* examples of Modus Tollens, as they do not precisely match the Modus Tollens pattern:

If A, then not-B	Not-R	If not-O, then not-Y
B	If not-D, then R	Y
∴ Not-A	∴ D	∴ O

Again, to be picky—and there’s little in life as picky as deductive logic—Modus Tollens makes use of two claims: one is a conditional, and the other is the negation of the consequent of that conditional; from these two we derive the negation of the conditional’s antecedent.

For those who are beginning to like the symbolic approach, Modus Tollens looks like this symbolically:

$P \supset Q$	$\sim\sim K$	$\sim A \supset B$	$K \supset \sim J$
$\sim Q$	$L \supset \sim K$	$\sim B$	$\sim\sim J$
∴ $\sim P$	∴ $\sim L$	∴ $\sim\sim A$	∴ $\sim K$

Recall that there are two formal fallacies that look like Modus Ponens and Modus Tollens. Review the differences here, as two are valid lines of inference, while the other two are invalid formal fallacies.

<u>Modus Ponens</u> ☺	<u>Affirming the Consequent</u> ??	<u>Modus Tollens</u> ☺	<u>Denying the Antecedent</u> ??
If P, then O	If P, then O	If P, then O	If P, then O
P	O	Not-O	Not-P
∴ O	∴ P	∴ Not-P	∴ Not-O

Disjunctive Syllogism

Disjunctive Syllogism is another common deductive pattern that produces a valid argument. A Disjunctive Syllogism is made up of two premises: a disjunction and the negation of one of the disjuncts. From those two statements we are guaranteed that the other disjunct is true. For instance:

A or B	Either R or Y	Not-K	Not-L or not-Q	Not-not-not-M
Not-A	Not-Y	K or W	Not-not-L	H or not-not-M
∴ B	∴ R	∴ W	∴ Not-Q	∴ H

English examples include:

* Either Obama is a Republican or he is a Democrat. But he’s not a Republican. Thus, he’s a Democrat.

* Either Lady Gaga is not a singer or she is dancer. But it is false that she is not a singer. Hence, she is dancer.

* I'm going to study for the test. For either I study for the test or I'll fail the thing, and I don't want to fail the thing.

For those who are interested, Disjunctive Syllogism would look symbolically like this:

$P \vee O$	$\sim I$	$\sim A \vee \sim Q$	$\sim\sim\sim\sim K$	$A \vee (H \cdot M)$
$\sim P$	$H \vee I$	$\sim\sim A$	$\sim\sim\sim K \vee P$	$\sim A$
$\therefore O$	$\therefore H$	$\therefore \sim Q$	$\therefore P$	$\therefore H \cdot M$

Hypothetical Syllogism

Hypothetical Syllogism is a pattern consisting of two premises, both of which are conditionals. It is required, though, that the two conditionals “match up kitty-corner.” That is, the antecedent of one must be exactly the same as the consequent of the other. If so, we can conclude with a third conditional. We argue this way all the time. For instance:

If Yogi is a bear, then Yogi is a mammal.
 If Yogi is a mammal, then Yogi is an animal.
 Thus, if Yogi is a bear, then Yogi is an animal.

Notice how “Yogi is a bear” matches up “kitty-corner” in the premises. The pattern would still be Hypothetical Syllogism if the premises are exchanged:

If Yogi is a mammal, then Yogi is an animal.
 If Yogi is a bear, then Yogi is a mammal.
 Thus, if Yogi is a bear, then Yogi is an animal.

The following are abbreviated examples of Hypothetical Syllogism:

If J, then L	If not-U, then K	If G, then V	$A \supset B$
If L, then B	If K, then W	If not-P, then G	$B \supset C$
\therefore If J, then B	\therefore If not-U, then W	\therefore If not-P, then V	$\therefore A \supset C$

****Practice Problems: Recognizing Propositional Patterns**

For each abbreviated argument below, determine if it's a precise example of Modus Ponens, Modus Tollens, Disjunctive Syllogism, or Hypothetical Syllogism. Or is it something else?

- If E, then K
Not-K
 \therefore Not-E
- J or Y
Not-Y
 \therefore J
- If M, then V

- If V, then J
 \therefore If M, then J
4. P
 If P, then D
 \therefore D
5. If N, then O
 Not-N
 \therefore Not-O
6. Not-H
 H or T
 \therefore T
7. If not-Q, then K
 Not-Q
 \therefore K
8. If A, then X
 X
 \therefore A
9. If not-I, then H
 If not-U, then not-I
 \therefore If not-U, then H
10. If A, then S
 If A, then J
 \therefore If S, then J

Answers:

1. Modus Tollens
2. Disjunctive Syllogism
3. Hypothetical Syllogism
4. Modus Ponens
5. Something else (it's the invalid formal fallacy Denying the Antecedent)
6. Disjunctive Syllogism
7. Modus Ponens
8. Something else (it's the invalid formal fallacy Affirming the Consequent)
9. Hypothetical Syllogism
10. Something else (it's an unnamed invalid inference)

Assessing Arguments

Let's now *do* something with our knowledge of these four propositional logic patterns. We're going to introduce a technique called *natural deduction*. This technique is a way of proving that valid arguments are indeed valid. We'll not look at the full system (you can do that in Symbolic Logic, i.e., PHIL& 120), but we'll get a taste of it here, and focus on the basic sort of argumentation we're likely to run into in ordinary, workaday situations.

We'll here work only with valid arguments. We'll also write them a little differently. Consider the following argument:

If Angie is from Albania, then Katie is not from Kenya. Angie is indeed from Albania. Katie is from Kenya or Phillip is from the Philippines. It follows that Phillip is from the Philippines.

We can abbreviate this argument as follows, lining the premises up vertically, and placing the conclusion to the right of the last premise, after a slash:

1. If A, then not-K
2. A
3. K or P / P

In natural deduction we use valid patterns of logic to deduce, or infer, the conclusion. Recall that with valid arguments, the truth of the premises will guarantee that the conclusion is true. So, if we can “get” the conclusion from the premises, we'll have shown that the premises guarantee the truth of the conclusion, and that the argument is valid.

To do this, we look at the premises and see if we can find any logical move to make. That is, can we see the possibility of applying any of the propositional patterns we have learned so far? Yes! If we focus on lines 1 and 2, we see that they form the Modus Ponens pattern, and they give us Not-K. So let's write that down, justifying the newly acquired statement by appealing to the lines used and the pattern (or “rule”) we used. Just so we don't have to write down the complete name of patterns, let's refer to Modus Ponens as *MP*, to Modus Tollens as *MT*, to Disjunctive Syllogism as *DS*, and to Hypothetical Syllogism as *HS*.

1. If A, then not-K
2. A
3. K or P / P
4. Not-K 1, 2 MP

We've yet to get the conclusion, so we've yet to prove this argument to be valid. But let's continue. We now have four lines available to us. Do we see another pattern we can use (or an additional use of MP)? Yes! We can use lines 3 and 4 with DS to get *P*, our conclusion! For those who are interested, to the right is the same proof using symbolic logic (it's not much different, just simpler).

- | | | | |
|-------------------------|--|-----------------------|---------|
| 1. If A, then not-K | | 1. $A \supset \sim K$ | |
| 2. A | | 2. A | |
| 3. K or P / P | | 3. $K \vee P / P$ | |
| 4. Not-K 1, 2 MP | | 4. $\sim K$ | 1, 2 MP |
| 5. P 3, 4 DS | | 5. P | 3, 4 DS |

Once we deduce (or “get”) the conclusion, we're done! We have shown that if the premises are true, then the conclusion *must* be true. The argument is therefore *valid*.

Here's another example that's already abbreviated (with another example of how such a deduction would look using symbolic logic):

- | | | | |
|-----------------|---------|------------------|---------|
| 1. A or D | | 1. $A \vee D$ | |
| 2. If A, then M | | 2. $A \supset M$ | |
| 3. Not-M / D | | 3. $\sim M / D$ | |
| 4. Not-A | 2, 3 MT | 4. $\sim A$ | 2, 3 MT |
| 5. D | 1, 4 DS | 5. D | 1, 4 DS |

We begin here by seeing that lines 3 and 4 fit the Modus Tollens pattern, so we use MT to get *Not-A*. That's not the conclusion, so we trundle forward. Scanning all four lines at that point, we see that lines 1 and 4 fit the Disjunctive Syllogism pattern, so we use DS to get *D*, which *is* the conclusion. We are then finished, and we've shown the argument to be valid.

Here's one more example.

1. If A, then not-G
2. If not-G, then K
3. If K, then either O or B / If A, then either O or B
4. If A, then K 1, 2 HS
5. If A, then either O or B 3, 4 HS

We began by seeing that lines 1 and 2 fit the Hypothetical Syllogism pattern to give us line 4: *If A, then K*. We then saw that the consequent of line 4 matched up with the antecedent of line 3, providing us another opportunity to use Hypothetical Syllogism, this time to give us the conclusion.

Some deductions (or *proofs*) take one or two steps, but others can take much longer. It all depends on how complex the argument is and how many patterns we have at our disposal. If we learned an infinite number of patterns, each deduction would take only one step. But who wants to learn that many patterns? If we learn only a small number of patterns, many of the deductions will be quite long and difficult. That doesn't sound very good, either. So, for our introductory purposes, we'll learn a small number (we'll examine only four more) and keep the deductions reasonably short.

****Practice Problems: Deductions Using the First Four Patterns**

Use the first four propositional patterns and natural deduction to prove the following arguments to be valid.

1. 1. If A, then if B then C
 2. B
 3. A / C

2. 1. D or not-H

2. L and K
3. If both L and K, then not-D / Not-H
3.
 1. If A, then G
 2. If not-A, then M
 3. Not-G / M
4.
 1. If D, then C
 2. If it is the case that if D then N, then F
 3. If C, then N / F
5.
 1. Not-Q
 2. If R, then Q
 3. If not-R, then if A then Q / Not-A
6.
 1. G, or B or M
 2. Not-B
 3. If A, then not-G
 4. A / M
7.
 1. If M, then if N then O
 2. If P, then if O then S
 3. M
 4. P / If N, then S
8.
 1. If N, then B
 2. N
 3. If B, then not-Q
 4. Q or A / A
9.
 1. G or S
 2. If A, then B or M
 3. Not-B
 4. A
 5. If M, then not-G / S
10.
 1. If B, then Q
 2. Not-A
 3. Not-Q
 4. If F, then A
 5. F, or if not-B then D / D

Answers:

1.
 1. If A, then if B then C
 2. B

3. A / C
 4. If B, then C 1, 3 MP
 5. C 2, 4 MP
2. 1. D or not-H
 2. L and K
 3. If both L and K, then not-D / Not-H
 4. Not-D 2, 3 MP
 5. Not-H 1, 4 DS
3. 1. If A, then G
 2. If not-A, then M
 3. Not-G / M
 4. Not-A 1, 3 MT
 5. M 2, 4 MP
4. 1. If D, then C
 2. If it is the case that if D then N, then F
 3. If C, then N / F
 4. If D, then N 1, 3 HS
 5. F 2, 4 MP
5. 1. Not-Q
 2. If R, then Q
 3. If not-R, then if A then Q / Not-A
 4. Not-R 1, 2 MT
 5. If A, then Q 3, 4 MP
 6. Not-A 1, 5 MT
6. 1. G, or B or M
 2. Not-B
 3. If A, then not-G
 4. A / M
 5. Not-G 3, 4 MP
 6. B or M 1, 5 DS
 7. M 2, 6 DS
7. 1. If M, then if N then O
 2. If P, then if O then S
 3. M
 4. P / If N, then S
 5. If N, then O 1, 3 MP
 6. If O, then S 2, 4 MP
 7. If N, then S 5, 6 HS

8. 1. If N, then B
 2. N
 3. If B, then not-Q
 4. Q or A / A
 5. B 1, 2 MP
 6. Not-Q 3, 5 MP
 7. A 4, 6 DS
9. 1. G or S
 2. If A, then B or M
 3. Not-B
 4. A
 5. If M, then not-G / S
 6. B or M 2, 4 MP
 7. M 3, 6 DS
 8. Not-G 5, 7 MP
 9. S 1, 8 DS
10. 1. If B, then Q
 2. Not-A
 3. Not-Q
 4. If F, then A
 5. F, or if not-B then D / D
 6. Not-B 1, 3 MT
 7. Not-F 2, 4 MT
 8. If not-B, then D 5, 7 DS
 9. D 6, 8 MP

Simplification

We'll now look at four more propositional patterns. Two are super easy, one is weird, and one is a mild pain in the neck. *Simplification* (abbreviated as *Simp*) is one of the easy ones. What Simplification says is if a conjunction is true, then each conjunct is true by itself. This is another case where talking about it makes it sound more complicated than it is. Here's what Simplification can do:

Mark is a medic and Mark is an athlete. Thus, Mark is a medic.

Or

Mark is a medic and Mark is an athlete. Thus, Mark is an athlete.

Obviously, if Mark is both a medic and an athlete, then he's a medic...or an athlete.

The pattern is so simple that it's hard to come up with a variety of illustrations of its use. But here are some:

A and G	A and G	Not-T and K	Not-R and either H or E	L, and B and W
$\therefore A$	$\therefore G$	$\therefore \text{Not-T}$	$\therefore H \text{ or } E$	$\therefore B \text{ and } W$

If you have a conjunction, you can think of the two conjuncts as two pieces of ripe fruit ready to be plucked. Simplification makes a somewhat complex conjunction simpler by reducing it to one of its conjuncts. Simplification can now be added to our growing list of patterns used in natural deduction.

1. A and G		1. N or J	
2. If A, then G / G		2. Not-N and G	
3. A	1 Simp	3. If J, then Q / Q	
4. G	2, 3 MP	4. Not-N	2 Simp
		5. J	1, 4 DS
		6. Q	3, 5 MP

Conjunction

Conjunction is another easy pattern to recognize. Conjunction (*Conj* for short) says that if you know any two statements to be true, then they are both true. That may seem *way* too obvious, but it's an important pattern in logic. Here's how it can work:

- * Canada is north of the USA. The USA is north of Mexico. Thus, Canada is north of the USA, and the USA is north of Mexico.
- * Theresa is tall. Theresa is a logician. Thus, Theresa is tall and Theresa is a logician.
- * Theresa is tall. Theresa is a logician. Thus, Theresa is a logician and Theresa is tall.

We might abbreviate the second argument above this way:

T
L
 $\therefore T \text{ and } L$

The individual statements can be simple or compound (with a complex symbolic example thrown in for those relishing such things):

K or B	If J, then Y	$A \bullet (B \vee \sim O)$
If G, then B	If not-P, then D	$\sim H \supset L$
$\therefore K \text{ or } B, \text{ and if } G \text{ then } B$	$\therefore \text{If } J \text{ then } Y, \text{ and if not-P then } D$	$\therefore [A \bullet (B \vee \sim O)] \bullet (\sim H \supset L)$

Here are two natural deduction proofs using Conjunction:

1. If both A and B, then C	1. K and G
----------------------------	------------

2. A		2. Not-L and E / K and not-L	
3. B / B and C		3. K	1 Simp
4. A and B	2, 3 Conj	4. Not-L	2 Simp
5. C	1, 4 MP	5. K and not-L	3, 4 Conj
6. B and C	3, 5 Conj		

Addition

This pattern may feel like cheating. *Addition* allows you to take any complete statement, and add any other statement to it to make a disjunction. Examples include:

A	G	U	If P, then N	Not-Y
$\therefore A \text{ or } B$	$\therefore G \text{ or not-G}$	$\therefore U \text{ or } U$	$\therefore \text{If P then N, or not-J}$	$\therefore \text{Not-Y or E}$

In English, this might look like this:

Dogs are animals. Thus, dogs are animals or whales fly through the air with pink wings.

As long as the first statement is true (and it is here), then it really does not matter what statement (true or false) we add to it; the disjunction as a whole will be true.

Here are examples of using Addition (*Add* for short) in natural deduction.

1. K		1. If A, then B	
2. If either K or G, then P / P		2. A / B or W	
3. K or G	1 Add	3. B	1, 2 MP
4. P	2, 3 MP	4. B or W	3 Add
1. Not-J		1. $\sim K \cdot Z$	
2. If either not-J or not-M, then Q		2. $(\sim K \vee R) \supset \sim O$	
3. If Q, then P / P, or both Y and E		3. $O \vee (K \vee H) / (H \vee I) \vee \sim P$	
4. Not-J or not-M	1 Add	4. $\sim K$	1 Simp
5. Q	2, 4 MP	5. $\sim K \vee R$	4 Add
6. P	3, 5 MP	6. $\sim O$	2, 5 MT
7. P, or both Y and E	6 Add	7. $K \vee H$	3, 6 DS
		8. H	4, 7 DS
		9. $H \vee I$	8 Add
		10. $(H \vee I) \vee \sim P$	9 Add

Constructive Dilemma

Well, one had to be the worst, and here it may be: *Constructive Dilemma* (*CD* for short). This pattern appeals to *three* statements to make an inference. Two are conditionals, and one is a disjunction made up of the antecedents of the two conditionals. What we can infer from this mess is another disjunction made up of the consequents of the two conditionals. Maybe looking

at some abbreviated examples will make more sense than verbally trying to describe it (the order of the three premises is irrelevant, of course). Let's also give those growing number of symbolic logic fans a peek at how Constructive Dilemma can look.

If P, then Q	Not-L or G	$P \supset Q$
If R, then S	If not-L, then D	$R \supset S$
P or R	If G, then not-K	$P \vee R$
$\therefore Q$ or S	$\therefore D$ or not-K	$\therefore Q \vee S$

Perhaps two English examples might help?

If I eat good food, then I'll be healthy.	If Joe studies tonight, then he'll pass the test.
If I eat junk food, I'll be unhealthy.	If Joe plays games tonight, then he'll have fun.
Either I'll eat good food or junk food.	Joe is either going to study or play games tonight.
Thus, I'll either be healthy or unhealthy.	Thus, Joe will either pass the test or have fun.

Here's how Constructive Dilemma might show up in a natural deduction proof:

1. If K then Y, and if J then B	1. If M, then B
2. K or J / Y or B	2. M or H
3. If K, then Y 1 Simp	3. If H, then Q
4. If J, then B 1 Simp	4. If B or Q, then A and E / E or W
5. Y or B 2, 3, 4 CD	5. B or Q 1, 2, 3 CD
	6. A and E 4, 5 MP
	7. E 6 Simp
	8. E or W 7 Add

****Practice Problems: Natural Deduction**

Use the eight propositional patterns and natural deduction to prove that each of the following arguments is valid. [We'll give the last beefy problem to those who have embraced the optional joys of symbolic logic.]

1. Not-Z, and either H or W
2. If H, then Y
3. If W, then Z / Y or Z
1. A and not-P
2. B and J / A and J
1. L and Q
2. K and not-W
3. If both Q and K, then D / D
1. If M then B, and P
2. If S, then I

3. S or M / I or P
5.
 1. A
 2. If either A or B, then S / S
6.
 1. Not-G
 2. G or not-D
 3. If F, then D / Not-F
7.
 1. T, and M or S
 2. If T, then not-M / T and S
8.
 1. If not-A, then B
 2. Not-A or D
 3. If D, then H / B or H, or not-M
9.
 1. Not-B
 2. H
 3. If both not-B and H, then either B or N / N
10.
 1. If A, then G
 2. If it's the case that if A then not-R, then it's the case that if F then G
 3. If G, then not-R / If F, then not-R
11.
 1. If B then G, and F
 2. M and S, and if A then D
 3. B or A / G or D
12.
 1. E and F
 2. S
 3. If both S and F, then H / H or not-F
13.
 1. If L, then J
 2. Not-J
 3. L or O
 4. If O, then either J or M / M and not-J, or J
14.
 1. It is false that either A or B
 2. A / Z
15.
 1. A
 2. S
 3. If both A and S, then P / P and S
16.
 1. Not-D and R

2. D or L
 3. If L, then S / S and R, or not-L
17.
 1. If G, then H
 2. Not-H and M
 3. If both not-G and M, then either H or P / P
 18.
 1. If R then S, and if S then not-D
 2. If it's the case that if R then not-D, then R
 3. D, or if S then M / If R then M, or if M then R
 19.
 1. A and not-C
 2. If A, then C or not-D
 3. If F, then D / Not-F or not-A, and A
 20.
 1. Not-A and not-I
 2. If not-Q, then A
 3. If not-A, then X
 4. If both X and not-not-Q, then K / K and not-I
 21.
 1. $(R \cdot P) \supset (Q \vee I)$
 2. $(\sim Q \vee A) \supset R$
 3. $(\sim Q \vee J) \supset P$
 4. $\sim Q \cdot N / [I \cdot (\sim Q \vee A)] \vee (P \supset I)$

Answers:

1.
 1. Not-Z, and H or W
 2. If H, then Y
 3. If W, then Z / Y or Z
 4. H or W 1 Simp
 5. Y or Z 2, 3, 4 CD
2.
 1. A and not-P
 2. B and J / A and J
 3. A 1 Simp
 4. J 2 Simp
 5. A and J 3, 4 Conj
3.
 1. L and Q
 2. K and not-W
 3. If both Q and K, then D / D
 4. Q 1 Simp
 5. K 2 Simp
 6. Q and K 4, 5 Conj
 7. D 3, 6 MP

4.
 1. If M then B, and P
 2. If S, then I
 3. S or M / I or B
 4. If M, then B 1 Simp
 5. I or P 2, 3, 4 CD

5.
 1. A
 2. If either A or B, then S / S
 3. A or B 1 Add
 4. S 2, 3 MP

6.
 1. Not-G
 2. G or not-D
 3. If F, then D / Not-F
 4. Not-D 1, 2 DS
 5. Not-F 3, 4 MT

7.
 1. T, and M or S
 2. If T, then not-M / T and S
 3. T 1 Simp
 4. M or S 1 Simp
 5. Not-M 2, 3 MP
 6. S 4, 5 DS
 7. T and S 3, 6 Conj

8.
 1. If not-A, then B
 2. Not-A or D
 3. If D, then H / B or H, or not-M
 4. B or H 1, 2, 3 CD
 5. B or H, or not-M 4 Add

9.
 1. Not-B
 2. H
 3. If both not-B and H, then either B or N / N
 4. Not-B and H 1, 2 Conj
 5. B or N 3, 4 MP
 6. N 1, 5 DS

10.
 1. If A, then G
 2. If it's the case that if A then not-R, then it's the case that if F then G
 3. If G, then not-R / If F, then not-R
 4. If A, then not-R 1, 3 HS
 5. If F, then G 2, 4 MP
 6. If F, then not-R 3, 5 HS

11. 1. If B then G, and F
 2. M and S, and if A then D
 3. B or A / G or D
 4. If B, then G 1 Simp
 5. If A, then D 2 Simp
 6. G or D 3, 4, 5 CD
12. 1. E and F
 2. S
 3. If both S and F, then H / H or not-F
 4. F 1 Simp
 5. S and F 2, 4 Conj
 6. H 3, 5 MP
 7. H or not-F 6 Add
13. 1. If L, then J
 2. Not-J
 3. L or O
 4. If O, then either J or M / M and not-J, or J
 5. Not-L 1, 2 MT
 6. O 3, 5 DS
 7. J or M 4, 6 MP
 8. M 2, 7 DS
 9. M and not-J 2, 8 Conj
 10. M and not-J, or J 9 Add
14. 1. It is false that either A or B
 2. A / Z
 3. A or B 2 Add
 4. A or B, or Z 3 Add
 5. Z 1, 4 DS
15. 1. A
 2. S
 3. If both A and S, then P / P and S
 4. A and S 1, 2 Conj
 5. P 3, 4 MP
 6. P and S 2, 5 Conj
16. 1. Not-D and R
 2. D or L
 3. If L, then S / S and R, or not-L
 4. Not-D 1 Simp
 5. L 2, 4 DS

- | | | |
|--|----------------------|-----------|
| | 6. S | 3, 5 MP |
| | 7. R | 1 Simp |
| | 8. S and R | 6, 7 Conj |
| | 9. S and R, or not-L | 8 Add |
- 17.
- | | | |
|--|--|-----------|
| | 1. If G, then H | |
| | 2. Not-H and M | |
| | 3. If both not-G and M, then either H or P / P | |
| | 4. Not-H | 2 Simp |
| | 5. Not-G | 1, 4 MT |
| | 6. M | 2 Simp |
| | 7. Not-G and M | 5, 6 Conj |
| | 8. H or P | 3, 7 MP |
| | 9. P | 4, 8 DS |
- 18.
- | | | |
|--|--|---------|
| | 1. If R then S, and if S then not-D | |
| | 2. If it's the case that if R then not-D, then R | |
| | 3. D, or if S then M / If R then M, or if M then R | |
| | 4. If R, then S | 1 Simp |
| | 5. If S, then not-D | 1 Simp |
| | 6. If R, then not-D | 4, 5 HS |
| | 7. R | 2, 6 MP |
| | 8. Not-D | 6, 7 MP |
| | 9. If S, then M | 3, 8 DS |
| | 10. If R, then M | 4, 9 HS |
| | 11. If R then M, or if M then R | 10 Add |
- 19.
- | | | |
|--|---|-----------|
| | 1. A and not-C | |
| | 2. If A, then C or not-D | |
| | 3. If F, then D / Not-F or not-A, and A | |
| | 4. A | 1 Simp |
| | 5. C or not-D | 2, 4 MP |
| | 6. Not-C | 1 Simp |
| | 7. Not-D | 5, 6 DS |
| | 8. Not-F | 3, 7 MT |
| | 9. Not-F or not-A | 8 Add |
| | 10. Not-F or not-A, and A | 4, 9 Conj |
- 20.
- | | | |
|--|--|---------|
| | 1. Not-A and not-I | |
| | 2. If not-Q, then A | |
| | 3. If not-A, then X | |
| | 4. If both X and not-not-Q, then K / K and not-I | |
| | 5. Not-A | 1 Simp |
| | 6. X | 3, 5 MP |
| | 7. Not-not-Q | 2, 5 MT |

- | | | |
|-----|--|------------|
| | 8. X and not-not-Q | 6, 7 Conj |
| | 9. K | 4, 8 MP |
| | 10. Not-I | 1 Simp |
| | 11. K and not-I | 9, 10 Conj |
| 21. | 1. $(R \cdot P) \supset (Q \vee I)$ | |
| | 2. $(\sim Q \vee A) \supset R$ | |
| | 3. $(\sim Q \vee J) \supset P$ | |
| | 4. $\sim Q \cdot N / [I \cdot (\sim Q \vee A)] \vee (P \supset I)$ | |
| | 5. $\sim Q$ | 4 Simp |
| | 6. $\sim Q \vee A$ | 5 Add |
| | 7. $\sim Q \vee J$ | 5 Add |
| | 8. R | 2, 6 MP |
| | 9. P | 3, 7 MP |
| | 10. $R \cdot P$ | 8, 9 Conj |
| | 11. $Q \vee I$ | 1, 10 MP |
| | 12. I | 5, 11 DS |
| | 13. $I \cdot (\sim Q \vee A)$ | 6, 12 Conj |
| | 14. $[I \cdot (\sim Q \vee A)] \vee (P \supset I)$ | 13 Add |

Chapter 11: Causal Arguments

We have looked briefly at causal arguments twice in this text. We first met them as a pattern of inductive argumentation. Secondly, we saw them in False Cause and Slippery Slope informal fallacies. Now it's time to look at causal argumentation more closely, and to see how one might reason well in drawing a conclusion about the cause of an event.

Necessary and Sufficient Causes

First, we need to be aware of some different kinds of spatio-temporal causation. Philosophers and scientists often appeal to two kinds of causes in particular: necessary and sufficient. A *necessary* cause is a state of affairs that is *needed* for another state of affairs to take place. Without the necessary cause, the second event will not take place. For instance, oxygen is necessary (i.e., needed) for a match to catch fire, but it is not sufficient. The match can be surrounded by oxygen, but still not catch fire. However, the match *needs* oxygen to catch fire. Other examples of necessary causes include the following:

- * Water is a necessary cause of plant health.
- * Force is needed for a coiled spring to be lengthened.
- * Paint is needed for an artist to paint a portrait.
- * Laborers are needed for farm crops to be harvested.
- * Fertilization is necessary for pregnancy to occur.
- * People need air to live.
- * Having power will cause a radio to play.

A *sufficient* cause is a state of affairs that is *enough* for another state of affairs to take place. There may be more than one factor that can cause an event to take place, but a sufficient cause is one of them. If a man is thirsty, water is usually *enough* to quench his thirst. He does not *need* water, as other liquids might do the job. Water is thus a sufficient—but not a necessary—cause for quenching a man's thirst. Other examples of sufficient causes include:

- * Placing a loaf of bread in one's home freezer will cause that loaf of bread to freeze.
- * Heating a pot of broth on a very hot stove is sufficient to boil the broth.
- * Exercising more and eating less is enough to lose weight.
- * Breaking a leg playing football is enough to make someone injured.
- * Eating cheesecake will give most logic instructors pleasure.
- * Having a billion dollars causes a person to be financially rich.
- * Dropping a radio from a high-flying airplane will cause the radio to break.
- * Lack of gasoline is a sufficient cause of a car not running.

If a causal relation is claimed—as in “C causes E”—then ask if C is *needed* or merely *enough* for E to take place. If the former, then C is a necessary cause of E; if the latter, then C is a sufficient cause of E.

Some causes are both necessary and sufficient for something to *obtain* (that is, to be the case as an event, occurrence, or state of affairs). For example, a logic instructor might say, “Our only course assignments are three tests. To get an A in this course, you will need to get an A on those three tests. Moreover, getting an A on those three tests is enough to guarantee that you’d get an A in the course, regardless of what else you may or may not do here.” This instructor is outlining the necessary and sufficient causes for getting an A in his class. Getting As on all three tests is—he claims—both needed and enough to get an A for the course.

The topic of causality can be vastly complex, and philosophers, scientists, doctors, and others are still looking into it. A paleontologist may want to know what caused a dinosaur to die; an epidemiologist may want to know what is causing a skin irritation; a sociologist may want to know what causes a culture to adopt a ritual; an environmental activist may want to know how best to cause people to use automobiles less. Some of this discussion hinges on what we mean by *cause*. Necessary and sufficient causes are often relevant, but a full discussion can hardly stop there. *Contributory* causes may be neither necessary nor sufficient, but causally relevant to a state of affairs obtaining. For instance, eating leafy greens—nutritionists tell us—in some sense causes bodily health. Yet leafy greens are not *necessary* for such health (there are people in the world who are healthy but who have no or little access to leafy greens), nor are they *sufficient* (you can have all the leafy greens imaginable, but still be unhealthy because you lack other important foods).

Australian philosopher J. L. Mackie (1917–1981) provides a more complex analysis of causation. In *The Cement of the Universe: A Study of Causation* (Oxford: Oxford University Press, 1980) he refers to *INUS conditions*. Mackie argues that what most of us have referred to as necessary or sufficient causes are often a combination, or plurality, of causes. An effect, he holds, can be produced by a variety of distinct groups of factors. Each group is sufficient to cause the effect, but no particular group is itself necessary to do so.

Normal events like a house catching fire, the initiation of a street riot, the learning of a new language, or the hitting of a baseball with a bat involve complex causal relations. An event E is caused not by one specific event A, but more likely by a group of conjoined events A+B+C+D. But, Mackie continues, E might also have been caused by a different set of conjoined causal factors: G+H+I or H+A+K+L. One can be guilty of the informal fallacy of False Cause if one “oversimplifies” a complex causal relationship issue by saying that E was caused by one specific causal factor (e.g., A).

Mackie refers to each individual factor of any group of causal factors as an *INUS condition* for E. The acronym refers to his description of an INUS condition as an *insufficient* but *nonredundant* part of an *unnecessary* but *sufficient* condition for E. He illustrates his meaning with a house fire caused (in part) by a short circuit. The short circuit is *insufficient* by itself, since it alone would not have started the fire. The short circuit was *nonredundant* (or needed) in the group of causal factors, however, as without it, the other factors would not have been enough to initiate the fire. The set of factors that includes the short circuit is together *sufficient* to start the fire, but *unnecessary*, as some other group of factors (e.g., ones involving lightning or an arsonist) could account for the blaze.

Classes in Metaphysics (the philosophical study of the ultimate nature of reality) and Philosophy of Science are probably the best places to tackle the intricate field of causation fully. For our purposes here, we'll limit our inquiry to necessary and sufficient causes.

****Practice Problems: Necessary and Sufficient Causes**

Are the following causes necessary only, sufficient only, both necessary and sufficient, or neither necessary nor sufficient for the state of affairs noted to obtain? Imagine ordinary circumstances for each case.

1. A large rock thrown through a window causes the window to break.
2. Diesel fuel causes Bob's diesel truck to run.
3. Hitting someone's leg with a small marshmallow causes her leg to be injured.
4. Getting the stomach flu causes one to feel poorly.
5. A rise in temperature will cause the mercury to rise in a thermometer.
6. Covering a campfire with dirt will cause it to go out.
7. Natural or artificial sunlight will cause photosynthesis to take place in plants.
8. Placing blue litmus paper in acid will cause the paper to turn red.
9. An ordinary lamp must have a bulb for the light to shine.
10. Eating five large hamburgers will cause a hungry person to feel full.

Answers:

- | | |
|-------------------------------------|----------------------------------|
| 1. Sufficient only | 6. Sufficient only |
| 2. Necessary only | 7. Necessary only |
| 3. Neither necessary nor sufficient | 8. Both necessary and sufficient |
| 4. Sufficient only | 9. Necessary only |
| 5. Both necessary and sufficient | 10. Sufficient only |

Mill's Methods

John Stuart Mill (1806-1873) was an important British philosopher famous for his books *Utilitarianism* (1861; read in many college Ethics classes) and *On Liberty* (1859; read in many Social Philosophy classes). He also wrote a book on logic: *System of Logic, Ratiocinative and Inductive* (1843; rarely read in logic classes today). The part of this logic book that logicians continue to draw upon is Mill's discussion of causal arguments. Although Mill's analysis has been improved upon, most logic and critical thinking textbooks produced today give credit to Mill by presenting "Mill's Methods." Those "methods" are five ways by which we can argue that C is a cause of E.

As it turns out, Mill did not really discover anything; ordinary people like your auto mechanic and doctor use these lines of reasoning all the time to try to determine the cause of your engine or stomach trouble. Most of this will seem like perfectly common sense, and it is. We'll just be making our reasoning about causation a little more precise.

Method of Agreement

Imagine that five students leave the school cafeteria one day, and all immediately begin to get physically, grossly sick, seemingly from something they ate. Given the high quality and social consciousness of the food offered at our cafeteria, this may be hard to imagine, but give it a try nonetheless. You are a doctor assigned to determine the likely cause of the public malady. A janitor is waiting in the wings, mop and pail in hand, to see if additional students become ill. What would you do?

The first and obvious response is to ask each of the five students what he or she had just eaten. You are hoping that there is a food item they each ate. Let's imagine this is what you found out:

- Al ate potato chips and french fries.
- Barbara ate french fries, a taco, and a donut.
- Charlie ate pizza, a corn dog, and stole some of Barbara's french fries.
- Debbie mixed her french fries in a side of gravy.
- Ellen is a vegetarian and ate only french fries, a donut, and potato chips.

What is the likely cause of the illness? French fries! Why? Because it is the one food that all five students ate, or *agreed* upon. It is the one factor in agreement among all the various foods they ate that woeful afternoon. It's not, of course, guaranteed that it was the french fries that caused the illness (it might have been the plates the fries were served on), but this is inductive reasoning, and guaranteed conclusions are not expected. Still, the fries are the likely suspect, and further inquiry or testing should be directed their way.

We can create a table to visually show the data collected. Place the various cases to the left of the table, and on top of the table place the various possible causes of the phenomenon in question (i.e., the illness). At the top right we'll note the phenomenon in question. For convention's sake, we'll use an *x* for when the cause obtains, and a *dash* for when it does not.

	Potato chips	French fries	Taco	Donut	Pizza	Corn dog	Gravy	<i>Became ill</i>
Al	x	x	-	-	-	-	-	x
Barbara	-	x	x	x	-	-	-	x
Charlie	-	x	-	-	x	x	x	x
Debbie	-	x	-	-	-	-	x	x
Ellen	x	x	-	x	-	-	-	x

It is only the french fries column that agrees perfectly with the column under the phenomenon in question. It's *possible* that Al had an allergic reaction to the potato chips he ate, and each of the others responded individually and uniquely to some food or combination of foods they ingested, but the likely cause of the group's illness was the one thing they all ate in common: the fries. It

looks like, moreover, that eating the fries was *needed* for them to get sick. Donuts were not needed, for instance, because Al, Debbie, and Charlie got sick without eating them. French fries were thus likely a necessary cause for these five students becoming ill in this manner this day. We can go so far as to say that if the Method of Agreement gives clear results, it can point to a *necessary* (but not sufficient) cause of an event.

The Method of Agreement works particularly well in hindsight, or retrospectively. If we have a group of events and want to know their cause, we can go back, so to speak, and try to determine what causes obtained for each of them. The Method of Agreement works better if there are more examples of things experiencing the phenomenon in question. So, if ten students got sick this woe begotten day, and all ten ate french fries, then we'd have stronger reason to believe it was indeed the fries that caused the illness. If we had only two students who became ill, and fries were the only thing they *both* had eaten, we could still conclude it was probably the fries that were the problem, but the argument would be weaker. Ideally, we'll have at least three cases to compare—with more being better—when appealing to the Method of Agreement. Anything less than three will make for a fairly weak inference; it would not be enough for us to see a solid pattern.

Method of Difference

Another line of causal reasoning is called the *Method of Difference*. Whereas the Method of Agreement is usually retrospective (i.e., looking back for a cause of an effect that already took place), the Method of Difference is usually forward looking and more controlled. In the Method of Difference, we take two things and make sure they are exactly the same in all relevant respects, except for one key difference. If one thing experiences the phenomenon in question and the other doesn't, then that one key difference is likely the cause of the phenomenon.

For instance, let's say you want to determine whether Kwik-Gro fertilizer stimulates plant growth. We might take two plants of the same variety, age, and health, and place them in similar pots, using similar soil. We give them exactly the same amount of sunlight, air circulation, and water. Everything is controlled to be the same, *except* we feed Plant A with Kwik-Gro, but do not give the amendment to Plant B. If Plant A grows significantly more vigorously than Plant B, we are justified in concluding that Kwik-Gro fertilizer is the cause of the extra growth.

Note that we can do this test in parallel, taking 200 similar plants and treating them all exactly the same, except we give Kwik-Gro to 100 plants but not to the others. This is using the Method of Difference in a 100-pair series. Such duplication of the method helps avoid unexpected differences in the plants, soils, water, air, or other factors. If 90 percent of the plants treated with Kwik-Gro showed substantial increase in growth, and none of those not treated with Kwik-Gro did, then we could say that there was approximately a 90 percent chance that Kwik-Gro will assist growth. Testing with more pairs of plants or finding a higher percentage in the results will make our conclusion stronger.

The Method of Difference works well on inanimate objects and plants, but is morally problematic when used on sentient animals; it's even more problematic when used on humans.

Humans are highly complex creatures, and no two are close to being exactly alike. Rats from the same parents, houseflies, and other simpler critters can be enough alike so that we can readily limit the test to one controlled difference in an experiment. Humans are too complex, and additional differences become apparent and undercut the conclusion that the one *intended* difference is the cause of the phenomenon in question. Also, it is often immoral to control humans to such an extent that only one difference is allowed. It would take caging a pair of humans and forcing upon them the same food, water, air, social companionship, freedom, rational discourse, etcetera to attempt the use of Method of Difference. The Nazis did this with unwilling humans, and the world still refuses to make use of the findings.

We can set up a chart for the Method of Difference:

	1 pint tap water / day	Southern exposure to sun	Soil X	Kwik-Gro	Variety of plant	1 week old at start	Healthy at start	Constant air circulation	<i>Extra growth</i>
Plant A	x	x	x	x	x	x	x	x	x
Plant B	x	x	x	-	x	x	x	x	-

This chart shows that the two plants were the same—as best as we could determine—in all relevant ways except that one received Kwik-Gro and the other did not. The one that received Kwik-Gro experienced extra growth; the other did not. Since Kwik-Gro was the only known relevant difference, we can conclude that it likely is the direct cause of the extra growth.

The results of the Method of Difference can be the inverse of this, though. Instead of the one subject that has the difference experiencing the phenomenon in question, it could be that the subject failing to have the differing trait experiences the phenomenon in question. For instance, suppose that two people (we *can* use people if we take moral considerations seriously) with similar tendencies for headaches are—as far as we can tell—the same in every relevant respect, except that for two weeks we give one daily doses of pain reliever Z. As it turns out, the person who takes Z does not get headaches over this time period, while the other person does get the regular headaches. Since pain reliever Z was the only known relevant difference, we can conclude that taking Z helps people from getting (at least those kinds of) headaches.

	Food	Drink	Exercise	Social life	Work pressure	Relaxation and recreation	Pain reliever Z	<i>Headaches</i>
Subject A	x	x	x	x	x	x	x	-

Subject B	x	x	x	x	x	x	-	x
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Here the Method of Difference is working in an inverse manner, but it shows what is enough to prevent headaches. We can also note that pain reliever Z may not be *necessary* to stop headaches; pain reliever Y or a good neck massage might work well, too. Still, Z appears to be *sufficient* to do the job. So, to speak more generally, the Method of Difference—when the results are clear—can provide a *sufficient* cause for a phenomenon in question.

Thus, the Method of Difference differs from the Method of Agreement in a variety of ways. The Method of Agreement looks backwards at what already happened, and tries to discover the cause of that event. The Method of Difference is usually looking forward at what will cause a specified phenomenon. The Method of Agreement works best appealing to three or more cases, while the Method of Difference works with two (or a multiple series of two). The Method of Agreement is often not controlled (since it is usually retrospective), while the Method of Difference is almost always highly controlled to ensure only one relevant difference. Finally, the Method of Agreement—if the results are clear—can point to a necessary cause, while the Method of Difference—if the results are clear—can point to a sufficient cause.

Joint Method

The Joint Method of Agreement and Difference—or more simply, the *Joint Method*—looks to be a bit like a combination of the Method of Agreement and the Method of Difference. It’s a broader ranging line of reasoning than either by itself, but it might be better understood as an expansion on the Method of Agreement. Whereas the Method of Agreement looks at a group of events (or individuals, or states of affairs) that exhibit some specific phenomenon (e.g., five students getting sick from eating food in the school cafeteria), the Joint Method takes into consideration both those who exhibit the phenomenon and those who don’t (or at least multiple instances of each). Consider the following dining fiasco:

Andy, Bella, Chikka, David, Ellen, Fred entered the Chunk-O-Cheese Pizzeria together in seemingly fine health, and ate dinner there. Soon afterwards, Bella, David, and Ellen became quite ill, apparently from something they ate. Andy had taken Critical Thinking at Bellevue College, so he knew to ask what each had eaten. He even drew up a table showing the information he collected:

	Spaghetti	Cheese pizza	Pepperoni pizza	House salad w/ ranch dressing	House salad w/ Italian dressing	Got sick
Andy	x	x	x	-	x	-
Bella	-	-	-	x	-	x

Chikka	x	-	-	-	x	-
David	x	x	-	x	-	x
Ellen	-	x	-	x	x	x
Fred	x	-	x	-	x	-

What is the likely cause of the illness? The ranch dressing! Everyone who got sick ate the ranch dressing, and no one who did not get sick ate it. It appears—we can conclude inductively—that the ranch dressing was *enough* to make Bella, David, and Ellen sick (because anyone who ate it got sick), and it was also (among the foods they ate) *needed* to make them sick (because nothing else seemed to make anyone sick). Thus—speaking more generally—the Joint Method can point to a necessary and sufficient cause of a specified phenomenon when the results are clear. That is, if the occurrence of the phenomenon agrees perfectly with the occurrence of a specific causal factor, then the method can provide a necessary cause. If the lack of occurrence of the phenomenon matches up with a lack of occurrence of the same causal factor, then the Joint Method can provide a sufficient cause.

The Joint Method as used here cannot *prove* deductively that the ranch dressing is the culprit, for it *may* be that each of the three ailing diners has a unique disposition toward various foods, and it was just dumb luck that they each ate something that did not sit well with him or her. Still, given the information we now have, the restaurant manager has good reason to take a careful look at the ranch dressing. If she had little by way of moral scruples, she might give samples to a variety of other customers, and refuse to serve it to a variety of others. If all those who eat it get sick, and none of the others do, she'd be using the Joint Method further to provide added evidence for the ranch dressing's gastrointestinal turpitude.

If the restaurant manager was an inquisitive moral monster, she might kidnap two people of similar genetic structure and heritage, confine them in similar cages, provide them the same water, food, air, light, and other bodily needs, and force only one to ingest a house-salad portion of ranch dressing. Here she's using the Method of Difference to help confirm the conclusion derived from use of the Joint Method. If the dressing-eater gets sick, but the other caged soul does not, then that gives her added reason to believe that something about the ranch dressing is causing the illness.

The Joint Method, like the Method of Agreement, may also be used retrospectively. Here, however, we have access to people who got sick and people who did not get sick. Therein we find the added strength to this line of reasoning. Ideally, you'll have at least two cases that exhibit the phenomenon in question (e.g., getting sick) and two that do not. Anything less will make for an ill-defined pattern and a fairly weak argument. Obviously, the more you have of each, the stronger the argument can be, all else being equal.

Concomitant Variation

The following two kinds of causal reasoning are quite different from the previous three. *Concomitant Variation* appeals to the related correlation of two rates of change, and concludes that there is a causal relationship between them. Examples are easy to find:

Your grandfather gives you some vinyl records and an old LP stereo system. It's so old, that all the letters and numbers have been worn off the knobs. By trial and error you figure out how to turn the stereo on, and throw on a near-mint-condition copy of *Meet The Beatles*. When "I Want to Hold Your Hand" comes on, you want to raise the volume to decibels that could neuter frogs at 100 yards. But you don't know which knob will do the trick. You take hold of one, and at the rate at which you turn it you hear the volume of The Beatles' lilting refrains rise. As you turn the knob back the other direction, the song fades in volume. You conclude that you have found the cause of the volume level; that is, you've found the volume knob.

Or, you are a precocious, inquisitive, and logically-minded four-year-old child. You are sitting in the front seat of your mother's idling car (she's away for the moment to deliver a Senate document to her Justice Committee secretary). You don't know how to drive, but are aware that the vehicle has the capacity to go quickly. Pushing aside "Re-elect Senator Sunny Shine" flyers from the driver's seat, you start playing around with pedals on the floor, and note that as you press one, the car begins to move. As you press it further, the car accelerates and moves faster. When you press the pedal to the floor, you whisk across the parking lot, slamming into other cars, barely missing the now-terrified ambassador from Estonia. You let up on the pedal, and the car slows to a halt. You logically conclude that you have discovered the cause of the car's acceleration and rate of speed. You have just used the method of *Concomitant Variation*.

In both examples above, the concomitant (i.e., naturally accompanying, associated, attendant) variation is direct, or parallel. As one thing goes up, the other thing goes up. As one thing goes down, the other goes down. Sometimes the relationship may be inverse, as when one thing goes down, the other thing goes up. For instance, as employment rates go up, crime goes down. This indicates that there is a likely causal connection between crime and employment rates. It is beyond the method of *Concomitant Variation*, however, to determine which is causing which. Is it the drop in crime that is causing the rise in employment rates, or is it the rise in employment rates that is causing the drop in crime? Common sense or further inquiry is needed to make that determination.

More complex cases of concomitant variation occur when, for instance, one rate of change is fairly steady, while another rate of change is different (either directly or inversely). For instance, imagine events A are changing at the rate of 1 unit per day: 0, 1, 2, 3, 4, 5, 6.... Let's say events B are also ascending (or descending) but more slowly, but at a different rate: 0, 0.5, 1, 1.5, 2, 2.5, 3.... There still is concomitant variation, as when A goes up one unit, B goes up half a unit. Since the rate of change (though different) appears to be related, we are justified in (at least tentatively) concluding that one set of events is causally related to the other set.

But consider this. It oddly is the case in the larger of U.S. cities that as the crime rate goes up, so too does the amount of ice cream eaten; and as ice cream is eaten less often, the crime rate drops. *Concomitant Variation* indicates that ice-cream-eating is causally related to crime. We thus

should ban all ice cream! Well, no. We might first ask which way the causal relation goes. Does eating ice cream cause crime, or does crime cause the eating of ice cream? Neither option seems to make much sense, so we need to look further. It won't take us long to consider a third factor: rising temperatures. As temperatures rise, so too does the eating of ice cream *and* social irritation and its attendant crime. It's the rise in temperature that drives up both crime rates and people's readiness to indulge in the butterfat-laden delights of vanilla, chocolate, or strawberry ice cream. Concomitant Variation can point to a causal relation, but reason and further inquiry often need to kick in afterwards to draw specific conclusions about what causes what. Also, Concomitant Variation by itself does nothing to indicate if the cause is necessary or sufficient.

Method of Residues

In our introductory exploration of causal reasoning, we'll look at one more method: the *Method of Residues*. This method works by determining that a specific state (or states) of affairs is caused by a fixed number of causes. After accounting for each individual cause's specific effect, we conclude the remaining (or residual) effect is caused by the remaining cause. Although the Method of Residues can help establish a cause, it does not do much to determine if that cause is necessary, sufficient, both, or neither. The pattern of the Method of Residues looks something like this:

A and B and C in some way cause effects X and Y and Z.

A causes X.

B causes Y.

Thus, it is likely that C causes Z (the residue, or what's left over of the total set of effects).

Here's an example. A logic instructor at a community college is beset by ten of his female peers. They all claim he's smart, handsome, and witty, and want to go on dates with him. The instructor determines that this unsolicited attention is due to a combination of his good looks, his wealth from the huge amount of money he makes teaching in the State of Washington, and his fancy new Subaru. Curious as to how many are interested in his dashing good looks, he borrows a friend's beater Dodge Dart and puts the word out that he no longer drives a hot Subaru wagon. Five of the women originally interested in him no longer even return his school-related emails. He then spreads the rumor that he has and will continue to donate half his pay to feeding starving whales off the coast of Mauritania, and two more once adulating peers drop off his radar screen altogether, never to be heard from again, except at longwinded division meetings. The logic instructor concludes that the remaining three peers are likely attracted to him due to his striking visage.

The Method of Residues may also appeal to percentages, arguing—for instance—that because Cause A produces 10 percent of a set of effects, Cause B produces 15 percent of those effects, and Cause D produces 50 percent of the effects, it follows that Cause E probably causes 25 percent (the remaining residue) of the effects.

For instance, imagine that Tiago has a chilly draft in his home's living room. He has determined that there are three causes of the draft: air coming in under his front door, a broken window, and

an open damper in his fireplace. Tiago closes the damper and 30 percent of the draft is stopped. He fixes his window, and that stops 20 percent of the original draft. Tiago concludes that the un-insulated door is allowing 50 percent of the original draft. That 50 percent is the residue of the effect (i.e., the original draft) left over after accounting for the other two causes.

And here's yet another example of using the Method of Residues. Sarah runs a grocery store and wants to slow down the shoplifting occurring there. She can afford only to pay for added security at one portion of the store's opening hours. So she carefully records all the stock, and determines that 10 percent of the theft is done in the mornings, and 15 percent of the thefts take place in the afternoons. She concludes that 75 percent of the thefts occur in the evening hours, and hires Buckkitt Security to patrol the store then.

****Practice Problems: Causal Arguments**

Answer the questions that follow each scenario below.

1. Sunny Shine decides that for this year's World Naked Gardening Day (held the first Saturday of each May) she would finally determine what effect Kwik-Gro fertilizer has on her primroses. She has two rows of these plants, each acquired from the same stock of primroses sold at the same store, and bought on the same day. She gives them all the same amount of water; the sunlight is the same for each row; the soil is the same. Dressed appropriately for the day, she applies Kwik-Gro to one row, but not to the other. The row receiving Kwik-Gro soon dries up and dies, leaving a grub-infested, gooey mess where once were thriving primroses. Shine concludes that Kwik-Gro kills primroses. Which of Mill's methods is Shine using? Which kind of cause is Kwik-Gro in killing Shine's plants: necessary, sufficient, both, or neither?

2. Pastor Bustle believes his church needs more funds so that he can give himself a raise in pay for preaching. He speculates that the length of his Sunday morning sermons is affecting the number of parishioners willing to tithe regularly to his church. For one month he preaches for 15 minutes. The next month he preaches for 30 minutes. The next month he preaches for 45 minutes. And finally, he preaches for 60 minutes each Sunday of a month. He notes that the longer he preaches, the less money comes in. He concludes that to get more money, he must shorten his sermon time. Which method is Bustle using?

3. Professor Kim wants to know what is a likely cause of student success (in terms of grades) in his Social Philosophy graduate seminar. He makes inquiries of his five students, and discovers that Angie reads anime comic novels regularly and studies each night for one hour; Bernardo studies only once a week for hours, is the only one to use flash cards, and drinks Red Bull before coming to class each day; Charisa does yoga before every class, but never studies, and reads anime comic novels regularly; Drew can't read, never studies, but does yoga before class with Charisa, and drinks Red Bull immediately afterwards; Ewan and Bernardo believe that reading is a sign of capitulation to intolerant power structures and refuse to do it; Angie, Charisa, and Ewan are all raw vegans who drink only water or freshly squeezed juice; Angie, Bernardo, and Ewan all think yoga is for sissies and scorn any suggestion that they should practice it; and Ewan studies each night for an hour. Only Angie and Ewan are getting high grades in Kim's seminar. Draw a table showing the information Kim has collected. Given this information, what is the

likely cause of receiving high grades in Kim’s seminar? What kind of cause (if determinable) is this (e.g., necessary, sufficient, both, neither)? Which method is Kim using?

4. Bob Shine likes horse racing, and given recent misfortunes at his architectural firm, wants to make some extra money. He finds the jockeys for the last three winning horses and bribes them into giving him some information to help him on future bets. All I Want to Be ate only oats the morning before the race, was never whipped by her jockey, and had Led Zeppelin played in her stall before the race. Badly in Need ate no oats before the race, had Barry Manilow played in her stall before the race, and was never whipped during the race. Only Can’t Bear to Lose and All I Want to Be received injections of horse steroids before the race. The jockey for Can’t Bear to Lose is opposed to all forms of harm to animals, refuses to uses whips, feeds his horse only oats, and has him listen to Led Zeppelin before races. Draw a table to show the information Shine received. Based on this information, what is the likely cause of these horses winning their races? Is the cause necessary, sufficient, both, or neither? Which method is Shine using?

5. Doctor Nan Kompos Mentis has good reason to believe that the swelling of Ellen Veegon’s limbs is due to three foods she eats each day—raw peanuts, tofu, and kale—and wants to know how much of a problem the kale in particular is causing. Mentis has Veegon stop eating peanuts, and the swelling goes down by half. Mentis then has Veegon also stop eating tofu, and the swelling drops down by half again. Mentis concludes that the kale is causing 25 percent of the original swelling. What method is Mentis using?

6. Given the table below, what is the likely cause of Z? What method is being used? Is the cause necessary, sufficient, both, or neither?

	A	B	C	D	E	F	G	Z
Case 1	x	x	x	x	x	x	x	x
Case 2	x	x	-	x	x	x	x	-

7. Given the table below, what is the likely cause of Y? What method is being used? Is the cause necessary, sufficient, both, or neither?

	A	B	C	D	E	F	G	H	Y
Case 1	x	x	x	x	x	x	-	x	x
Case 2	x	-	-	x	x	-	-	x	x
Case 3	x	x	-	x	-	-	x	-	x
Case 4	-	x	x	x	x	-	-	-	x

8. Given the table below, what is the likely cause of W? What method is being used? Is the cause necessary, sufficient, both, or neither?

	A	B	C	D	E	F	G	H	W
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Case 1	x	-	x	-	x	-	x	-	x
Case 2	x	x	-	x	-	-	x	-	x
Case 3	-	-	-	-	-	x	x	x	-
Case 4	x	x	-	-	-	x	-	x	x
Case 5	-	x	-	-	x	-	x	-	-
Case 6	-	-	-	x	-	-	-	-	-
Case 7	-	x	x	x	-	-	-	-	-

9. Given the table below, what is the likely cause of V? What method is being used? Is the cause necessary, sufficient, both, or neither? (This is a more challenging problem.)

	A	B	C	D	E	F	G	H	V
Case 1	x	-	-	x	x	x	x	x	x
Case 2	x	x	x	x	x	x	x	x	-
Case 3	x	x	-	x	x	-	x	x	x
Case 4	x	x	x	x	x	x	-	x	-
Case 5	x	x	x	x	x	x	-	x	-
Case 6	-	-	-	x	-	x	x	x	x
Case 7	x	x	x	-	x	x	x	x	x

10. Given the table below, what is the likely cause of U? What method is being used? Is the cause necessary, sufficient, both, or neither? (This is a similarly more challenging problem.)

	A	B	C	D	E	F	G	H	U
Case 1	x	x	-	-	x	-	x	x	x
Case 2	-	x	-	x	x	-	-	-	x
Case 3	x	-	-	-	-	x	-	x	x
Case 4	x	x	x	x	-	x	x	x	-
Case 5	x	x	-	-	-	x	x	-	-
Case 6	x	x	x	x	-	x	-	x	-

11. The morally-challenged manager of the Chunk-O-Cheese Pizzeria thinks her ranch dressing is causing an outbreak of illness among her customers. She kidnaps five adults presently dining in her establishment, and in the restaurant’s basement forces each to eat some of the dressing. One kidnapped customer is fed one tablespoon of dressing, the second is fed two tablespoons, the third three tablespoons, the fourth four, and the fifth five. The ones who eat more of the dressing get sicker than the ones who eat less. “Eureka!” she cries. “I’ve now confirmed that it’s the ranch dressing causing the illness!” Which method is the manager using?

12. Bob Shine’s accountant says his architectural company is losing money to theft, and that the thefts are limited to employees stealing office supplies, employees using company gasoline to drive to Reno for weekends, and employees using company vouchers for private dinners. Bob cancels all food vouchers and locks up the company gas tanks. Monetary loss due to stealing drops 75 percent. He concludes that 25 percent of the employ theft was from stealing office supplies. Which method is Shine using?

13. The cook at Joe’s Café wants to experiment with his meatloaf. He makes two batches, cooking both at the same temperature in the same oven and for the same length of time. The recipes are exactly the same except that he adds a quarter cup of bourbon to one batch. Everyone in the cafe likes the bourbon meatloaf better, and Joe concludes that bourbon makes his original recipe more popular. Which method is Joe using? Is the bourbon a necessary cause, sufficient cause, both, or neither?

14. Albert drank five Scotch and sodas, and got drunk. Barney drank five bourbon and sodas, and got drunk. Charlene drank five gin and sodas, and got drunk. Doris drank five rum and sodas, and got drunk. Tiago is watching all of this and concluded that soda causes people to get drunk. Which method is Tiago attempting to use (albeit, not very well)?

15. What might a logical and informed person do to show Tiago (from Problem #14) that his conclusion is mistaken and that the alcohol in the drinks caused the people to get drunk?

Answers:

1. Method of Difference; sufficient cause.
2. Concomitant Variation.
- 3.

	Reads anime comic novels	Studies each night for 1 hour	Studies once per week for hours	Uses flash cards	Drinks Red Bull	Does yoga	<i>Get high grades</i>
Angie	x	x	-	-	-	-	x
Bernardo	-	-	x	x	x	-	-
Charisa	x	-	-	-	-	x	-
Drew	-	-	-	-	x	x	-

Ewan	-	x	-	-	-	-	x
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Studying each night for one hour is the likely cause of the high grades; necessary and sufficient cause; Joint Method.

4.

	Ate oats	Ate hay	Not whipped	Led Zeppelin	Barry Manilow	Horse steroids	Won a race
All I Want to Be	x	-	x	x	-	x	x
Badly in Need	-	x	x	-	x	-	x
Can't Bear to Lose	x	-	x	-	x	x	x

Not being whipped by the jockey is the likely cause of the wins; necessary cause; Method of Agreement.

5. Method of Residues.

6. The likely cause of Z is C; Method of Difference; sufficient cause.

7. The likely cause of Y is D; Method of Agreement; necessary cause.

8. The likely cause of W is A; Joint Method; necessary and sufficient cause.

9. We're clearly not using Concomitant Variation or Method of Residues. That leaves the first three methods. But it's not Method of Difference, because there are not *two* cases being compared in which they are exactly the same except for *one* difference. If we were using the Method of Agreement, then all the cases would experience the phenomenon in question (i.e., V). So we are using the Joint Method, which makes sense since we are appealing to three or more each of cases that did and did not experience V. No causal factor (i.e., A-H) matches perfectly with the phenomenon in question, but factor G matches the best, so it is the most likely cause among these eight factors. Where G is experienced, V is uniformly experienced, too, so G "agrees" with H, thus providing a necessary cause. But the failure to experience G does not match up perfectly with the failure to experience V, so G does not quite provide a sufficient cause. This is a case of using the Joint Method, but it is not giving us everything we would like from it.

10. The likely cause of U is E; Joint Method; sufficient (but not necessary) cause.

11. Concomitant Variation.

12. Method of Residues.

13. Method of Difference; sufficient cause.

14. Method of Agreement. Obviously, though, it's not the soda that causes people to get drunk, but the Method of Agreement alone cannot tell us this. Now go to Problem #15.

15. One way to do this is to use the Joint Method. Have a variety of people drink a variety of liquids (five glasses each), some containing alcohol, while other drinks do not. If our hypothesis is correct (i.e., that the alcohol is the one and only cause of the people getting drunk), then all people guzzling drinks with alcohol will get drunk, and none of the people sipping alcohol-free

drinks will get drunk. The Method of Difference might be helpful, too. We can get a series of pairs of people of roughly the same height, body mass, gender, and age (factors that may play a role in how easily one gets drunk), and give half of them five drinks with alcohol, and have half of them five drinks exactly the same except without alcohol. If the alcohol drinkers get drunk and the others do not, then it looks like alcohol is a sufficient cause for getting drunk (i.e., from this information, it is possible still that some other drug can make people drunk, too).

Chapter 12: Hypotheses

We often face philosophically interesting questions, those whose proposed answers thoughtful and informed people are debating. Whether murdering people to steal their money is morally right or wrong is *not* philosophically interesting in this sense. All reasonable and informed people agree that it's wrong. *Why* it's wrong *is* philosophically interesting, though, because there remains thoughtful, informed debate on that more basic issue. Of continued philosophical interest include questions such as, "Is euthanasia always morally wrong?" "How did the dinosaurs die?" "Was O. J. Simpson guilty of murder?" "How should teachers respond to student cheating?" "Does God exist?" "What role does race play in U.S. politics today?" "Should baristas be able to serve lattes buck naked?" There are intelligent, informed people still disagreeing on such matters.

If there are two proposed theories—or hypotheses—answering a question, and one of them has a clear preponderance of evidence or good argumentation on its side, then rational people should embrace that better-supported theory. Having a clear superiority of evidence does not guarantee that one answer, theory, or hypothesis true, but as rational beings, we want—and probably should—embrace the view that is most justified or warranted. But what should we do if there are multiple opposing answers to an important question, and each answer lacks any clear evidence, or any clear preponderance of evidence? What if each answer is equal in terms of evidence and support? It may be that we could ignore the situation, and put the question "on a shelf," so to speak, not worrying about it until more evidence comes in directing us to embrace rationally one over the other. But sometimes the question is vital enough that we need to move forward and work with a viewpoint. Guessing or simply going for the answer that "feels" right will hardly do for rational people, at least if there is more that can be said in favor of one hypothesis over another. Fortunately, there *is* a set of principles that can help in this regard, and before giving in to guesswork or personal, arbitrarily biased feelings, these principles are worth considering. They form criteria articulating what we'd want out of any well functioning hypothesis, that is, out of any viable answer to a puzzling question.

An *hypothesis* is a proposed answer to a puzzling question. If a police detective finds a dead man with a knife in his back, the puzzling problem might be "Who killed the man?" or "Why was this man killed?" An archeologist might ask at an excavation site, "Why and how did this group of people construct this stone building?" A medical researcher might ask, "What can cure this particular form of cancer?" A philosopher might ask, "What is the ultimate nature of who we are?" A theologian might ask, "Why would an all-good God allow evil in the world?" All are philosophically interesting questions, and multiple hypotheses have been offered for each. Sometimes empirical evidence or logical analysis shows an hypothesis to be unworthy of serious consideration. Sometimes, however, two or more hypotheses have just as much—or just as little—evidence backing them up, and it's not clear which one (or ones) is true.

What we might do in such bothersome situations is consider what we want out of an hypothesis. There are certain character traits that an ideal hypothesis will have, just like there are ideal character traits a hammer will have. If you are in need of a hammer, but don't know at this point

which kind you need (small or large, framing, ball-peen, claw and nail, dead-blow, sledge, masonry, or rock pick), you at least want a hammer that does the job of a hammer. If you *had* to select a hammer for some use, but did not know what kind to use, and had to choose from a poorly crafted masonry hammer or a superbly crafted claw and nail hammer, you'd be advised to select the latter, better constructed tool. It may turn out that it's *not* the best hammer for the job, but for the moment, you are rational in *tentatively embracing* the better crafted hammer.

This embracing of a hammer or hypotheses is tentative and non-dogmatic, as being well-crafted provides no evidence that it's the best hammer for the job or the correct answer to your puzzling question. Still, we may need a working hypothesis to march forward to do medical research, or to inquire further into the philosophic question at hand, or to guide the police detective in searching for a killer. If evidence or arguments turn up that make our hypothesis look bad, then we are ready to give it up, as we've not committed ourselves too strongly to it; we are embracing it only tentatively, until more information comes in.

We'll consider five criteria for superior hypotheses. If we find ourselves with two hypotheses to choose from, and if we are in need of adopting an hypothesis to move forward to do the work we need to do, then we will be advised to adopt tentatively the hypothesis that meets these criteria the best. This is not, however, an argument for the truth of the hypothesis. The "better crafted" hypothesis may still be incorrect, and the one with virtually no evidence and lacking in some important character traits may still be the true one, but we can use the appeal to these principles to say that one hypothesis is better—as an hypothesis—than the other, and *that* warrants us in tentatively working with it for the time being. We'll need to wait until new evidence or argumentation comes in to justify our saying that the hypothesis is actually true.

Criteria for Hypotheses

An ideal hypothesis will have five character traits. The hypothesis will exhibit:

- Explanatory power
- External consistency
- Internal consistency
- Fruitfulness
- Simplicity

If we are considering two hypotheses, one may look good regarding a couple criteria, while the other may look good in reference to two or three others. There is no way to quantify precisely how well one hypothesis meets these criteria when compared to another. We need to consider each hypothesis, and then make a judgment as to which one overall—and all else being equal—functions best as an hypothesis. We'll then tentatively embrace that one.

Explanatory Power

The purpose of an hypothesis is to provide an answer to a potentially puzzling question. If the hypothesis doesn't do that, it's not functioning very well. Let's say that Bob stubs his toe on the

bedpost one night, and wonders why that happened. He might hypothesize that God caused it to happen. As an hypothesis, this doesn't explain much. It doesn't explain why God wanted Bob to stub his toe, or how God caused the event to take place. It may be true that God exists and caused Bob to stub his toe, but a thinking person will likely—all else being equal—be inclined to look for another theory that provides a more detailed *explanatory* answer.

What we want out of hypotheses is as much relevant information as possible. If the question pertains to the cause of a mysterious event, ideally we'd like the hypothesis to tell us what happened, what caused it to happen, why it happened the way it did, how it happened, and to do so for as many of the details of the event as possible. Moreover, an ideal hypothesis will do all this with clarity, detail, and precision. Of course, that may be too much to ask for in certain situations, but if we have two purported answers to a question and the first hypothesis answers more of the question and in greater detail than the second, then all else being equal the first is the better hypothesis.

Consider a medical researcher who seeks the cause of a form of paralysis. Hypothesis A says the illness is caused by breathing polluted air. Hypothesis B says that drinking tap water from rusty pipes over the course of one year will damage nerves, cause tingling in the extremities initially, and eventually produce paralysis. Hypothesis B answers more of the question than Hypothesis A, and does so with far more detail. If the evidence and argumentation for A is equal to that of B, then because B has more explanatory power than A, we should prefer it over A...at least tentatively. We must keep in mind that just because an hypothesis provides broad-ranging and detailed answers to a question, does not itself show that the answer is correct. It just means that the hypothesis is doing what hypotheses are supposed to do.

We also want an hypothesis to fit the facts in question as precisely as possible. If the rusty water pipe hypothesis merely accounted for "a number" people becoming ill, while the less encompassing bad air hypothesis explained why exactly 25 people in a given community would become ill, then in *that* regard the bad air hypothesis is the better of the two. As it stands now in our scenario, there is reason under the criterion of being explanatory to favor both hypotheses. That's no surprise as thinking people rarely consider woefully bad hypotheses for very long; on the flip side, viable hypotheses tend to have *something* going for them to keep them under consideration.

Moreover, the hypothesis will ideally explain other analogous questions equally well. For instance, if beekeepers find that their hives of bees in one Washington county are sick and come up with an hypothesis that accounts for it well, then that hypothesis should—in principle—work for other *similar* cases of bee hive illnesses. Hypotheses should thus not be ad hoc and arbitrary; if they answer one puzzling question well, they should be of use with other relevantly similar puzzling questions.

External Consistency

A good hypothesis will also be *externally consistent*. By that we mean that the hypothesis does not conflict with what we know (or think we know) about the world, and that it conforms to the

facts as we understand them. That is, all else being equal, we don't want to embrace an hypothesis that forces us to give up well-established beliefs. For instance, if you follow the motion of Mars in the sky over the course of many nights, you might observe it moving in one direction, then switching back for a time, then moving back in its original direction. This observed oddity of planetary movement is called retrograde motion. What causes it? One less-than-sure-footed astronomer might hypothesize that five-year-old boys are floating in space pushing Mars around. Obviously, this is an embarrassingly ridiculous hypothesis, but it's ridiculous because it patently goes against what we know to be true about little boys: their lack of existence in space and their lack of power to move planets. We reject the hypothesis out of hand because it lacks external consistency, and—all else being equal—this is enough to make us smile upon a more externally consistent alternative hypothesis.

Two competing hypotheses might both be externally consistent; that is, they match up equally with what we are confident is true about the world. For instance, if a mysterious light appears in the sky at night, and we want to theorize about its cause, two people might come up with differing hypotheses: the light is caused by either a military plane or a distress flare shot up by someone on the ground. Both hypotheses are consistent with what we know to be true of the world. In this case, we either need to find additional evidence or argumentation for embracing one hypothesis over the other, or, short of that, one hypothesis needs to meet the other criteria better.

A potential problem lies in that what we are psychologically confident in may actually be false. Belief and perspective do not by themselves determine truth or reality. We *can* be mistaken. This perfectly reasonable and humble position is called *fallibilism* (i.e., the acknowledgement that we are fallible, or able to make errors in judgment about the world). So, just because an hypothesis is inconsistent with something we take to be true about the world, it does not follow with certainty that the hypothesis is false. It *may* be that we are mistaken about the world and the hypothesis is true. To the degree we can be justifiably confident in our belief about the world, to that degree we can be confident that an hypothesis inconsistent with this belief has something seriously wrong with it. One important lesson to learn here is that when making hypotheses, we need to be aware of the assumptions we make in the process. Our assumptions *might* be mistaken. None of this warrants becoming overly skeptical about every little belief about the world, but we need to be careful nonetheless.

Internal Consistency

An hypothesis must also be *internally consistent*. All the parts of an hypothesis must be consistent with and not contradict each other. A self-contradictory hypothesis has something seriously wrong with it. All else being equal, if one hypothesis is more internally consistent than another, the more consistent one is to be preferred.

An hypothesis to a complex problem will usually have multiple parts. For instance, if we are interested in why the dinosaurs died, one might come up with a “meteorite hypothesis” having numerous parts. For instance, (a) a meteorite hit the earth causing (b) a huge dust cloud that (c) blocked sunlight and (d) killed plants which caused (e) dinosaurs to starve to death. There are at

least five major elements to this hypothesis. Here, so it seems, each part is consistent with the others.

The highly influential French philosopher Rene Descartes (1596-1650) is said by many scholars to have offered an hypothesis regarding the nature of reality that was internally *inconsistent*. Descartes hypothesized that reality is made up of two radically distinct substances: mind and body. What is true of one, is not true of the other. Mind is immaterial, takes up no space, is indivisible, and thinks. Body is material, spatial, divisible, and does not think. We call this two-substance view *Dualism*. Descartes also hypothesized that with humans the mind and body have a causal relation with each other. The view is called *Interactionism*. For instance, one can cause one's arm to move by using one's mind to think, "Raise my arm!" Also, the body can cause mental events in the mind. If someone kicks your physical knee, that causes the mental event of a pain. A problem facing Descartes is that the combination of Dualism and Interactionism really seems to be internally inconsistent. For it is inexplicable how a non-material entity like Descartes's mind can have any physical effect on his material body, and vice-versa. The mind has no substance or energy (note that energy and matter are related, and are part of the bodily world); it would be like an immaterial ghost trying to punch a living person. The ghost's "fist" would go right through the living man's face. (If you've seen the movie *Ghost* (1990) starring Patrick Swayze and Demi Moore, you will be aware of this problem.)

Fruitfulness

An hypothesis that will make most inquirers jump with joy will be what we'll call *fruitful*. One way an hypothesis can be "fruitful" is to provide opportunities to test it. Some hypotheses are simply impossible to verify or falsify; there's no way even in principle to tell if they're true or false. To wax theological once again, if a family member gets sick and we want to know why, if we hypothesize that it's God's will, the hypothesis is completely untestable. There is nothing we can do to show if the hypothesis is correct or not. We can pray and ask God to give us a verifying sign of event X, Y, or Z, but if the event does not occur, then we *still* won't know that God's will was not causally involved with the sickness, because God could have decided—for whatever good reason—not to provide the requested sign. If the sign *does* appear, we'll not be sure that it actually came from God, or came from Him for that purpose. Maybe He's "testing" us. There's no way to know.

Scientists sometimes seem to be at odds with believers on various issues. The origin of life, changes within a species, the death of the dinosaurs, the plausibility of a global flood, the existence of God, and similar puzzles often find each party with different (sometimes conflicting) hypotheses. There are many intelligent, informed scientists who believe in God, and as a matter of their faith believe various things that may not be empirically verifiable or falsifiable. Still as scientists, they will also likely seek—when possible—hypotheses that can be tested. That's what in large part makes them scientifically minded. They understand that a true, accurate hypothesis may be untestable, but as long as the possibility remains, they will want—as scientists—to examine other hypotheses that *can* be tested. All else being equal, an hypothesis that can be tested is preferable to one that cannot. At least, that's the sort of hypothesis we'll

want to tentatively embrace until a preponderance of evidence or argumentation comes in favoring one hypothesis over another.

We also want hypotheses to be fruitful in another way. We want them to lead us to further inquiry, to be springboards, as it were, for continued discussion. If an hypothesis gives no hint as to what we might do or develop for further research or testing, or if the hypothesis in principle suggests no further avenue of discussion, then again it's not doing much for us. It *may* be true and accurately answer the question at hand, but as an hypothesis, it's largely useless to us. For example, someone might attribute a bout of stomach flu to the karmic effects of bad actions performed in a previous life. The law of karma can have explanatory power, be internally consistent, and externally consistent, and but it is not a very fruitful hypothesis, as (a) it's not at all testable in any practical way, and (b) it gives little or no new insight as to how to test a purported karmic law or how to think or inquire about it further.

Simplicity

The Principle of Simplicity urges us to adopt—all else being equal—the *simpler* of two hypotheses. Called “Ockham’s Razor” in Europe’s Medieval Period, and referred to in India since its Classical Period as “The Principle of Lightness,” this criterion does not suggest we tentatively adopt the hypothesis that is easiest for us to understand. We don’t mean “simple” in this sense. Rather, the hypothesis that appeals to the fewest number of entities, or the least complex set of entities is less likely to be mistaken.

Imagine two automobiles of equal quality, but one has twice as many moving parts as the other. Which is more likely to break down most often? The more complex car, as it has more ways to break. So too with hypotheses. One making ten claims has more ways to be inconsistent with the world or internally inconsistent with itself, than an hypothesis making only three claims. Years ago, the Honda automobile company ran ads singing, “Honda...we make it simple.” They were saying that Hondas were so simple that they would last a long time without breaking down. Many people liked that idea, bought the cars, and made Honda a major car producer internationally.

Another way to look at simplicity is to think of an hypothesis’s “modesty.” A modest hypothesis claims only what is needed to answer the puzzling questions powerfully. That is, the ideal hypothesis is trimmed down to the bare essentials, so that it can do the job of explaining in detail what needs explaining, but makes no extra claim that could end up being false. *Simplicity* in this sense can initially appear to be in conflict with the robust demands of *explanatory power*, but it’s not. The best hypothesis will be as streamlined as possible, yet still do their designated job.

By way of illustration, let’s go back in time to when the Italian scientist and philosopher Galileo Galilei (1564-1642) was challenging the dominant geocentric view of the universe devised by Ptolemy centuries beforehand. Galileo believed that the Sun was the center of the universe (i.e., Copernicus’s heliocentrism), while the Church defended the Earth-centered system long and hard. Technology had not advanced yet to the degree that could provide detailed evidence for one view or to show that one hypothesis was clearly the best. For a time, both sides had little to

appeal to other than these five criteria. We might imagine the following conversation between Galileo and his geocentric opponent. (See Galileo's *Dialogue Concerning the Two Chief World Systems: Ptolemaic and Copernican* for his actual arguments along these lines.)

Galileo: Well, my development of telescopes still leaves something to be desired, so I can't yet use precise, detailed measurements of the starry heavens to show that the heliocentric system offered by Copernicus is measurably best, but my hypothesis is better than your Ptolemaic geocentric one, at least as far as hypotheses go.

Opponent: Bah! My theory explains things well. It explains why planets and stars appear to move across the night sky in the directions they do, and how they are arranged.

G: So does my heliocentric theory. I'll admit that your picture explains as much as mine, but that's not where my hypothesis shows its strength.

O: Strength? Strength? You can't handle strength! You want strength? Take a look at the external consistency of geocentrism. If your doofus hypothesis was true, the Earth would be moving through space. But does it *feel* like the Earth is moving? No! Also, if the Earth is moving rapidly through space, wouldn't we notice a strong wind constantly coming from one direction? But we don't! *And*, if I hold this small rock over this X marked on the table, and drop the rock, if the Earth is moving, the X should slide over to one direction, and the rock shouldn't hit it. But it *does* hit it...dead on every time! So your hypothesis is not nearly as consistent with what we know about the world. Plus, the Bible says the Earth sits still and the Sun moves around it. It's in the book of Joshua or somewhere.

G: Well, I'm not going to argue today with the Bible, and I've got to admit that much of what we normally take to be obviously true about the world is pretty consistent with geocentrism, but my hypothesis is better nonetheless.

O: Humph!

G: My hypothesis is internally consistent. There are no parts to it that contradict one another.

O: So too with mine.

G: Yah, I suppose so. I don't believe your hypothesis, but it at least seems to hold together internally.

O: Now look at the fruitfulness of my geocentric picture. Sailors can assume that it's true and use it to navigate across vast seas and get exactly where they wish to go. Also, we can test my hypothesis. For instance, if my hypothesis is true, then—given careful calculations—we can predict accurately where Mars will be in the night sky weeks from now, and do so with a high degree of accuracy.

G: Yah, but my heliocentric hypothesis can do the same thing. So it's a tie as far as fruitfulness goes.

O: So what's your big strength? Why on our geocentric Earth should anyone think that your hypothesis—as an hypothesis—is better than mine?

G: Because mine is more simple. To make yours work, that is, to make yours help sailors get across the sea or to draw implications as to where Mars will be later this week, you have to

appeal to gobs and gobs of epicycles, deferents, and equants. My Copernican system of heliocentrism appeals to far fewer such entities. Thus my hypothesis is simpler. Thus my hypothesis is—all else being equal—to be preferred, at least tentatively, until I improve my scientific instruments and show that I can predict planetary motion far more accurately and with more detail than you can with your soon-to-be outmoded geocentrism.

O: I doubt it. You're always talking about new technology. Our hypotheses appear close to a tie at this point, and given that the Church says the Earth sits still, I think it's best to go with that view.

G: Shall we then talk about these craters I recently found on the Moon?

O: Ack! Heresy!

****Thinking Problems: Criteria for Hypotheses**

Consider each of two competing hypotheses, and write a mini-dialogue in which characters appeal to the five criteria for hypotheses to support tentatively embracing one hypothesis over the other.

1. The doctrines of karma and reincarnation are true vs. Life's a beach, then you die...period
2. Jesus was resurrected from the dead vs. No way José!
3. Evolution vs. Creationism
4. Free will vs. Determinism

Answers:

Obviously, there are hundreds of ways to write dialogues for each debate. This is a rare time when “answers” will not be provided in this text. But ask yourself: As hypotheses, which criteria does each side meet well? As an hypothesis, does one side meet more of the criteria, or meet some of them better? If evidence for one outweighs the evidence of the other, then we don't need to worry too much about these criteria; so for the sake of discussion, assume the empirical evidence or strength of argument is either equal or equally non-existent for each.

****Practice Problems: Criteria for Hypotheses**

Which one of the five criteria for hypotheses is the speaker below most clearly considering?

1. “My hypothesis is better because mine appeals to A and B, while yours appeals to A, B, *and* C.”
2. “Your hypothesis is worse than mine, because yours appeals to A and B, but A and B can't both be true.”
3. “How on Earth could we *ever* determine if your hypothesis is true or not?”
4. “Your hypothesis is so vague that it fails to answer any of the question with any precision.”
5. “Your explanation for the origin of life on Earth appeals to a giant silicon-based being deep in space sending a rocket with sperm cells on the nose cone. But we have no reason to believe that there is any such intelligent life out there.” [Note: This was an hypothesis of a Nobel Prize-winning astrophysicist.]
6. “Your hypothesis says that the dinosaurs died due to starvation, but it also claims they died due to a virus. Which was it?”

7. "You hypothesize that Bob died from suicide. But we who knew him best knew him to be happy and quite content with life."
8. "You theorize that Sunny Shine's proclivity to garden and cycle naked is due to a complex set of early maladjusted family relationships that produced neuroses resulting in mannerisms conjoined with environmental factors producing her deviant social behavior. Isn't it more likely that she enjoys being clothes-free just because it feels good?"
9. "The resurrection hypothesis accounts for the apostles' later purportedly first-hand testimony of it and willingness to die for it better than any other hypothesis regarding the days following Jesus' death. There's no way the apostles would die brutally for what they knew to be a lie."
10. "I can't test my theory that after we die life is over any more than you can test your theory that the law of karma forces us to be reincarnated. For once we die, either way, we're not going to be able to let others know what happened."
11. "Your hypotheses consists of claims A, B, C, and D. But we know from experience that C is false. Thus we should reject your hypothesis."
12. "Your hypothesis regarding the murder implies both that Albert hated his wife and that he cared for her. No way!"
13. "Evolution makes no appeal to a divine intelligence, yet accounts for life as we know it. Thus it's a better hypothesis than Creationism."
14. "Free will matches up with the fact of moral responsibility and moral deliberation better than your determinist hypothesis saying that all acts are fully caused by antecedent conditions. Thus the free will hypothesis is better than the determinist hypothesis."
15. "My hypothesis explains ten aspects of the puzzling question at hand; yours explains only five of them. Thus my hypothesis is a better hypothesis, all else being equal."

Answers:

- | | | |
|-------------------------|-------------------------|--------------------------|
| 1. Simplicity | 6. Internal consistency | 11. External consistency |
| 2. Internal consistency | 7. External consistency | 12. Internal consistency |
| 3. Fruitfulness | 8. Simplicity | 13. Simplicity |
| 4. Explanatory power | 9. External consistency | 14. External consistency |
| 5. External consistency | 10. Fruitfulness | 15. Explanatory power |

Inference to the Best Explanation

A growing number of philosophers and other thinkers are appealing to these five criteria for good hypotheses to engage in an inductive line of reasoning: Inference to the Best Explanation. It's worth seeing how this form of critical thinking can be used.

Once again, consider the situation in which there are two or more opposing answers to a question, yet there is insufficient evidence or argumentation to clearly point to one hypothesis being better supported than another. Once the equally-supported (or equally-unsupported) hypotheses are on the table, so to speak, we might try arguing by claiming that our hypothesis is better than any of the others, and on that basis should be accepted as most likely the true one. The way we establish that our hypothesis is better than the others is by showing that it best conforms to the criteria above. Perhaps—all else being equal—it answers more of the puzzling question, or perhaps it's simpler, or perhaps it matches up with established beliefs better. None

of this goes to prove that our hypothesis is true, but it does argue that in comparison to the other offered hypotheses, ours is a better-formed answer to the question.

Users of Inference to the Best Explanation must be careful to avoid certain pitfalls, though. One obvious potential problem for us is to view our hypothesis as the best because we merely think it the best. That would clearly be begging the question. We can't say our hypothesis is better than the others without some sort of good reason. If there is objective reason to say our hypothesis is true, then we don't need Inference to the Best Explanation; that objective reason itself gives adequate justification for believing our hypothesis. What we are limited in doing with Inference to the Best Explanation is arguing that our hypothesis is better formed as an hypothesis than the others, and somehow that makes it more likely for it to be true. This is a tenuous bit of reasoning, but it should at least justify everyone giving a serious look at our hypothesis.

Another potential problem to avoid when using Inference to the Best Explanation is a common (perhaps all-too-human) tendency to believe one's proposed hypothesis, and to see more readily how *it* conforms to the criteria above, while ignoring how the other hypotheses do so. Comparing competing hypotheses by appealing to the five criteria is not an exact science, and often cannot be quantified precisely. It's usually a matter of considering informally the merits of each hypothesis, weighing the results, and making a thoughtful judgment when possible. Rarely will assessment of opposing hypotheses presented by intelligent and informed people result in a case of one being clearly the "best explanation," unless, of course, there is some underlying begging of the question.

A third potential problem is to inadvertently embrace the informal fallacy of False Dichotomy. Inference to the Best Explanation argues this way:

1. There are two (or three, or four, etc.) opposing hypotheses.
2. My hypothesis fits the criteria for hypotheses best of the two, making it the best explanation.
3. Thus, it is likely that my hypothesis is true.

False Dichotomy occurs when someone offers a disjunctive syllogism (i.e., A or B; Not-A; Thus, B) when there is an additional option not being considered. Recall the teenager argument from our section on Informal Fallacies. "Mom, either you let me go to the dance with the biker gang, or my life will be ruined. Surely you don't want my life to be ruined. Thus, you should let me go to the dance with this gang." Mothers somehow intuitively know bad arguments, at least when presented by teenagers. They recognize that there is at least a third option not being considered in the daughter's first premise: She will *not* go to the dance with the biker gang, and her life will *not* be ruined. So, no, permission to boogie with the leather-clad ruffians is denied.

So too with Inference to the Best Explanation. We must be sure—and have some reason to offer for believing so—that the hypotheses we are comparing are the only ones meriting consideration (and not beg the question in our hypothesis's favor in doing so). The burden of proof is on our shoulders to show that there are no additional hypotheses worth considering; and if we cannot offer such justification, then we should limit ourselves to concluding that our hypothesis matches the criteria for hypotheses better than do the others we are considering, and on that basis we are

justified in taking it seriously as a viable hypothesis. But this falls somewhat short of being able to claim that we have via this inference pattern given reason to believe our hypothesis is true.

The long and the short of it may be that when well-established evidence or cogent/sound reasoning does not clearly establish that one thoughtful hypothesis is more likely to be true over a competitor, Inference to the Best Explanation may be the best line of reasoning we have. Be careful not to overstate its powers, though.

Testing Hypotheses

One of the criteria for an ideal hypothesis was its fruitfulness, which included its ability to be tested. We now need to explore how we might reasonably test an hypothesis. The procedure is not obvious, and it was only “discovered” a few hundred years ago. With that process, scientists in Europe were able to make startlingly fast advances in knowledge about the natural world and in technological development. Those advances gave Europe the opportunity to do much good and bad in the world, and to make “the West” a powerful force internationally. *Hypothetical reasoning* or the *Scientific Method* was not the only way of pursuing knowledge, but it has proven to be an effective and productive way of sorting out viable from non-viable hypotheses, and thus paves the way for even more advances in theory and practical development.

Hypothetical reasoning is useful particularly when we cannot find an answer directly to a puzzling problem. Let’s say we find blue polluting goo floating down a stream, and we want to find the cause of this goo. All we need to do is walk up the stream. If we find a large pipe with this goo pouring out of it, and we see no blue goo flowing down from upstream, we’ve pretty well established where the goo came from. End of inquiry. But if the puzzling question pertains to events that happened in the past (e.g., How *did* those dinosaurs die?), or far away (e.g., Are mountains forming on Mars the same way they form on Earth?”), then we may not be able to look or test directly to determine the answer. Here we’d need to form an hypothesis, and devise a clever way of testing it *indirectly*. Therein lies the brilliancy of hypothetical reasoning, aka “the Scientific Method.”

Hypothetical Reasoning involves a series of steps:

1. Articulate a puzzling *problem*
2. Collect *information*
3. Form an *hypothesis*
4. Draw an *implication* to the hypothesis
5. *Test* the implication
6. Draw a *conclusion*

1. Articulate a puzzling *problem*

We begin the process with a puzzling *problem* that cannot be answered or explained with direct observation or testing. For instance, if Bill wants to know why Mary is angry (a puzzling problem, at least for Bill), he might ask her. If she’s willing, she’ll tell him: “You bumped into

my car when you backed out of our driveway, you idiot!” Bill has no need of hypothetical reasoning here, as he can get his answer directly. Sometimes, however, there is no one to ask and no opportunity to observe to get an answer. The cause of events in the distant past can be like this (e.g., a puzzling migration of an ancient civilization from one site to another), or events can occur in unobservable locations (e.g., on other planets, or at the center of the Earth, or at the atomic level). Here we can make use of hypothetical reasoning.

2. Collect *information*

Before we are in a position to make a good hypothesis, we’re going to need some *information*. If we want to know what caused the dinosaurs to die, we’ll need to know what dinosaurs are, how they lived, and how they might die. We’ll need to draw from our knowledge about the world to form a reasonable answer to the question. If a police detective had the puzzling problem of determining who killed a man lying on the floor, he’d walk around the room looking at every detail he thinks might be relevant. That Barack Obama was the U.S. president in 2010 is probably not relevant here, so the detective would not consider it. He’d likely consider a note clutched in the dead man’s hands and a knife in his back as relevant. Note that the detective will need to approach this stage of the inquiry already having a general idea as to what is likely to count as relevant information. This takes some previous background knowledge that not everyone has. A good detective will know enough and be experienced enough to make good judgments as to what might be important information regarding a murder. The detective will be observant enough not to let little details slide by his or her observation. Think of the television detective Adrian Monk walking around a crime scene with his hands squared up to help focus his search for details that can help him later form an hypothesis.

Two points are worth noting already. First, to form a viable hypothesis, one usually needs to have a good deal of background knowledge about the situation. If we know nothing about how dinosaurs lived and how they might die, we’ll be hard pressed to come up with a reasonable answer to our question. Also, assumptions grounded in our world view will impact what information we consider to be relevant to the question. If we mistakenly assume, for instance, that all murders are caused by anger, then we’d miss or ignore any information that might prod us to hypothesize that a murder took place because of greed. If we assume that animals die only from Earth-caused forces (e.g., disease, animal attacks, drowning, earthquakes), then we might miss or ignore information that might warrant our considering a cause of death from space (e.g., a meteorite carrying a deadly virus). We should thus be aware as best we can of any biases or assumptions we bring to the inquiry, and be aware that they can limit what information we collect in the process of forming an hypothesis.

3. Form an *hypothesis*

Forming an *hypothesis* takes a certain amount of creativity, and that’s why some people are better at it than others. Two police detectives can enter the same room, collect the same information at the crime scene, but only one may pull it all together, see a pattern, and visualize what might have happened. Two auto mechanics examine the same stalled engine, but only one comes up with a reason to explain the problem. Only one, then, might have the creativity—or

genius—to form a viable answer to the question at hand. Some scientists, doctors, police detectives, and auto mechanics are better than others at this task.

The hypothesis will ideally meet the five criteria discussed above. It will explain why the dinosaurs died in some detail, or who killed the man on the floor, how, and why. In both cases, we cannot simply look to see if the hypothesis is true. We can't go back in time to see what happened to the oversized lizards or to the hapless sap lying on the carpet. The next step really is the key to hypothetical reasoning.

4. Draw an *implication* of the hypothesis

We next draw one or more *implications* of the hypothesis. This is where many students get confused, so we're going to take this part of hypothetical reasoning slowly and proceed thoroughly. We can say that H implies I (or I is implied by H) if and only if the occurrence of H guarantees that I takes place. As there are different levels of guarantees, there will be different levels of strictness in implication. A *logical implication* will be the strictest. If "H implies I" is meant as a logical implication, then it is absolutely impossible for H to be true and I to be false. It would somehow result in a logical contradiction to say that H is true and at the same time from the same perspective say that I is false. For instance, Ann's having exactly three coins (H) implies that Ann has an odd number of coins (I). It is impossible for Ann to have exactly three coins *and* not to have an odd number of coins. Other examples of logical implications include:

If Bob is taller than Sam, then Sam is shorter than Bob.

If Maria is the sister of Juan, then Juan is the sibling of Maria.

If Aarav is the father of Krishna, then Krishna is the child of Aarav.

If Daria is older than Eva, and Eva is older than Sofia, then Daria is older than Sofia.

If the rag is moist, then the rag is damp.

If A and B are true, then B is true.

If A is true, and B is true, then A and B are true.

If all dogs are animals, then it is false that some dogs are not animals.

In each case above, if the first claim is true, it is logically impossible for the second claim to be false. There is a logical contradiction in saying otherwise. Claiming that Bob can be taller than Sam *and* that Sam can fail to be shorter than Bob contradicts itself. Hypothetical reasoning, however, does not require quite this strict a level of implication. Consider the following implications (again presented as "If..., then..." statements):

- * If a small boy eats five large hamburgers in one sitting, then he will afterwards feel full.
- * If a radio is tossed out of a high-flying airplane and falls to the ground, then it will break.
- * If a large meteorite lands on an animal, then that animal will be harmed.
- * If a man has his head cut off, then he will die.
- * If a dinosaur fails to eat for 1000 days straight, then it will starve to death.

In each case, it is logically *possible* for the first part of the conditional statement to be true and the second part of the statement to be false, but it is so incredibly unlikely that for all intents and purposes we can say that the second is pretty well guaranteed.

Now we can come to the key point pertaining to hypothetical reasoning. An implication we draw from an hypothesis must meet two criteria: (i) the implication must be testable directly, and (ii) it must be pretty well guaranteed by the hypothesis. That is, we'll not demand a *logical implication*, but for the process of hypothetical reasoning to continue effectively, the hypothesis must give us extreme confidence that the implication will be true. We'll discuss the first criterion shortly, but here are some hypotheses with suggested "implications" that are *not* truly implications. That is, the purported "implication" does not follow from the hypothesis; the hypothesis does not guarantee in any practical manner that the purported implication will obtain.

- * If brontosaurus used to live in what is today Bellevue, Washington, then we'd today find brontosaurus bones in every backyard there.
- * If Bob killed the man lying on the floor, then Bob's fingerprints will be on the knife in the man's back.
- * If this afternoon is warm and sunny, then Sunny Shine will be tending her garden today.
- * If a bear messed up the campsite, then there will be bear fur all over the place.
- * If God exists, then my request offered in prayer will be granted.
- * If my request offered in prayer is not granted, then God does not exist.

For an implication to do anything in hypothetical reasoning, we must be able to say confidently that if the hypothesis is true, then that lets us know that the implication is *surely* true. The implication can't just be consistent with the hypothesis; and we can't be satisfied with saying that the implication *might* be true if the hypothesis is true.

****Practice Problems: Hypotheses and Implications**

For each hypothesis and implication suggested below, determine whether the purported implication is truly implied by that hypothesis (it need not be a *logical implication* as defined above).

1. H: A lion killed the dead wildebeest lying here before us. I: The wildebeest carcass will show signs of claw or teeth marks.
2. H: The wind blew a tree over a power line causing the blackout in our house. I: Other houses nearby using the same power line will exhibit blackouts, too.
3. H: The maid killed the butler in the kitchen with a gun. I: The maid will admit to the killing.
4. H: Your car won't run because you are out of gasoline. I: The car will run after you fill the gas tank with gasoline.
5. H: Your radio isn't working because it's not plugged in. I: By plugging the radio in, it will start working.
6. H: This society is generally opposed to murder. I: People will never murder one another in this society.
7. H: Jan studied for her logic test. I: Jan did well on her logic test.

8. H: Mehdi moments ago swallowed the packet containing the secret formula. I: If we looked inside Mehdi's body, we'd find the packet.

9. H: Fatima is a college student who wants to be an anthropologist. I: Fatima will major in Anthropology.

10. H: Malaya is a woman. I: Malaya wears what she believes to be traditional women's clothing.

Answer:

- | | |
|-----------------------|------------------------|
| 1. Implication | 6. Not an implication |
| 2. Implication | 7. Not an implication |
| 3. Not an implication | 8. Implication |
| 4. Implication | 9. Not an implication |
| 5. Implication | 10. Not an implication |

5. *Test* the implication

As we said, there are two character traits any good implication must have in hypothetical reasoning. For all practical purposes, the implication must be guaranteed by the hypothesis, and it must be testable directly. It's because the hypothesis itself cannot be tested directly that we even go through the hypothetical method process. If an implication we draw is not itself testable, then we are back in the woeful state where we started. Of course, we might draw implications that are guaranteed by the hypotheses, but which are not themselves testable. Imagine the following line of reasoning that does so:

"You want to know why the dinosaurs died? I'll tell you! My hypothesis is that ancient aliens landed on Earth and sucked all the cosmic vital energy from the behemoths, leaving them dead on the ground where they lay. An implication to my hypothesis, is—of course—that these aliens understood a lot about cosmic vital energy."

Aside from the many problems with the hypothesis itself (as an hypothesis), the implication that is drawn is not testable (although it is likely guaranteed by the hypothesis). There is nothing we can do to verify or falsify that ancient aliens knew a lot about this so-called "cosmic vital energy." Imagine a slightly revised pronouncement:

"You want to know why the dinosaurs died? I'll tell you! My hypothesis is that ancient aliens landed on Earth and injected the dinosaurs with a virus that gave the behemoths arthritis, and then not being able to move easily, the dinosaurs died. An implication to my hypothesis, is—of course—that the bones of these dinosaurs will show traces of arthritis."

Here at least we have an implication that is testable. We can dig up dinosaur bones, and see if they show signs of arthritis (a process that is quite do-able in many cases). Whether the bones do or do not show such signs will tell us something important about the hypothesis. So, we may not be able to test directly the hypothesis itself, but for hypothetical reasoning to work, we *must* be able to test to see if the *implication* (which is different from the hypothesis) will come out to be true or false.

6. Draw a *conclusion*

If the test of the implication comes out negative (i.e., false), that tells us one thing about the hypothesis. If the test of the implication comes out positive (i.e., true), that tells us something else about the hypothesis. Let's think abstractly for a moment. Let H stand for the hypothesis, and I stand for the implication we draw from it. Then after testing the implication, let's imagine we get a negative result; the implication turns out to be false.

If H, then I [that's the implication we draw from the hypothesis]
Not-I [the test of the implication comes out negative]

What should we conclude? The deductive propositional logic pattern of Modus Tollens tells us:

If H, then I
Not-I
Thus, not-H

If I is truly an implication of H, and if I tests out as false, then Modus Tollens tells us that the hypothesis must be false. We thus have used hypothetical reasoning to disprove an hypothesis. For instance:

If a meteorite hit the Earth causing dust that blocked enough sunlight to kill plants and starve dinosaurs, then there should be uniform a layer of dust (allowing for changes in geographical topography) underground around the world. [For the sake of this example, let's pretend that this is a good implication.] The chief scientist in the process gets underpaid graduate students to go around the world and dig holes looking for that layer of dust. And now let's imagine that no such layer of dust is found. What's the result? Well, if the hypothesis is true, there *should* be a layer of dust that can be found. But there is *no* such layer of dust. So the hypothesis must be false.

Another example: The detective hypothesizes that Andy killed the man on the floor by picking up a knife and stabbing the man with it. One implication would be that Andy has the strength to use a knife. The detective goes to Andy to ask him some questions, and finds out that Andy has been in a coma for the past month, and could not have used the knife. The detective concludes that his hypothesis about Andy killing the man with a knife must be rejected.

Hypothetical reasoning is thus pretty adept at disconfirming, falsifying, or ruling out bad hypotheses. That is, this line of reasoning is a powerful tool in showing that a theory or hypothesis should be rejected. Things are little different when we use it to confirm hypotheses or to show that they are true. Consider the following abstract scenario:

If H, then I [we draw an implication from our hypothesis]
I [the implication tests positively, that is, the implication comes out to be true]

What can we conclude here? That the hypothesis is true? That would look like this:

If H, then I

I

Thus, H

But this is an example of the formal fallacy known as Affirming the Consequent. Any argument fitting this pattern will be invalid. The first two claims do not guarantee the conclusion; that is, it is possible for the first two claims to be true and the conclusion false. So—and here’s a subtle point about much of the science and police work based on hypothetical reasoning—hypothetical reasoning can disprove an hypothesis, but it can’t really prove an hypothesis to be true.

That said, hypothetical reasoning can give us good reason to be happy with an hypothesis. We might say that if the procedure is used well, it can *confirm* an hypothesis. Going back to the dinosaur/meteorite hypothesis/implication, if those underpaid graduate students dug holes all over the world and *did* find a uniform layer of dust, that would not *prove* the meteorite hypothesis to be true, but it would confirm it somewhat. It would give us reason to think that we might be on the right track. So what should we do then? Come up with another implication!

If the full meteorite hypothesis is true, then there should be an extra thick layer of dinosaur bones near the layer of dust, because it’s the dust that was the indirect cause of the dinosaurs’ death. So the primary investigating scientist sends her graduate-student minions out to re-dig those holes and to look this time for an extra thick layer of dinosaur bones. Eureka! They find them! The scientist is really happy about her hypothesis at this point, because two things the hypothesis pointed to showed up to be true. Can she further confirm her hypothesis and get even more grant money? Yes! She needs to draw yet another implication. Hm. “If my hypothesis is true,” she might reason, “there should also be an extra thick layer of plant fossils near the extra thick layer of dinosaur bones we just found, since the plants’ death is what caused the dinosaurs to starve. Minions! Dig once again!” If the now weary graduate students find the expected plant fossils, then that adds further confirmation to the hypothesis. This procedure will never deductively prove the hypothesis to be true (for that would engage the hypothesis’s proponents in the fallacy of Affirming the Consequent), but it *can* provide so much confirmation that only drooling idiots will say, “Well, you haven’t *proven* your point; and besides, it’s just a *theory*.” It’s responses like this that can drive otherwise stable scientists to drink.

Oh, if it were only that easy...

...life would be pleasant, carefree, and long, instead of nasty, brutish, and short. You didn’t really think science was this straightforward, did you? If it were, someone would have discovered a cure for every form of cancer by now, and we’d understand how your least favorite U.S. President got elected. We’ve already hinted at many potential problems in using hypothetical reasoning. (i) We may be dumb as dirt and not even understand the question pertaining to the puzzling phenomenon. (ii) We may not know enough about the world or have enough intellectual creativity to come up with a viable hypothesis answering the question. (iii) We may be unable to see what would be implied by our hypothesis. (iv) The “implication” we come up with may not be truly implied (or guaranteed) by our hypothesis. (v) The implication

we come up with may be useless because it's not testable. (vi) The underpaid graduate students we use to test our good implication may do poor work (due to laziness, inebriation, human error, or having slacked off during their training in lab science classes) and not get accurate test data. (vii) We may overstate our results by saying that we've *proved* our hypothesis to be true when all we really should have said is that we've *confirmed* it.

The additional complexity we now need to understand pertains to the last part of the process in which we draw a conclusion from the results of our test of the implication. So far, we've been thinking in fairly simple *and* simplistic terms:

If H, then I	or	If H, then I
I		Not-I
Thus, H is confirmed		Thus, not-H

On the left, we conclude that we've confirmed our hypothesis; on the right we conclude that we've disproved our hypothesis. Neither assessment is quite correct because the vast majority of philosophically interesting hypotheses are complex critters consisting of more than one claim. Earlier, when we were looking into the enviable character traits of being *internally consistent* and *simple* we noted that many hypotheses make more than one claim. More often than not, the hypothesis is actually a conjunction of claims such as $A+B+C+D+E$. The dinosaur/meteorite hypothesis had about that many parts to it. So, in a sense, $H=(A+B+C+D+E)$. So, what is actually happening in hypothetical reasoning is this:

If $A+B+C+D+E$, then I	or	If $A+B+C+D+E$, then I
I		Not-I
Thus, $A+B+C+D+E$ is confirmed		Thus, not- $(A+B+C+D+E)$

Consider the line of reasoning above and to the right. What is it that will make you reject as false $A+B+C+D+E$ as a whole? If any one or more parts of it are false, then you'd say that the *conjunction* (an "and" statement) as a whole is false. For instance, figure out when you would agree or disagree with the four statements below as wholes:

1. Elephants fly, Oregon is a state in the USA, and $2+2=4$.
2. Mermaids exist, Oregon is south of California, and the Seattle Mariners are a baseball team.
3. Paris Hilton is a space alien, the Earth has three Moons, and $2-1=1$.
4. Circles are round, Ronald Reagan was a U.S. president, and $2+3=5$.

You disagree with numbers 1, 2, and 3 because there is at least one claim in the conjunctions that is false. The only way a conjunction can be true is if each of its parts (i.e., its *conjuncts*) is true, as with number 4. So, if a conjunction is found to be false, then all we really know at that point is that one or more of its elements is false. So for the use of hypothetical reasoning above and to the right, all we can say is that we've disproved $A+B+C+D+E$. But it might be that only *A* (or *B*, or *C*, or *D*, or *E*, or $A+B$, or $A+B+C$, etc.) is false. Actually, the key part of the hypothesis might be true, and some relatively unimportant part of the hypothesis is false. The point is that we have

only gotten started in narrowing down the core problem of our hypothesis. We might still have hope that the key element is true, regardless of what we found out above to the right.

An analogous problem lies in using hypothetical reasoning as we did above to the left where we concluded that we'd confirmed our hypothesis. Since Affirming the Consequent is an invalid inference, we are not justified in saying that the truth of I confirms *all* of $A+B+C+D+E$. It may be that part of this complex hypothesis is enough to guarantee I, and that's why the results of the test on I came out positive. I being true really only confirms *something* about H (i.e., A, or B, or C, or $A+B$, or $A+B+C$, etc.), but further work is needed to confirm that H as a whole is true or that only certain parts of H are true.

So it may be overstating things to even say that the hypothetical method disproves or confirms much of anything. Still, it's a powerful tool for considering hypotheses, and if we pay enough attention to the details, we can warrant enough confidence in an hypothesis to make looking elsewhere for others a seemingly pointless task. Sometimes rational inquiry can give you enough to send a rocket to the Moon and have it land safely in exactly the intended spot, without deductively proving any part of the inquiry and discovery process along the way. Such is life with inductive reasoning.

****Thinking Problems: Hypothetical Reasoning**

1. Your desktop computer suddenly stops working. Think of an hypothesis to explain this phenomenon, and draw one good, testable implication to this hypothesis.
2. Imagine that you just baked a dozen chocolate chip cookies, placed them atop a counter, and told your precocious five-year-old son to stay away from them until after dinner. You go upstairs to print some more re-election fliers for your Senate race, and hear a crash in the kitchen. You run down to find a half dozen warm cookies and the plate smashed on the floor. Your son sits in the corner of the kitchen by the swinging door, and says, "Mommy, a ghost came in, ate some cookies, and dropped the plate. He just left." How might you use hypothetical reasoning to determine what really happened?
3. Read about planet Neptune, and briefly explain how the method of hypothetical reasoning played a role in its discovery. (http://en.wikipedia.org/wiki/Discovery_of_Neptune)
4. Read the Sherlock Holmes short story by Sir Arthur Conan Doyle titled, "The Adventure of the Redheaded League." Briefly explain one way Sherlock Holmes uses hypothetical reasoning in the story. (<http://168.144.50.205/221bcollection/canon/redh.htm>)

Answers:

1. Many good responses are possible. For instance: H1: the building's power is suddenly shut down. I: the room lights won't work either. H2: you accidentally unplugged your computer from the wall. I: the plug will not be plugged in.
2. Multiple scenarios are possible. For instance: Puzzling problem: missing cookies [we might tackle one puzzling problem at a time; the breakage of the plate is a separate puzzling problem, although it's probably related to the loss of the cookies]. Hypothesis: the son got up on the counter and ate some cookies. Implication: the son would have cookie crumbs and melted chocolate in his mouth. Test: the mother looks in the son's mouth and finds no cookie crumbs or

chocolate. Results: Her hypothesis is disproved, as the kid did not have enough time to rinse his mouth clean; the missing cookies must be due to something other than the son eating them.

3. Puzzling problem: Uranus had strange, unexpected perturbations in its orbit. Hypothesis: an unknown planet existed whose gravitational pull was affecting the orbit of Uranus. Implication: this previously unknown planet would be in spot X in the night sky at a particular time. Test: a planet was found in exactly the spot predicted. Results: the “new planet” hypothesis was confirmed.

4. Multiple answers are possible. Look for a specific question Holmes is trying to answer. He will then form an hypothesis, draw an implication from it, and then test that implication. Why, for instance, do you think he was tapping his stick on the ground outside the Redheaded League’s office?

****Practice Problems: Hypothetical Reasoning**

Are the following claims true or false?

1. An hypothesis can have at most one implication.
2. Only one hypothesis may accurately explain a given puzzling problem.
3. In hypothetical reasoning, the hypothesis must be testable directly.
4. In hypothetical reasoning, the implication must be testable directly.
5. In hypothetical reasoning, the test of an hypothesis will either confirm or disprove it.
6. In hypothetical reasoning, the test of an implication will either confirm or disprove a simple, one-part hypothesis.
7. In hypothetical reasoning, the puzzling problem must imply the hypothesis.
8. If an hypothesis says a liquid is an acid, then an implication would be that placing blue litmus paper in the liquid will cause it to turn red.
9. The person forming the hypothesis and implication must be the person performing the test on the implication.
10. One’s worldview can impact what hypotheses one is willing to entertain in answering a puzzling problem.

Answers:

- | | |
|----------|----------|
| 1. False | 6. True |
| 2. False | 7. False |
| 3. False | 8. True |
| 4. True | 9. False |
| 5. False | 10. True |

Chapter 13: Definitions and Analyses

Far too often when people are debating the answer to some question, they end up talking at cross purposes. Someone listening in might say, “Oh, they’re just wrapped up in semantics” or “They aren’t even talking about the same thing.” The likely problem in this debate is that each side is using the same word but meaning something different by it. Imagine a dispute about God’s existence in which one woman is referring to the supremely perfect being of traditional theism while the other is talking about a lesser being like Zeus of ancient Greece. The two interlocutors could bound on indefinitely without ever gaining ground, all because neither took the time to define her terms. Definitions thus are an important part of reasoning, and are worth exploring in some detail.

Definitions are usually desired for a *word*. We might, for instance, want to know someone’s definition (or meaning) of the word ‘God’. Once we understand each other, we can then move forward to examine reasons for believing in the existence or non-existence of such a being. If Sue is trying to demonstrate that a supremely perfect being exists, and Mandy is trying to show that a lesser god (like Zeus) does not, their conversation will be surreal at best.

Definitions assist in avoiding two problems in rational conversations: ambiguity and vagueness. A term can be *ambiguous* when it has multiple clear meanings. The word ‘bat’, for instance, may refer to a lathed piece of lumber intended to smack baseballs, or it can refer to a cave-dwelling flying mammal. If someone uses the word ‘bat’ in a conversation and I cannot detect the intended meaning, I may need to ask for a definition.

Also, some words are *vague*; they have a generally understood meaning, but their specific meaning remains unclear. Imagine someone explaining that bonsai are not tall trees. (We refer here to the Japanese art form of pruned trees growing in small pots or trays.) The word “tall” here is vague. We know generally speaking what it means to be tall, but what counts as “tall” in the context of bonsai? If the bonsai enthusiast explains further that by “tall” he refers to trees over six feet in height, the vagueness dissipates, and the conversation or debate may continue with all involved knowing what the other is talking about.

The word ‘poor’ can be both ambiguous *and* vague, as in “Shahd is poor.” Are we saying that Shahd is poor in spirit? Poor in skill or quality? Poor financially? Let’s disambiguate the word, and say that we are referring to financial status. She lacks money in some sense. Still, the word is vague, as the specific meaning remains unclear. Compared to many villagers of poorer countries, Shahd might be quite rich, but compared to Bill Gates, she might be woefully impoverished. If I ask, “What do you mean by ‘poor’?” I’d be asking for clarification. “I mean she has less annual income than 90 percent of the people in the town she lives in,” the other might reply. “Okay,” I can continue, “I now know what you mean by ‘poor’ in this context.”

Let’s also introduce some vocabulary. A *definition* is made up of two things: the *definiendum* (the word to be defined) and the *definiens* (the words doing the defining).

Definition = Definiendum + Definiens

An example of a definition is, “The word ‘ice’ means frozen water.” Here, “Ice” is the definiendum, and “frozen water” works as the definiens.

Another technical nicety pertains to proper use of quotation marks. If we are quoting someone or using a word, we’ll use double quotation marks (unless we are in a British or British-aligned country, in which case we’ll do everything backwards compared to how the USA does it). Note that in the USA, periods and commas are placed *before* (i.e., inside) double quotation marks. If we are referring to or mentioning a word (not quoting it), then we use single quotation marks. For instance, if we want to say that the word ‘snorkel’ has seven letters in it, we’d punctuate the word exactly as we just did. Also—in the USA—when we use single quotation marks, commas and periods are properly placed *after* the quotation mark. For the following examples pay attention to the placement of quotation marks, periods, and commas:

- * Bob said, “I love snorkels!”
- * Sally said, “Snorkel.”
- * Sally said, “I love to say the word ‘snorkel’.”
- * Sally said, “Bob whispered with glee, ‘Snorkel.’”
- * Hamza said, “I use a snorkel when swimming.”
- * “I too like snorkels,” Youssef said.
- * Bob said, “Snorkels are fun to use”; later he said, “They are particularly fun to use in the ocean.”
- * The word ‘snorkel’ has seven letters in it.

Quotation marks are used so haphazardly, what with newspapers throwing punctuation out the window in grappling with the confinements of the fourth-grade reading level of their intended readership, and of the narrow size of their text columns. British magazine and book editors will do one thing, Americans something else, and far too many don’t seem to care. Philosophers and linguists pay the most careful attention to these things, so it’s little wonder that students have seen a dizzying variance in punctuation styles. In this text, too, there has been an effort to keep things simple, and to use double quotation marks (or italics) to quote, set apart, or emphasize words or phrases. For the most part, this text will continue in this vein, but the rigors of critical thinking as it relates to definitions demand that we be at least aware of the issue.

Purposes of Definitions

There are many reasons for wanting clarification on a word or phrase. Let’s consider the following purposes of definitions:

- Lexical
- Precising
- Stipulative
- Persuasive
- Theoretical

Lexical Purpose

A *lexical* definition provides the general meaning of a word or phrase as understood by the majority of ordinary people. Dictionaries are sometimes called “lexicons,” and attempt to provide exactly this meaning. If I want to know what most people mean by the word ‘snorkel’, I can look in a dictionary and find something akin to the following:

1: any of various devices (as for an underwater swimmer) to assist in breathing air while underwater; 2: a tube housing air intake and exhaust pipes for a submarine’s diesel engine.

If there are multiple definitions, then the first listed in the dictionary will be the most popular or most common (at least at the time of that edition’s publication). Lexical definitions thus give a general, sometimes overly simple meaning understood by most “people on the street.” If you do not know what a word means, a lexical definition may be enough to give you a basic facility in using the word. Meanings can, of course, change over time, and that is why—at least in the USA where language tends to be rather fluid and ever-changing—it’s important to have access to the latest edition of a good dictionary. The most common meaning of ‘pot’, ‘lid’, and ‘gay’ may have meant one set of things 100 years ago, but it’s quite possible the most common meanings have shifted today. We thus *can* be mistaken in our belief about a lexical definition, and reliable dictionaries help us understand the meaning words more accurately.

Precising Purpose

Sometimes we have a general understanding of what a word means, but need a more precise definition of it in a particular circumstance. Here is where we’d want a *precising* definition. Such a definition has as its purpose to make a generally understood meaning more specific to a particular situation. Precising definitions often have a clause pointing out their precising nature: “*For the purposes of X, Y means Z.*”

For instance, we all know what it means to be financially *poor*, but imagine the problem if a bank acquires access to \$50,000,000 for loans to “poor” people, and advertised the opportunity by saying, “Loans available to the poor!” One person in the bank’s Bellevue, Washington neighborhood might take himself to be poor, when he’s actually filthy rich compared to many people around the world, but impoverished compared to some of the upper-level Microsoft administrators living in Medina on Lake Washington. What the bank needs to do is provide a precising definition of “poor,” and say, perhaps in small print, that “For the purposes of this bank loan, ‘poor’ means having a net annual family income of no more than \$10,000.” Now we know what ‘poor’ means in this specific situation.

Sometimes a word has more than one meaning, and we need a precising definition to tell us the context for the particular intended meaning. The word ‘strike’, for instance, has a variety of meanings pertaining to baseball pitches, bowling scores, labor disputes, the lighting of a match, or a blow to the face with a fist. If Theresa simply says, “A strike is a good thing,” we’ll not know if we should agree with her or not. But if she defines her terms as follows, then all is clear: “A strike—in the context of a miner looking for gold—is a good thing, at least for the miner.”

Precising definitions sometimes overlap with lexical ones, as a word might have multiple ordinary meanings, but each is used in a distinct circumstance. The lexical definitions may need to provide context for each partial definition. The lexical definition of 'strike' might say, in part, "1: in the context of labor disputes, an organized halt of labor; 2: in the context of baseball, a pitch that is delivered past the batter over the plate, below the batter's shoulders, and above the batter's knees..." It's no surprise that people might have more than one purpose for a definition.

Stipulative Purpose

A *stipulative* definition is offered when there is a need for a new word to name a new or selected object, or when a word with an established meaning is given a new usage. For instance, if I invent a machine that washes dishes and tells me the time, I get to choose what to call it, and I might stipulate that it be called a "washclock." If zoologist Barry has discovered how to mate a horse with a gorilla (even though none of us can think of a good reason to do so), and the tender moment produces offspring, Barry gets to stipulate what to call the ensuing critters. Perhaps he'll call them "horsillas." That would be a stipulation on his part. Or Barbara could decide that she wants to call the dandelions in her front lawn "imps." We can't say that she's wrong. The word 'imp' may already have an established meaning (an imp is a devilish little creature), but if Barbara wants to stipulate that *that's* what she means by 'imp', then that's her business.

In a sense, stipulative definitions can't be wrong. If Stan decides to call the wrinkles in his elbows "glips," there's no one who can tell him not to do so. Any use of language may be counter to accepted meaning, but if Stan wants to call those fissures glips, he can. He *stipulated* it; he decided that that's what he wished to call them. If, moreover, the word catches on and works its way into day-to-day parlance, then if the editors of dictionaries are doing their job well, the lexical definiens of 'glip' may very well become "elbow wrinkles." That is, what started out as a rather arbitrary stipulation can become so popular as to have a commonly shared meaning ripe for a lexical definition.

Persuasive Purpose

If what we wish to do with a definition is to persuade emotively someone to a given position, then we want a *persuasive* definition. Persuasive definitions use emotive, affective language to try to sway people to one side or another. They do not usually tell us what people normally mean by the word, so they are not lexical. Nor are they usually new words selected for a new or subjectively selected object, so they are not stipulative. Usually the person offering the definition is trying to avoid using logic or evidence to convince another; he's trying to stir emotions (positively or negatively) so that the other will feel good or bad about the thing being defined. For instance:

- * 'Abortion' refers to the senseless slaughter of innocent babies for self-centered motives.
- * 'Abortion' refers to the natural right of all women to experience liberty and self-autonomy.

Well, who would want to slaughter an innocent baby? And who would not support women's rights to autonomy? But neither of these "definitions" explains what abortion is. They provide no cognitive information; they simply stir emotions. Note the use of words and phrases that we associate with bad (or good) things. A more informative definition would use language that is emotionally neutral. Here are a couple more examples:

- * The word 'chess' refers to a silly and puerile game fit only for the socially inept
- * The word 'chess' refers to a noble game fostering the highest intellect, uniting cultures, and giving rise to clear, methodical thinking.

Theoretical Purpose

Finally, a *theoretical* definition attempts to provide a developed, full understanding of what a word means. Usually, theoretical definitions are desired when the meaning as embraced by most "people on the street" is insufficient. Philosophically or scientifically complex words often require such definitions. Philosophers, for instance, will often discuss the nature of "moral goodness." "What does 'morally good' mean?" they might ask. Appealing to a dictionary will hardly help, nor can a philosopher simply say, "I stipulate that 'moral goodness' means X! There, the conversation is over."

Theoretical definitions are an attempt at giving a true, informative understanding of potentially complex words. From the philosophy of Britain's John Stuart Mill (1806-1873), we might define 'good act' as an act that maximizes happiness for all involved. The German philosopher Immanuel Kant (1724-1804) might have defined it as that which is consistent with the Categorical Imperative. Both of these definitions need explanation to be fully informative, but that's what a theoretical definition aims to do.

Theoretical definitions of words referring to philosophically interesting concepts are decidedly open to intelligent debate. There have been a number of viable attempts at defining 'good act' over the millennia. Such debates are more often over the better analysis of a concept than over the meaning of the related word, so perhaps we should defer further discussion along these lines until we look at analyses more closely.

If a word refers to a fairly simple item, like a square, then a theoretical definition may be quite similar to a lexical definition: 'Square' means an enclosed, planar geometric figure with four equal sides and four right angles. But try getting a definition of an electron from people walking the streets, and it will surely be unserviceable for a physicist studying subatomic particles. Such scientists need a richer, more informative definition than will likely be found in a lexicon. We'll revisit the deeper needs of theoretical definitions shortly when we discuss analysis.

****Practice Problems: Purposes of Definitions**

For each definition below, determine whether its single clearest purpose is lexical, precisising, stipulative, persuasive, or theoretical.

1. 'Crosshair' means a fine wire or thread in the focus of the eyepiece of an optical instrument used as a reference line in the field or for marking the instrumental axis.
2. 'Golf' refers to a ridiculous "sport" in which grown men dress up like idiots and chase a little ball around on overly fertilized fields of unnatural grass.
3. Bernadette just developed a new variety of cannabis. She's going to call it 'Bellevue Blitzkrieg'.
4. 'Magnetic' refers to a magnet's moment (also called magnetic dipole moment and usually denoted μ) that is a vector characterizing the magnet's overall magnetic properties. For a bar magnet, the direction of the magnetic moment points from the magnet's south pole to its north pole, and the magnitude relates to how strong and how far apart these poles are. In SI units, the magnetic moment is specified in terms of $A \cdot m^2$.
5. 'Magnetic' means 1. a. Of or relating to magnetism or magnets. b. Having the properties of a magnet. c. Capable of being magnetized or attracted by a magnet. d. Operating by means of magnetism; 2. Relating to the magnetic poles of the earth; 3. Having an unusual power or ability to attract.
6. Euthanasia is the cold-hearted murder of helpless souls who can't cry for help.
7. For the purposes of a baseball game, the word 'pitch' refers to the throwing of a ball by the pitcher to the batter.
8. The Chinese philosopher Kongzi (551-479 BC) used the word *ren* to refer to the fullness of developed human nature in which people can and wish to empathize with others, and wish to seek after the well-being of others.
9. "Motion sickness" means sickness induced by motion (as in travel by air, car, or ship) and characterized by nausea.
10. The word 'pin', in golf, refers to the flag pole placed in each hole's cup.
11. "To make my discussion simpler, I'm going to refer to contributory causes as 'INUS conditions'."
12. 'Strife' means a bitter, sometimes violent conflict or dissention.
13. 'Communism' refers to a godless ideology festering in intellectually feeble countries and oozing its way to mindless, lazy masses.
14. A 'valid' argument is one in which it is impossible for the premises to be true and at the same time and from the same perspective the conclusion be false.
15. 'Knowledge' means adequately justified cognitive assent to a proposition that corresponds truly with the world.

Answers:

- | | | |
|----------------|----------------|-----------------|
| 1. Lexical | 6. Persuasive | 11. Stipulative |
| 2. Persuasive | 7. Precising | 12. Lexical |
| 3. Stipulative | 8. Theoretical | 13. Persuasive |
| 4. Theoretical | 9. Lexical | 14. Theoretical |
| 5. Lexical | 10. Precising | 15. Theoretical |

Types of Definitions

Definitions that attempt to provide the cognitive meaning of a word fall into two basic camps: *extensional* and *intensional*. An extensional (aka denotative) definition provides examples or a

list of members of the group referred to by the word being defined. The list might be complete—as with “‘Highest mountain in the world’ means Mt. Everest” or “‘Ocean’ means the Atlantic, Pacific, and Indian”; the list might instead be partial, as with “‘Country’ is something like Peru, Angola, or France” or “‘Baseball team’ means something like the Giants, Mariners, Yankees, or White Sox.”

Intensional (aka connotative) definitions provide a description, or character traits of the thing referred to by the word being defined. For instance,

- * ‘Ice’ means frozen water.
- * ‘Wife’ means married woman.
- * ‘Jolly’ means jovial.
- * ‘Philosophy’ comes from two Greek words together meaning love of wisdom.
- * ‘Hot’, for spas, refers to the water’s ability to raise the mercury in a thermometer to over 95 F.

Here the definitions are attempting to provide the meaning of the definiendum. With extensional definitions, all we get are examples, or a list of what counts as being a member of the definiendum. Examples may help, but they do not really provide an explanation of what the word means. At best, they may serve as illustrations for the meaning conveyed by a more informative intensional definition.

There are at least three kinds of extensional definitions:

Demonstrative definitions
Enumerative definitions
Definitions by sub-class

A *demonstrative* (aka ostensive) definition simply points to an example of the thing referred to by the word being defined. If a woman wishes to provide a demonstrative definition of ‘chair’, she might point to or direct our attention toward a chair. “What does ‘chair’ mean?” we might ask. She points to one, and ideally we nod in understanding. “Oh, it’s one of those things.” She could also draw a picture of a chair and direct our attention to that. If limber enough, and perhaps majoring in performance art, she might hunch down, make herself look like a chair, and point to herself.

A more common extensional approach is the offer of *enumerative* definitions. These provide specific, named examples of the things the word refers to. For instance:

- * ‘Baseball player’ is someone like Willie Mays, Babe Ruth, or Hank Aaron.
- * ‘U.S. State’ means something like Alabama, Oregon, Indiana, or Hawaii.
- * ‘Mountain’ refers to things like Mt. Si, Mt. Rainier, and Mt. Everest.

A more general extensional approach is to use a definition *by sub-class*. Here, we don’t provide specifically named examples; we instead provide classes or groups that exemplify the thing referred to by the word we’re trying to define. For instance:

- * ‘Baseball player’ is something like a second baseman, an outfielder, or a pitcher.
- * ‘College major’ is a focused study in areas like psychology, philosophy, or chemistry.
- * ‘Machine’ means something like a computer, automobile, threshing mill, or radio.

Extensional definitions, again, offer no explanation of a word. Examples may help illustrate a more informative definition, but if you really do not know what the word ‘mountain’ means, hearing that it is something like Mt. Si, Mt. Rainier, or Mt. Everest could just as easily tell you that ‘mountain’ means something people climb on, or something with rocks and trees on it, or something found on maps, or something tall. In a day-to-day setting, an extensional definition may be all we need to get a basic idea of what a word is likely to mean, and we can usually ask for clarification if we need more information. It’s with intensional definitions, however, that the most cognitive information is conveyed, and that’s the main purpose of most definitions.

There are many ways of attempting to convey the meaning of a word. Some are more useful in some contexts; others are more useful in other contexts. Let’s examine the following kinds of intensional definitions:

Etymological definitions
 Operational definitions
 Synonymous definitions
 Analytical definitions

An *etymological* definition appeals to the etymological roots of a word or phrase to be defined, explains what it means in the original language, and hopes *that* will help people understand the present meaning of the word. Many words used in English, for instance, have roots in Latin, Greek, Sanskrit, or Arabic. So if we wish to define ‘ignition’, we might say “‘Ignition’ comes from the Sanskrit word ‘Agni’, which is the name of the Hindu fire god.” Other examples of etymological definitions include the following:

- * ‘Gymnasium’ comes from two Greek words meaning place of naked training.
- * ‘Karma’ comes from the Sanskrit verb *kr*, which has the double meaning of to do and to make.
- * ‘Pornography’ comes from the Greek word *pornographos*, which means depicting prostitutes.
- * ‘Algebra’ comes from the Arabic word *al-jabr*, which means completing or restoring broken parts.

Etymological definitions can be interesting, and they *can* give some insight (historical or otherwise) into a word’s use and meaning, but they often are of limited use when the primary goal is to convey today’s meaning as used in ordinary or technically precise discussion. Every philosophy instructor feels compelled to tell his or her class that the topic under discussion comes from the Greek words *phileo* and *sophia*, and that Philosophy is thus the love of wisdom. But does “love of wisdom” really do much to explain what students will be doing in a philosophy class for the next ten weeks? And what about the meaning of ‘gymnasium’? It could put a whole new slant on co-ed PE.

Operational definitions provide a test by which to determine if something is accurately referred to by the word being defined. For instance, if we wanted to define ‘hot’ in the context of ironing, we could say “‘Hot’—in the context of an iron used for pressing clothes—refers to a temperature high enough that when you lick your finger and quickly touch the iron the iron sizzles.” Other examples include:

- * Peanut oil is ‘hot’ in a wok when it begins to shimmer and just before it starts to smoke.
- * A liquid is ‘acidic’ if blue litmus paper turns red when it touches the liquid.
- * ‘Passing’ means—in the context of Smith’s Logic class—receiving a GPA of 0.85 or better on the tests.
- * ‘Tall enough’ for this roller coaster means you stand above the hand held out by this wooden figure.

A *synonymous* definition provides a synonym for the definiendum. As long as the person asking for clarification of a word understands the synonym, such a definition can be of practical use. For example:

- * ‘Physician’ means doctor.
- * ‘Damp’ means moist.
- * ‘Arid’ means dry.
- * ‘Jocular’ means jovial.

World language students are often quite satisfied with synonymous definitions. Oftentimes, though, a word referring to a complex thing may not have an exact synonym in the language used. We’d be hard pressed to find exact, accurate synonyms for ‘love’, ‘justice’, and ‘time’; and if we *did* find one, it may not be of much help, since if people do not know what the original word means, they may not know what the synonym means either.

The most informative type of intensional definition is what we can call *analytical*, as it gives an analysis of what the term means. Finally, among all the types of definitions we’ve looked at so far, this one attempts to explain accurately what characterizes the things referred to by the word being defined. A common, effective, and highly informative way of doing this is to provide the *genus* and *difference* of the word. The genus refers to the general kind of thing the word refers to. For instance, consider, “‘Skyscraper’ means tall building.” Here “building” is the *genus*, as that word refers to the general kind of thing a skyscraper is. But what kind of building? An igloo? A mud hut? A doghouse? A two-story apartment? No, skyscrapers are *tall* buildings. “Tall” here functions as the *difference* in the definiens. Scientists use the method of genus and difference to name animals and plants. The genus points to the kind of thing they are, and the difference differentiates them from all the others within that genus. Of course the word ‘tall’ can be vague, so further elaboration may be necessary, but we now have a basic cognitive *understanding* of what the word ‘skyscraper’ means, and this is far more than extensional or other intensional definitions are likely to do for us.

Both the genus and the difference may contain multiple traits. “‘Son’ means male offspring” has only one trait (and one word) per genus and difference, but “‘Boy’ means young human male”

(intending “human male” for the genus) has two traits for the genus (human + male). Of all beings that are both human and male, boys are the *young* ones.

Also, when presented in English, a genus and difference definition need not state the genus first. English is flexible enough not to require that one comes before the other. Note, moreover, that sometimes it’s not clear which of two traits is the genus and which is the difference. That’s okay! Consider “‘Ice’ means frozen water.” We could intend to get across the idea that of all frozen things (the genus), ice is *water* (as opposed, for example, to frozen hydrogen or yogurt); or we could intend to get across the idea that of all forms of water, ice is the *frozen* form (as opposed, for example, to water vapor). Either way, we are conveying what ‘ice’ means, and were doing so accurately and informatively.

Here are some more examples of definitions attempting use of the analytical method of genus and difference (with the intended genus underlined and the intended difference in italics):

- * ‘Husband’ means married man. [The genus and difference could easily be switched here.]
- * ‘Square’ means enclosed geometric figure *with four equal sides and four right angles*.
- * ‘Hammer’ means a tool *used for pounding*.
- * ‘Student’ means a person *who studies*.
- * ‘Biology’ means the study *of life*.
- * ‘Coffee mug’ means a handheld container with a handle *used for holding and drinking coffee*.
- * ‘Logic teacher’ refers to an instructor *who wishes to torment students with useless information*.

Note that some analytical definitions may be false, as these are not mere stipulations stating how someone arbitrarily wishes to use a word. This is an attempt to explain what the word actually means, given current usage in a given context. To define ‘circle’ as a flying elephant would be false, because circles are enclosed geometric figures, not aerial pachyderms.

****Practice Problems: Types of Definitions**

For each definition below, determine which type it most clearly illustrates. Do not be concerned with whether the definitions are accurate or not.

1. ‘Calculator’ means a machine that can add and subtract numbers.
2. ‘Flower’ is something like a rose, lily, daisy, or tulip.
3. ‘Tool’ means instrument.
4. ‘U.S. President’ refers to people like George Washington, Abraham Lincoln, and George W. Bush.
5. ‘Civilized’ comes from the Latin word *civis*, referring to living in community.
6. ‘Sentence’ means the kind of thing you are presently looking at.
7. Cooking oil is said to be ‘hot’ if you toss in a few drops of water and the water splatters.
8. ‘Tiger’ means a cat that is large and striped.
9. ‘Company’ means something like Microsoft, Starbucks, or Boeing.
10. ‘Book’ refers to things like novels, collections of short stories, and atlases.
11. ‘Frigid’ means cold.
12. ‘Building’ means something like this [as the definer points to a building].

13. 'Igloo' means house made of ice.
14. 'Heavyweight' refers in boxing to people who step on a scale and the scale indicates over 200 pounds.
15. 'Pen' means an instrument with ink used for writing and drawing.

Answers:

- | | | |
|----------------------|-----------------------|------------------------|
| 1. Analytic def. | 6. Demonstrative def. | 11. Synonymous def. |
| 2. Def. by sub-class | 7. Operational def. | 12. Demonstrative def. |
| 3. Synonymous def. | 8. Analytical def. | 13. Analytical def. |
| 4. Enumerative def. | 9. Enumerative def. | 14. Operational def. |
| 5. Etymological def. | 10. Def. by sub-class | 15. Analytical def. |

Analyses

Closely related to analytical definitions of words are analyses of concepts or kinds of things. We might want to know what the word 'Justice' means, or we might want to know what Justice is. They are related questions, to be sure, and if we know the answer to one, we may be on the road to knowing the answer to the other. The Greek philosopher Plato (428-347 BC) wrote dialogues featuring his real-life teacher Socrates (469-399 BC) as the main protagonist. Socrates wandered about Athens in real life and in Plato's dialogues asking for accurate, detailed analyses of moral virtues like courage, wisdom, and justice. He wanted to understand what these virtues were—in part—so that he could more readily adopt them into his own character. Socrates rarely found anyone who could provide a good analysis, and philosophy students around the world today cut their teeth on these dialogues to fine-tune their analytic skills.

When Socrates appeared to be asking for definitions for words, he was actually looking for a cognitive understanding of the necessary and sufficient conditions for something being the kind of thing it was. If he had asked about the nature of a triangle, he'd be searching for something like "A triangle is an enclosed geometric figure with three sides." If he had asked for the definition of 'triangle' (i.e., the word), he'd likely have been happy with "'Triangle' means an enclosed geometric figure with three sides." The responses are similar, but it's a somewhat different challenge to determine the essential nature of a thing as opposed to what is accurately meant by the word used to refer to that thing. Justice is what we want for our society; 'Justice' is a word with seven letters.

We've just now referred to the *essential nature* of a thing. That needs a little explanation. A thing's essential nature—in the present context—refers to the character traits or properties that thing must have to be the kind of thing it is. That sounds more complicated than it is. Think of a square. What traits *must* it have to be a square? For one, it must be an enclosed geometric shape. Having an enclosed geometric shape is true of some things—like rectangles and circles—but false of others—like desires for cheeseburgers, the possibility Bob might fall in love with Mary, and Justice. What's true of a thing can be said to be one of its properties. You can't be a square unless you're an enclosed geometric shape. Thus having an enclosed geometric shape is an essential property of a square. But you've also got to have four equal sides. And have four right angles! Each of those traits is necessary for you to be a square. As it is, that list of conjoined

character traits—enclosed geometric figure composed of four equal sides and having four right angles—is *sufficient* for you to be a square. If you have all of those traits (or properties), then that’s enough to guarantee your squareness. A complete and accurate analysis of a square—or squareness—will thus provide the traits that are necessary and sufficient for squareness.

Objects in our day-to-day, ordinary world will also have *accidental* character traits. These traits are characteristics that the thing has but does not need to be the kind of thing it is. A table—as a table—will have a flat, raised surface; that will be an essential trait of a table. But the table might also be colored blue. Blueness is not essential to the table, as we can paint the object completely red, and it will still be a table. Blueness or redness are thus accidental traits of a table.

The concept of a *condition* is more general than that of a *cause*, as all spatio-temporal causes are conditions of effects, but not all conditions are spatio-temporal causes. The presence of gasoline is a necessary cause for a standard automobile to run, and we can also say more generally that the presence of gasoline is a condition that must obtain for the car to run. However, a fistful of an odd number of coins is a necessary condition for the fist to hold exactly three coins, but being odd in number is not exactly a cause of one’s holding three coins. Also, having a flat surface will be a condition that *must* obtain for an object to be a table, yet it will sound peculiar to refer to having a flat-surface as a *cause* of a thing being a table.

In an analysis of a thing or concept, we are looking for conditions that obtain that make the thing or concept what it is. We do not want to appeal to accidental conditions, for a thing does not need those traits to be the kind of thing we’re analyzing. We want to refer to all and only the essential character traits, that is, to the necessary and sufficient conditions for an X being an X. And this is rarely easy, especially with philosophically interesting and scientifically complex concepts. Heck, it’s even a challenge to analyze what a *chair* is. Let’s try.

Attempt #1: A chair is something like that over there [as we point to a chair].

But, for all I know, what you’re pointing to is wooden things, or things painted brown, or things owned by humans. To what exactly are you intending to draw my attention? I still don’t know what a chair is.

Attempt #2: A chair is something like a barber’s chair, a beach chair, a director’s chair, a recliner, or a child’s high chair.

But none of this tells me what a chair *is*. I continue to be ignorant about the nature of chairs.

Attempt #3: A chair is something that is accurately referred to by the word ‘chair’.

You’re kidding, right?

Attempt #4: A chair is something we can sit on.

But that includes pillows and the floor. That may be a necessary condition for a thing being a chair, but it’s not sufficient. Your fourth attempt doesn’t help much.

Attempt #5: A chair is a raised seat we can sit on.

But that includes stools, which are not chairs. Arg! Try again.

Attempt #6: A chair is a raised seat that we can sit on, with a back.

But that includes couches, doesn't it? And couches are distinct from chairs.

Attempt #7: A chair is a raised seat with a back upon which only one person can sit.

Why couldn't that describe a bar stool with a back? Should I be going to *Wiki* for a definition? Huh?

Attempt #8: A chair is a seat with a back, where the seat is raised to approximately knee level of an average adult so that he or she can sit on it.

But that excludes two-inch chairs for doll houses, and small though they may be, they're still chairs. Yes? Egad! I thought I was asking a simple question!

Let's give up and leave this task to chair experts. We're not saying it's impossible to analyze the nature of a chair, but it's often more challenging than it looks. No wonder the people Socrates conversed with long ago had such trouble analyzing more complex things like Knowledge, Justice, Goodness, and Courage.

And yes, there's vocabulary distinct to analyses! Just like a definition is made up of a definiendum and a definiens, so too does an analysis consist of an *analysandum* (the concept to be analyzed) and an *analysans* (the words doing the analysis of the analysandum).

To summarize, we can say that an accurate and complete analysis of a concept or thing will provide all and only the essential properties of that thing. In other words, such an analysis will provide the necessary and sufficient conditions for something being the kind of thing it is. If I am analyzing what it is to be a chair, then I will provide all the essential character traits of chairs (as chairs), and nothing else. And that's not always easy; and according to some philosophers, it's sometimes impossible. But let's go as far as we can with analysis. Let's not give up simply because the going gets rough. Clarity is almost always a good thing.

****Practice Problems: Essential and Accidental Properties**

Are each of the properties below essential or accidental to the kind of thing in question?

1. Three-sidedness is a property of triangles.
2. Having a four-inch-long hypotenuse is a property of triangles.
3. A yardstick must have measuring marks.
4. Water possesses hydrogen.
5. Senator Sunny Shine gardens clothes-free whenever she can.
6. Pastor Bustle is a Central Baptist minister.
7. Dictionaries contain definitions of words.
8. Oil-based paint has oil in it.
9. This pencil is made of yellow-colored wood.
10. This pencil has the property of being usable for drawing.
11. The line making up this circle has the property consisting of points.
12. This circle has the property of being one foot in diameter.
13. All dogs have the property of being animals

14. No cats are fish.
15. Some birds are green.

Answers:

- | | | |
|---------------|---------------|----------------|
| 1. Essential | 6. Accidental | 11. Essential |
| 2. Accidental | 7. Essential | 12. Accidental |
| 3. Essential | 8. Essential | 13. Essential |
| 4. Essential | 9. Accidental | 14. Essential |
| 5. Accidental | 10. Essential | 15. Accidental |

****Thinking Problems: Essential and Accidental Properties**

1. What are the essential properties of the Supremely Perfect Being (i.e., God)?
2. What are the essential properties to being a human?
3. What are the essential properties of a *just* social system?

Answers:

1. Traditional theism includes omnipotence, omniscience, omnibenevolence, and eternity. Philosophical theology is filled with discussions about this topic. The topic can be of interest whether you are a theist or not. Even an atheist might wish to experiment with the hypothetical question, “If God exists, what might be an *accidental* character of such a being?”
2. The Aristotelian tradition points to the use of reason; John Stuart Mill—at least in his *On Liberty*—seems to favor free will as an essential feature of humanity; the Confucian tradition focuses on our social nature and capacity to form key relationships. If you can come up with a better answer, publish it, get rich, and pave the way for a truly just political environment.
3. There is no way this text is even going to try to answer this one. That’s what Social Philosophy and Political Philosophy classes are for.

Mistakes in Definitions and Analyses

The problems we had above trying to give an accurate, full analysis of a chair or definition of ‘chair’ points to a number of common mistakes. Let’s look at some in detail. Obviously, any definition or analysis needs to be accurate. If we define the word ‘T-shirt’ to mean “a large striped cat living in India,” we’d be mistaken, wrong, and muddle-headed. Some people simply misunderstand or are ignorant of the meaning of certain words, or cannot successfully and accurately describe the essential character traits of a thing. What we are looking for here are basic traits any good, useful definition should possess, when they are intended to convey cognitively the accurate analysis of a thing or the meaning of a word. A good analysis of a kind of object and a good analytical definition of a word should *not* be the following:

- Merely extensional
- Too broad
- Too narrow
- Circular
- Negative
- Unclear

Merely Extensional

Since extensional definitions only provide examples, they are unlikely to be a satisfactory definiens or analysans. Socrates was constantly running into this problem in Plato's dialogues. He'd want an informative analysis of a moral virtue, and usually the first thing he'd hear was an example of that virtue. In the dialogue *Euthyphro*, he wanted to know what piety was, but was initially told that it's prosecuting wrongdoers. Even if true, this hardly gave Socrates an understanding of the essential nature of the virtue. Although extensional definitions may provide useful illustrations for more informative intensional definitions or analyses, they are our first mistake: When offering a definiens or analysans, avoid being *merely extensional*. For example:

- * 'Rock-n-Roll band' means groups like the Rolling Stones, AC/DC, and Black Sabbath.
- * 'Rock' means materials that are igneous, sedimentary, or metamorphic.
- * Courage is being willing to obey orders on the battlefield when under heavy fire.
- * Honesty is saying "No" to one's boss when she asks you to lie to a customer.

Too Broad

A common mistake in analyses and definitions, and one that is often a challenge to avoid is being *too broad*. Here, the analysans or definiens covers more than it should; it's too broad in scope; it covers what it's supposed to, but more. For instance, "A tiger is a large, fierce cat." As an analysis of a tiger, "large, fierce cat" covers all tigers, but also includes lions, cheetahs, and leopards. Better would be "A tiger is a large, fierce, striped cat." Such an analysis has the advantage of not including lions, cheetahs, and leopards.

If a proffered analysis is too broad, then what we can do to relieve our poor benighted friend of his befuddlement is to point out examples of things that are clearly included in the concept being analyzed, but which sadly fall outside his analysans. "A truck is a vehicle," our confused friend might say by way of a brief analysis of his favored means of transportation. In the spirit of *bon ami*, we might reply, "Your analysis falls just shy of coherent, for cars, motorbikes, and hovercrafts are a vehicles, too, yet only the delusional perceive them as trucks." Other analyses that are too broad include the following:

- * A ketch is a sailing vessel.
- * A boy is a young human.
- * A pencil is an instrument used for writing or drawing.
- * A chainsaw is a tool used for cutting wood.

Definitions that are too broad include, analogously, the following:

- * 'Ketch' means sailing vessel.
- * 'Boy' means a young human.
- * 'Pencil' refers to an instrument used for writing or drawing.
- * 'Chainsaw' means a tool used for cutting wood.

Too Narrow

The flip side of being too broad is being *too narrow*. Here the definiens or analysans covers too little; the analysandum or definiendum refers to things that are not included. For instance, “A tiger is a large, fierce, striped cat presently living in India.” The most obvious problem here is that there are tigers that presently live outside of India, perhaps in zoos. The analysans here is too finely focused; it’s not broad enough; it’s too narrow. Other examples of analyses and definitions illustrating this problem include the following:

- * An abode is a three-bedroom house with two baths.
- * A birdhouse is a dwelling for wrens.
- * A musical instrument is something producing sound for a piece played in an orchestra.
- * ‘School’ means an environment in which history is studied formally in classroom settings.
- * ‘Soccer’ refers to a competitive sport played with a ball.
- * ‘Battleship’ means a large, sea-going vessel.

Circular

For obvious reasons, we don’t want to assume people understand the word or concept being defined or analyzed when we offer our definition or analysis. If we define a word by using that same word—or a near variant of it—we are not helping folks much. Examples of egregiously *circular* definitions include:

- * ‘Musical instrument’ is an instrument used to play music.
- * ‘Loneliness’ is the state of being lonely.
- * ‘Happy’ means not being unhappy.
- * ‘Woodcutter’s ax’ refers to an ax used by a woodcutter.

Sometimes definitions and analyses can be circular in more subtle ways. Imagine a person who for whatever reason does not know what money is. You get air-lifted to her isolated village, strike up a conversation, and use the word ‘money’. She asks, “What do you mean by the word ‘money’?” As a committed advocate of capitalism, you welcome the chance to afford this childlike soul insight into your society’s highest value. “The word ‘money’ means the thing made by a mint.” The problem is that if this otherwise fulfilled woman does not know what money is, she can hardly be expected to know what a mint is. You need to know what money is in order to make any sense of an institution whose primary function is to make money.

The same problem can occur in analyses. When Socrates was hunting around for an analysis of piety in Plato’s *Euthyphro*, one response was “Piety is that which pleases the gods.” Since that did not really tell Socrates what piety *is*, he asked what it was that pleases the gods. It turned out to be piety. So, piety is that which is pious. Sigh.

Negative

A simple problem to recognize in definitions and analyses is being *negative* instead of affirmative. That is, when we can, we want to explain what a word means rather than what it does not mean. When possible, we want to provide the necessary and sufficient conditions for a concept, and not say what they are not. To do the latter in each case fails to explain the meaning of the word or the nature of the thing being analyzed. Note how the following definitions fail to explain what each word means:

- * ‘Tack hammer’ is a tool that is not used for setting screws.
- * ‘Sloop’ is a boat that is not a ketch.
- * ‘Harmony’ is not a state of discord.

In each case, we could have done better by trying to say what the word *does* mean:

- * ‘Tack hammer’ is a tool used for pounding small nails, brads, or tacks.
- * ‘Sloop’ is a one-masted sailing vessel.
- * ‘Harmony’ is a state of concord.

Some words require a negative definition, and in such cases, there’s nothing to be done than to define them in negative terms. For example:

- * ‘Bald’ means having no hair.
- * ‘Darkness’ means absence of light.
- * ‘Vacuum’ refers to an absence of air.

The same mistake can be found in unsatisfactory analyses:

- * Corporeal substances are those having no thought, will, or consciousness.
- * Being good is to avoid acting in selfish ways.

As with definitions, some concepts require a negative analysis. The concept of darkness, for instance, will be analyzed in terms of absence of light. Still, when we can, we should try to analyze a concept in terms of what it is, rather than in what it is not.

Unclear

We place here at the end a hodgepodge list of problems that boil down to making the definition or analysis unclear. This is more often than not due to poor writing skills, and less often to a lack of commitment to presenting a clear explanation of a word or concept to help convey cognitive meaning. A definition or analysis may be unclear due to poor grammar, or by being vague, figurative, emotive, or needlessly complex. We’ll look briefly at each, focusing on definitions, but each problem can apply to analyses, too.

It takes careful writing to select words with precision, and *poor grammar* can get in the way. Students in a Critical Thinking class should know that arguments contain premises and conclusion. But consider this definition: “The word ‘argument’ means where you have a set of

statements, one or more of which are premises, and one other of which is the conclusion that follows from the premises.” The problem here is with the use of ‘where’. An argument is not a place, so “where” makes no sense here. Better would be this: “The word ‘argument’ refers to a set of statements, one or more of which are premises, and one other of which is the conclusion that follows from the premises.”

Here are two more examples of poor grammar getting in the way of a definition being successful:

- * A ‘statement’ is when a sentence is true or false. [Statements are not a time.]
- * Bill spoke about the word ‘square’. Meaning a four-sided enclosed geometric figure with four right angles. [Fragmented sentences convey no meaning.]

Another problem with grammar can reflect a definition’s unwanted ambiguity. An ambiguous definiens has two clear meanings, and thus the intended meaning of the definiendum is unclear. For instance,

- * In the game of chess, a player is said to be “mated” when one player moves pieces so that the other player’s king is in check but cannot move it legally.

Here, we cannot tell who is mated. Is it the player who can still move his or her queen, or the other player who cannot do so? A better definition that avoids this problem would be, “In the game of chess, a player is said to be ‘mated’ when his or her king is in check but cannot move it legally.”

If a keyword in our definiens is *vague*, then we’ll fail to convey in an informative fashion what the word means that we are trying to define. Recall that a word is vague if even with a general understanding of its meaning, we still don’t know what the word is supposed to mean in the present context. For instance:

- * ‘Bonsai’ means a small tree planted in a tray. [The word ‘small’ is vague here. Better would be “‘Bonsai’ refers to a tree less than six feet in height planted in a tray.”]
- * ‘Desert’ refers to a dry place. [‘Dry’ is vague here; it would be better to refer to maximal inches of annual rainfall.]

We also want to avoid merely *figurative* language. Here we refer to poetic, whimsical, or metaphoric forms of expression. It may make for entertaining writing or discourse, but it rarely conveys cognitive meaning. In other words, it can be fun—and even funny—but it does not tell us what the word actually means. For instance:

- * ‘Puritan’ refers to one who fears that someone, somewhere is having fun.
- * ‘Love’: a rose with gentle blooms and painful thorns.
- * ‘Chess’ refers to the game of kings.

Since we usually want our definitions to convey cognitively the meaning we intend, we also want to avoid using *emotive* or affective language. Such language may psychologically persuade

and stir feelings—and there may be nothing wrong with that—but a roiling of one’s emotions is not the same thing as getting across clearly what we mean by a word or concept. Note the emotionally charged words used in each definition below. They are intended to sway readers to a point of view, rather than to educate them regarding our intended meaning of a phrase.

- * ‘Free speech’ refers to the natural right of every citizen to speak his or her mind, openly and unafraid of tyrannical censure, on matters great and small, for the health of a thriving democracy.
- * ‘Free speech’ is what racists, sexists, and homophobes dishonestly employ to spew their vile hatred upon those unjustly marginalized with less political power.

Finally, a definition can be unclear because it is *needlessly obscure*. Again, if the purpose of our definition (or analysis) is to explain the meaning of a word (or to provide its necessary and sufficient conditions), then speaking in a pointlessly obscure or overly complex style will get in the way of informing our listeners. Of course, some words and concepts are quite complex, and demand a complex definition or analysis. The word ‘dynatron’ probably needs something as complex as “a vacuum tube in which the secondary emission of electrons from the plate results in a decrease in the plate current as the plate voltage increases.” If there is a simple and straightforward way of defining a word accurately, however, then we’ll want to proceed in that fashion. There is no good reason to use fancy, bizarre language when normal, conversational wording will do. We should thus eschew obscurantism. Those offering the following definitions should be slapped upside the head:

- * ‘Dustbuster’ means a handheld, mechanical, motorize atmospheric pressure gradient creator for removal of particulate matter.
- * ‘Soap’ means a saponified glyceride intended as a sanitizing or emulsifying agent.

****Thinking Problems: Mistakes in Definitions and Analyses**

Read Plato’s delightfully short *Euthyphro* or Book I of *Republic*, and look for various bad definitions and analyses of piety and justice respectively. The works can be found online for free and in many good print editions.

****Practice Problems: Mistakes in Definitions and Analyses**

What is the single most obvious problem in each of the following definitions and analyses? (A) Too broad, (B) Too narrow, (C) Circular, (D) Merely extensional, (E) Negative, (F) Unclear. Some may be guilty of more than one problem, but select the most obvious mistake.

1. A trout is a fish.
2. ‘Happiness’ refers to the state of being happy.
3. ‘Gourmet chef’ does not refer merely to fry cooks.
4. A university is something like Harvard, Princeton, or Yale.
5. A fish is something with gills that swims in the Atlantic Ocean.
6. A test is when one demonstrates mastery of some knowledge or skill.
7. Justice is paying one’s debts and telling the truth.
8. A camel is a ship of the desert.
9. A guitar is a stringed musical instrument.

10. Intelligence is what intelligent people have.
11. 'Telephone' means a device used for communication.
12. Birds are things like parrots, eagles, and doves.
13. Football is a senseless sport watched by couch potatoes wishing to escape their sedentary lives.
14. A skeptic is a non-believer.
15. 'Sibling' means sister.
16. Socialist health care is bank-breaking, unwarranted give-away to loafers.
17. 'Deodorant' means a preparation for camouflaging the malodorous secretions of the apocrine sudoriferous glands.
18. Architecture is frozen music.
19. A musician is someone who plays music.
20. A bonsai is a small tree.
21. A statue is something like *that* [as he points to Michelangelo's *David*].
22. 'Democracy' refers to the self-affirming right of a people to chart their own destiny to greatness.
23. A 'sound' argument is where the premises would guarantee the conclusion *and* the premises are true.
24. A tiger is a carnivorous animal.
25. A bird is a feathered animal from South America.
26. A devout Christian is not agnostic.
27. Investing money in the stock market is legalized gambling.
28. 'Street cleaner' refers to a public thoroughfare sanitation engineer.
29. 'Sandwich' refers to two pieces of bread holding slices of roast beef and cheddar cheese.
30. 'Italian food' means items like spaghetti, lasagna, and tortellini.

Answers:

- | | |
|--|---|
| 1. Too broad | 16. Unclear (emotive) |
| 2. Circular | 17. Unclear (needlessly complex) |
| 3. Negative | 18. Unclear (figurative) |
| 4. Merely extensional | 19. Circular |
| 5. Too narrow | 20. Unclear (vague) |
| 6. Unclear (grammar; a test is not a time) | 21. Merely extensional |
| 7. Merely extensional | 22. Unclear (emotive) |
| 8. Unclear (figurative) | 23. Unclear (grammar; an argument is not a place) |
| 9. Too broad | 24. Too broad |
| 10. Circular | 25. Too narrow |
| 11. Too broad | 26. Negative |
| 12. Merely extensional | 27. Unclear (figurative) |
| 13. Unclear (emotive) | 28. Unclear (needlessly complex) |
| 14. Negative | 29. Too narrow |
| 15. Too narrow | 30. Merely extensional |

Chapter 14: Probability

Probability Theory

Inductive arguments purport to show that a conclusion is *probably* true given the truth of the premises. Strength of inductive arguments—as students will recall—comes in degrees. Some inductive arguments are strong, some are really strong, and some are really really strong. Weakness comes in degrees, too. Probability theory attempts to quantify probability, since the difference between really strong and really really strong is shy of transparent. Probability theory thus attempts to make the degree of probability clearer and more precise.

There is more than one way to understand probability, and different *probabilistic* claims will mean somewhat different things depending on what kind of probability is intended. You can even at this initial stage likely sense the difference between the following statements:

- * It is probable that a six-sided die rolled honestly will turn up with a number greater than one.
- * The probability of a 17-year-old male getting in an auto accident within ten years of driving is greater than that of a 17-year-old female.
- * It is probable that Dan and Sue will get married this year.

To determine the probability of these events requires our using different approaches to probability. We'll thus examine three theories of probability: the *Classical Theory*, the *Relative Frequency Theory*, and the *Subjectivist Theory*. Each is distinctly suited for specific kinds of circumstances, and each can thus help us make sense of the meaning behind "It is *probable* that X."

First, let's look at how probability is quantified. An absolutely guaranteed event has a probability of 1; an event that cannot possibly happen has a probability of 0. Thus probability can be expressed in terms of decimal numbers between and including 0.0 and 1.0. A probability of 0.5 indicates that there is a half chance of the event occurring. A probability of 0.25 indicates a one fourth chance. We can also see that probability can be expressed in terms of fractions. An honest flip of a coin will result in an even chance of either heads or tails, so there is a $1/2$ or 0.5 probability of getting heads. There are six sides to a normal die, so an honest toss will give us a $1/6$ or 0.166 probability of getting, say, a 2.

We can also express probability in terms of a percentage. If there is a $1/4$ chance of winning a bet, then we stand a 25% chance of winning that bet; if we stand a 0.17 probability of losing a bet, then we stand a 17% probability of losing that bet (to covert from a decimal number to a percentage, just shift the decimal two places to the right).

Finally, we can think of probability in terms of odds. While probability is often thought of as a fraction, odds are thought of as a ratio. If we have fifty/fifty chances, that gives us a probability of $1/2$. That converts to odds of 1:1. That is, there is one chance of winning (the left number in the ratio) to one chance of losing (the right number). A perfectly ridiculous way to convert from probability to odds, or from odds to probability, is to chant one easy mantra: "top-left, top-left,

top-left....” Notice that fractions (the probability) have top and bottom numbers (i.e., the numerator and denominator respectively), while ratios (the odds) have left and right numbers. The top number of the fraction and the left number of the ratio will be the same. So, let’s say you know that the probability of an event happening is $1/3$. The odds will be $1:x$. You don’t know what x is yet, but don’t sweat it; you’re half way there. Now think of the $:$ as if it’s a plus sign. The two odds numbers together add up to the bottom number of the fraction. Sooooo... $1+x=3$. That means the x is 2. BINGO! If the probability of an event is $1/3$, then the odds are $1:2$. You have one chance of winning against two chances of losing, which is exactly what you’d expect with a $1/3$ probability of winning.

For those who appreciate clarity and precision, the following formula expresses how odds work:

$$\text{Odds}(A) = f:u$$

In English, this says, “The Odds of event A happening can be stated in terms of a ratio of favorable outcomes (“winners”) to unfavorable outcomes (“losers”).

The odds-probability shift can work from odds to probability just as easily. Imagine you’ve got $2:3$ odds of winning a game. What would be the probability? Think “top-left, top-left- top-left....” The left number from the odds is 2, so the top of the fraction is 2. So far you’ve got $2/x$. The x will be the two odds numbers added together: $2+3=5$. So the bottom number of the fraction is 5, making the probability $2/5$!

2:3 2/5 The top-left numbers are the same.
2:3 2/5 2+3=5 The two odds numbers add up to the bottom fraction number.

A cleaner, more precise formula might look like this:

$$\text{Odds}(A) = x:y \text{ is equivalent to } P(A) = x/(x+y)$$

Here are some examples of equivalent quantified probabilities:

<u>Fraction</u>	<u>Decimal number</u>	<u>Percentage</u>	<u>Odds</u>
0/6	0.0	0%	0:6
1/2	0.5	50%	1:1
1/4	0.25	25%	1:3
1/3	0.33	33%	1:2
1/5	0.2	20%	1:4
1/8	0.125	12.5%	1:7
3/78	0.038	3.8%	3:75
17/91	0.187	18.7%	17:74
1/1	1.0	100%	1:0

****Practice Problems: Quantifying Probability**

Provide the requested probability or odds.

1. What is the probability of $\frac{4}{5}$ in terms of a decimal number?
2. What is the probability of $\frac{4}{5}$ in terms of a percentage?
3. What are the odds of an event with a probability of $\frac{4}{5}$?
4. What is the probability of 0.9 in terms of a fraction?
5. What is the probability of 0.9 in terms of a percentage?
6. What are the odds of an event with a probability of 0.9?
7. What is the probability of 75% in terms of a decimal number?
8. What is the probability of 75% in terms of a fraction?
9. What are the odds of an event with a probability of 75%?
10. What is the probability if the odds are 2:7?
11. What is the probability if the odds are 5:1?
12. What is the probability if the odds are 23:90?
13. What is the probability if the odds are 90:23?
14. What are the odds if the probability is $\frac{5}{6}$?
15. What are the odds if the probability is $\frac{94}{113}$?
16. What are the odds if the probability is $\frac{3}{19}$?
17. What are the odds if the probability is $\frac{2}{8}$?
18. What is the probability if the odds are 2:6?

Answers:

- | | | |
|-------------------|----------------------|---------------------------------|
| 1. 0.8 | 7. 0.75 | 13. $\frac{90}{113}$ |
| 2. 80% | 8. $\frac{3}{4}$ | 14. 5:1 |
| 3. 4:1 | 9. 3:1 | 15. 94:19 |
| 4. $\frac{9}{10}$ | 10. $\frac{2}{9}$ | 16. 3:16 |
| 5. 90% | 11. $\frac{5}{6}$ | 17. 2:6 = 1:3 |
| 6. 9:1 | 12. $\frac{23}{113}$ | 18. $\frac{2}{8} = \frac{1}{4}$ |

The Classical Theory

The *Classical Theory* of probability was developed in the 17th century to analyze the probabilities involved in games of chance. Two conditions must obtain for us to use this approach to determining how probable an event will be. First, the total number of possible outcomes must be known. Second, each outcome must have an equal chance. For instance, an honest toss of a coin has only two realistically possible outcomes: heads or tails. It is logically possible that it could land and remain on its edge, but for all intents and purposes, that *logical* possibility is not—in this sense—possible. Moreover, if it is an honest toss, then there is just as much chance that it will turn up heads as tails.

The same is true of a roll of an honest six-sided die. There are exactly six possibilities (it won't land and remain on an edge or corner): 1, 2, 3, 4, 5, and 6; and each side has an equal chance of appearing. If the die is "loaded" or weighted, it is not an honest toss, and the chances of any one number coming up are not equal to the others. The Classical Theory would then not give an accurate analysis of the probability of any one number coming up. Or, if we did not know how

many sides the die had (it might be a gamer's dodecahedron die with 12 sides), we'd not be able to calculate any probability of outcomes.

Even though it involves some rudimentary math, the Classical Theory is still inductive. There is not absolute certainty, for instance, that there are only two outcomes to a coin flip. It remains logically possible that it might land on its side, or that it will float in the air and never drop. It *could* (if we're talking *logical possibility* here) turn into an elephant and fly to the Moon. These bizarre, counter-intuitive scenarios are enough to keep probability calculations about ordinary events like die or coin tosses inductive operations.

The Classical Theory is easy to use on single, isolated events. We take the number of possible "winners," or favorable outcomes, and make that the "top" number (i.e., the numerator) in our probability fraction. We then take the total number of possible outcomes and make that the "bottom" number (i.e., the denominator) of our fraction. And that's it! That's the probability of the event taking place, or of "winning"! The formula looks like this:

$$P(A) = f/n$$

In English, that says, "The Probability of event A is the number of favorable outcomes over the number of total outcomes." Below are some illustrations. For the sake of the examples and practice problems in this text, unless it's stated otherwise, all dice will be six-sided, decks of cards will consist of the 52 normal playing cards (minus jokers) in a poker deck, and all events will be honest (e.g., no loaded or shaved dice).

What is the probability of rolling a 2 on one roll of a die? Well, there is only one "winning" or favorable outcome (i.e., 2), so 1 is the top number of our fraction: $1/x$. Since there are six possible outcomes to our toss of the die, we place 6 at the bottom of our fraction to get $1/6$. That's it! We're done.

What is the probability of rolling an even number on one roll of a die? There are three possible favorable outcomes (2, 4, and 6). So we place a 3 at the top of our fraction to get $3/x$. Moreover, there are six possible outcomes, so we place a 6 at the bottom of our fraction to get our answer: $3/6$, which reduces to $1/2$.

What is the probability of selecting a black jack on one honest blind draw from an ordinary deck of 52 playing cards? There are two favorable "winners" (i.e., the jack of spades and the jack of clubs), so we place a 2 at the top of our fraction. There are a total of 52 cards to draw from, so we place 52 at the bottom of our fraction to get $2/52$ or $1/26$.

Imagine that I toss out the face cards (kings, queens, and jacks) from my deck of cards. What is the probability that you will draw an ace on one blind draw? There are four favorable outcomes (i.e., the four aces), and a total of 40 cards to draw from (there were 12 face cards, leaving 40 numbered cards). So, the probability of drawing an ace from *this* smaller deck is $4/40$, which reduces to $1/10$.

An opaque urn has three green balls, two black balls, and five yellow balls in it. You reach in on a blind draw and select one ball. What is the probability that it is (a) a black ball? (b) a red ball? (c) any of the balls? Answers: (a) There are two black balls and a total of ten balls in the urn, so the probability of drawing a black ball is $2/10 = 1/5$. (b) There is no red ball in the urn among a total of ten balls, so the probability of drawing a red ball is $0/10$, which equals 0, which means it's impossible. (c) Here, any ball is a "winner," so the number of favorable outcomes is ten, and the total number of balls is still ten. So, the probability is $10/10$, which is 1, which means you are absolutely guaranteed to win (i.e., to draw a ball). Woo hoo!

The Relative Frequency Theory

The *Relative Frequency Theory* of probability was developed by insurance actuaries in the 18th century as they tried to determine the likelihood of groups of people living to a given age. The Classical Theory could not be used, because the chances of living to age 20, 30, 40, 50, and beyond are not equal. The Relative Frequency Theory is simple, though. The way it works is to observe a number (the larger the better) of outcomes and see how many of those exhibit the particular outcome you are interested in. The handy-dandy formula looks like this:

$$P(A) = f_o/n_o$$

In English, this says, "The Probability of event A is the number of favorable observed outcomes over the total number of observed outcomes." Don't get bogged down in what looks like weird math. This is even easier than the Classical Theory. An example will make this fairly clear.

Let's imagine you want to know the probability of 17-year-old males who have just received their driver's license getting into an auto accident during their first year of driving. What you do is observe as many 17-year-old males as you can who just received their driver's licenses, and watch them for one year. The more such males you watch for a year, the stronger and more reliable will be your inductive probability calculation. In the course of that year, you note how many of these males get into an auto accident (i.e., the "favorable" outcome; that is, the one you are interested in). If you were able to observe a total of 500 17-year-old males the first year they receive their driver's licenses, and you observed 25 of them getting in auto accidents, the probability you are looking for is $25/500$ or $1/20$. We can also present the results in other terms: 0.05, or 5%, or with odds of 1:19.

Here's another example. You want to know the likelihood of evening shoppers purchasing a brand of soap now that you—as store manager—have placed it at the end of a store aisle. You watch 1000 shoppers walk past during evening hours, and 25 of them pick up and purchase the soap. The probability is determined simply by making a fraction, placing the favorable observed outcomes number at the top (to get $25/x$) and placing the total number of observed outcomes at the bottom, to get $25/1000 = 1/40$. And again we can present the probability in other terms: 0.025, or 2.5%, or with odds of 1:39.

The Subjectivist Theory

The probability of some events occurring cannot be determined by either the Classical Theory or the Relative Frequency Theory. For instance, what if we wanted to know the probability of the Seattle Seahawks winning their first football game in the upcoming season? Or the probability of Tom marrying Sue this year? The Classical Theory cannot work in either case, because even though there seem to be two clear outcomes (win or lose, marry or not), the chances of either happening are surely not equal. Nor can we use the Relative Frequency Theory, because we do not have a background of a total number of observed outcomes; we have not seen the Seahawks play their first game of this upcoming season before (it's a unique event), and even if Tom and Mary have married in the past, we surely have not observed enough such legal unions between them to use this second theory. Again, this potential marriage is probably a unique event that has not been observed in the past. What's a gambler to do who wishes to bet on the Seahawks game or Tom and Sue's relationship?

The *Subjectivist Theory* of probability is the simplest yet, but may seem the least satisfying. What we do is go to an expert, someone who knows the Seahawks better than anyone else, or who has the closest ties to Tom and Sue; we'll then ask him (we'll imagine it's a guy here) what odds he'd honestly, sincerely give that the Seahawks will win or that Tom and Sue will marry. If his odds are sincere and based on his knowledge of the situation, then he *should* give opposite odds that the Seahawks will *lose* or that Tom and Sue will *not* marry. We then take those odds, convert them to a probability, and *voilà*, we have our probability. Simplicity itself! For formula fans, here is what the odds-probability equivalence looks like:

$$x:y \text{ is equivalent to } x/(x+y)$$

Let's have another football example. If we want to do the best we can at determining the probability of the Broncos beating the Raiders in the upcoming game, we go to a football expert who tells us that she'd give 2:1 odds that the Broncos will win (and she'd give 1:2 odds that they'd lose). We take those 2:1 odds and translate them into a probability fraction to get 2/3. We now have reason to believe that the Broncos have a 2/3 chance of winning that game!

This is called the "Subjectivist Theory" because it is highly subjective to someone's opinion. No one can know for sure if the Broncos will beat the Raiders, nor can anyone know the exact probability of it happening. There are too many variables involved (sick quarterbacks, psychotic tight ends, a last-minute jailing of the star running back, a drunken coach...and that's only some *bad* factors that most people will not know about; good factors can skew probability the other way). Still, some people are more knowledgeable about a team's chances than are other people, and the odds they honestly and sincerely provide count for more than do those of less informed people. If the experts truly have expertise in the matter, and if they provide us with odds they sincerely believe in, we may not have much else to go on. That is, our understanding of probability in such unique cases may not amount to much more than this.

Now let's have one more example of using the Subjectivist Theory. What's the probability of Jared getting accepted to Harvard Law School? He's applied there only this one time, so we have no body of observed outcomes with which to use the Relative Frequency Theory. And the chances of his being accepted or not are surely not dead equal, so we can't use the Classical

Theory. The best we can do is to find an expert on the matter. Perhaps that is a school councilor who knows Jared and his academic career in high school and college, and who knows something about law school admittance demands. If this expert offers an honest appraisal of 1:10 odds, then we can do the simple translation to a probability fraction to determine that from the subjective opinion of the best expert we can find, Jared has a $1/11$ chance of getting into Harvard Law School.

****Practice Problems: Single-Event Probability Calculations**

1. What is the probability of rolling an odd number other than three on one roll of a six-sided die? What are the odds?
2. What is the probability of drawing an even numbered card on one blind draw of a deck of 52 playing cards?
3. If we watched 200 Bellevue College students take a Symbolic Logic test, and 25 received an A, then on that basis what is the probability of a Bellevue College student receiving an A on a Symbolic Logic test? What are the odds? What is the probability of a Bellevue College student *not* receiving an A on a Symbolic Logic test? What are the odds?
4. Stan is Bill's best friend and has known him all Bill's life. Stan give 3:5 odds that Bill will buy a Ford USV within the year. What probability should we—on this basis—assign to this purchase taking place in this time frame?
5. Take what we know from problem #4, and include Andy's odds of 5:1 that Bill will buy a Ford SUV within a year. Andy has just met Bill, and is an enthusiastic, optimistic Ford dealer. What probability should we now assign to Bill's buying Ford SUV within a year?
6. Imagine an urn with four white balls, three black balls, and one green ball. What is the probability of selecting a white ball on one blind draw, when you can draw one and only one ball on any given draw? A black ball on one blind draw? A green ball on one blind draw? What are the odds for each case? What theory needs to be used here?
7. What is the probability of drawing a red ball from the urn in problem #6? What is the probability of drawing a ball on one blind draw from the urn? What are the odds in each case?
8. Janelle gives 10:1 odds that the Cowboys will beat the Broncos in their next game together. Given her odds, what is the probability that the Cowboys will beat the Broncos? What theory needs to be used in this problem?
9. Of 367 Bellevue College students who completed Critical Thinking last year, 192 required psychiatric care within six weeks. Based on those observations, what is the probability that a Bellevue College Critical Thinking student will require psychiatric care within six weeks of completing the course? What are the odds of this happening to such a student? What theory needs to be used in this problem?
10. Imagine an ordinary deck of 52 playing cards. What is the probability of getting a jack or a red queen on one blind draw? What are the odds?
11. Juanita believes that she has a 50% chance of getting an A in her Chemistry class. What probability in terms of a fraction, and then in terms of a decimal number, should she give herself assuming this estimate? What odds should she give herself assuming this estimate?
12. The Bellevue Alimentary Robustness Foundation observed 175 people eating lunch at the Bellevue College Cafeteria. Of those observed, 35 became ill immediately afterwards. BARF has good reason to believe that the food caused the illness. Given this finding, what is the probability

that a BC Cafeteria diner will get sick due to eating there? What is the probability in terms of a percentage? In terms of a decimal number? What are the odds of getting sick from eating there?

13. Imagine an urn with three green balls, two yellow balls, and ten brown balls. What is the probability of selecting a yellow ball on one blind draw? What would be the odds? What theory needs to be used to determine the probability?

14. Janet believes that the odds are 2:3 that the Eagles will win next week's football game. Given those odds, what is the probability that the Eagles will *lose* that game? What theory are you using here? What odds should Janet give that the Eagles will *lose*?

15. Julia's four friends and eight family members (who all took Critical Reasoning) have watched her meet ten football quarterbacks in the past two years, and she has dated seven of them within three weeks. Now that Julia has met the Seattle Seahawk's first-string quarterback, and given the information provided here, what probability and odds should her friends and family decide for Julia to date this quarterback within three weeks? What theory do they use here?

16. Jessica and her new boyfriend begin to play a rather frisky game of cards, but all of the kings and half of the jacks are missing. At one point, Jessica fans the deck of cards out to her friend, and asks him to select one blindly. He does so. What is the probability that he will select an ace? A jack? A king? Any card at all? What theory are you using here?

17. Janelle has gone to Death Valley 12 times in different summers, and nine times the daily mid-day temperature was over 110 F. She is going to Death Valley again this summer. What is the probability that the daily mid-day temperature will not reach 110 F? What are the odds? What theory are you using to determine the probability?

18. Jackie places a knight randomly on an empty chess board. What is the probability that she places it on a square on which a pawn usually sits at the beginning of a game of chess? What is the probability in terms of a decimal number? In terms of a percentage? What are the odds? What theory are you using to determine the probability?

19. Imagine that Jackie is playing white and is about to make her first move. She knows nothing about chess theory, so her move is legal but random. What is the probability that her first move will be to place one of her men on c3 (i.e., the square directly in front of her queen's bishop)? What are the odds?

20. Jillian had three shots of bourbon while playing craps at a Las Vegas casino on Tuesday and won money at the game. She had three shots of Scotch while playing roulette at the casino on Wednesday and won money at the game. On Thursday she drank three shots of rum and lost money to the casino at blackjack. On Friday she drank three shots of gin and won money at baccarat. It's now the following Monday, and she's about to play poker. Jillian quickly drinks three shots of tequila, using what she takes to be critical thinking skills and the Relative Frequency Theory to conclude that her probability of winning is greater with three shots of alcohol in her. What probability did she derive for winning money at poker while mildly besotted this evening? Why is her reasoning and use of the Relative Frequency Theory poor?

21. A dishonest gambler shaves a die slightly on one particular side so that it's more rectangle than square, making the die more likely to come up on one face rather than another. What theory would you use to determine the probability of the die coming up an even number?

22. Washington Senator Sunny Shine is entering her first re-election campaign. She thinks she's got a pretty good chance of retaining her Senate seat. Pastor Bustle and some of his parishioners are vehemently against Shine's public support of a clothing-optional beach at a remote

Washington State Park, so they form the Public Opposed to Reckless Nudity political action committee to oppose Shine's re-election. Bustle has stated in media interviews that "Shine has no chance at all of re-election!" How shall we best determine the probability of Shine getting re-elected?

23. You are playing some form of draw poker and see three aces, four kings, two queens (all face up), and ten other unknown cards face down on the table. Your three friends hold five cards each, none of which your friends let you see. You need a queen to fill out a straight (K-Q-J-10-9) you're seeking, so you discard one useless card and draw one card from the deck. What is the probability of getting your needed queen?

Answers:

1. $2/6 = 1/3$, 1:2

2. $20/52 = 5/13$, 5:8

3. $25/200 = 1/8$, 1:7, $175/200 = 7/8$, 7:1

4. $3/8$

5. It's still $3/8$, because Stan is more of an expert on Bill than is Andy.

6. $1/2$, $3/8$, $1/8$, 1:1, 3:5, 1:7, Classical Theory

7. $0/8$, $1/1$, $0/8$, 1:0

8. $10/11$, Subjectivist Theory

9. $192/367$, $192:175$, Relative Frequency Theory

10. $3/26$, 3:23

11. $1/2$, 0.5, 1:1

12. $1/5$, 20%, 0.2, 1:4

13. $2/15$, 2:13, Classical Theory

14. $3/5$, Subjectivist Theory, 3:2

15. $7/10$, 7:3, Relative Frequency Theory

16. $2/23$, $1/23$, 0, 1, Classical Theory

17. $1/4$, 1:3, Relative Frequency Theory

18. $1/4$, 0.25, 25%, 1:3, Classical Theory (there are 16 pawns at the beginning of a game, and a chess board has 64 squares of equal size)

19. $1/32$, 1:31 (she can move her queen's knight or her queen's bishop pawn there)

20. $3/4$. There are many problems with Jillian's thinking, but two major concerns are her embracing a False Cause fallacy (drinking alcohol does not cause people to win at games of chance), and the uncertainty that she has equal knowledge, skill, or experience with each game.

21. Since we do not know the exact amount shaved off the die, and thus cannot determine the likelihood of each face coming up, we cannot use the Classical Theory. We can, however, roll the die 100 times and see how many times it comes up even. We'd thus be using the Relative Frequency Theory to calculate a fairly strong probability. If we rolled the die 1000 times and counted the number of times it came up even, we'd have an even stronger argument for the probability.

22. There are two outcomes (re-elected or not re-elected), but the chances are not equal. So we can't use the Classical Theory. Shine has never entered a re-election campaign before, so we have no set of observed experiences with some favorable outcomes to draw upon. Thus we cannot use the Relative Frequency Theory. We are left with the Subjectivist Theory, but Pastor Bustle and PORN are likely unreliable sources of odds, especially as Bustle speaks emotively before the media. Shine's opinion of her chances may be somewhat biased or overly optimistic,

so she may not be a reliable source of odds, either. What we'd need to do is find an expert: someone who knows Shine well, the mood of Washington voters, and the circumstances underlying the re-election. We'd seek odds of Shine winning from this expert, and calculate Shine's probability of winning based on that. That may be the best we can do here.

23. We can use the Classical Theory here, but it's a bit complicated getting there. There are 14 cards you know the nature of: three aces, four kings, two queens, and the five in your hand (none of which is a queen). The ten unknown cards on the table, the five your friends are each holding close to their chests, and the remaining cards in the deck make up the pool from which you hope to get a queen (of course you can draw only from the deck). The deck itself thus now contains 13 cards ($52-14-10-5-5-5=13$). There are two queens left somewhere among the unknown cards, so you have a probability of $2/13$ of getting a queen. This assumes that none of your friends is neurotic about keeping or discarding queens, and that you have no good reason to think any friend is holding on to one to better his or her hand. Since queens are generally more valuable than most other cards, we might want to say that the Classical Theory can tell us that you have no better than $2/13$ chance of drawing a queen here. Gambling is clearly fraught with peril, and students would do best to avoid it at all costs. They will probably be safer watching television. Wanna bet on that?

Probability Calculations

Each probability problem we've looked at so far concerns the probability of a single result from a single event: one number coming up on a roll of a die, one draw from a deck of cards producing specified card, male drivers getting into an auto accident, Bob and Sue getting married, or the Seattle Seahawks winning their next game. Sometimes, however, we want to determine the probability of more than one event happening, or the probability of either of two events happening. For instance, we may want to know the probability of drawing an ace *and* then a king from a deck of cards, or we may want to know the chances of getting *two* sixes on a roll of a pair of dice. Or, we may want to know the likelihood of Sue marrying Bob *or* Tim, or the probability of getting a king *or* a queen on a draw from a deck of cards. These slightly more complex calculations require one or more additional rules. We'll examine a small handful here to help us with some fairly straightforward calculations. A wee bit of grade-school math is needed, but no more than to add or multiply some basic fractions. It's do-able. Feel free to keep a calculator handy for the practice problems, though.

Restricted Conjunction Rule

Let's imagine that we need heads on two tosses of a coin. We toss the coin once and get heads. Good so far! We toss it a second time and get heads again. We win! What was the probability of getting heads those two times in a row? To determine this, we make use of the *Restricted Conjunction Rule* (RCR). It's a "conjunction" rule because it's a rule about the conjunction of two events: we want to know the probability of getting the first heads *and* getting the second heads. To use the rule we simply determine the probability of getting heads the first time and multiply that by the probability of getting heads the second time: $1/2 \times 1/2 = 1/4$.

This result should sound correct, as there were four possible outcomes for the two coin tosses:

H-H
H-T
T-H
T-T

There was thus a one-in-four chance of getting two heads; that is, there was a probability of 1/4.

A formula for the RCR looks like this:

$$P(A \text{ and } B) = P(A) \times P(B)$$

This says—in fairly normal English—that the probability of events A and B both occurring equals the probability of A occurring times the probability of B occurring. We can use the Classical, Relative Frequency, or Subjectivist Theories to determine each individual probability (depending on which theory is called for), and then simply multiply them.

RCR works only when two or more events are *independent* of each other. Events are independent of each other when they do not impact or affect each other. Two coin tosses have no effect on the outcome of each other, nor do two separate rolls of a single die. Imagine, however, wishing to draw two aces from a deck of 52 playing cards when you do *not* replace the first card in the deck prior to making the second draw. The first draw would have an impact on the probability of getting that second ace because the deck now would have only three aces and a total of 51 cards. We'll need a second conjunction rule for this scenario. But first, let's look at some more examples of finding the probability of two events that are *independent*.

What is the probably of rolling a six-sided die twice and getting a five both times? We roll the die the first time with a 1/6 probability of getting a five. Let's say we get that five! We roll the die again and get the second five. The probability of getting *that* five was again 1/6. So, we multiply the two probabilities to get our final result: $1/6 \times 1/6 = 1/36$.

What's the probability of getting three heads in a row on three tosses of a coin? Each independent toss resulting in heads has a probability of 1/2. So we multiply each of the three probabilities to get our final result: $1/2 \times 1/2 \times 1/2 = 1/8$.

What is the probability of rolling one six-sided die four times, and getting a two each time? Well, the probability of getting a two on any one of those rolls is 1/6, so to find the probability of getting four winners in a row we multiple 1/6 four times: $1/6 \times 1/6 \times 1/6 \times 1/6 = 1/1296$.

Finally, since urns with balls in them are *so* cool, let's imagine that we have an opaque urn holding three red balls, two green balls, and five black balls. We want to know the probability of drawing two black balls, when we replace the first ball drawn before reaching in for the second ball. The draws are *independent* because we replace that first drawn ball before drawing the second. If we did not replace the first ball, the first draw would have an impact on the probability of the second draw (because there would be four black balls in the urn instead of the original

five, and there would be a total of nine balls in the urn instead of the original ten). So, we reach in and draw the first ball. The probability of getting a black ball is $5/10$, or $1/2$. We replace that ball, shake the urn a bit, reach back in and fortuitously draw a second black ball. The probability of drawing *that* ball is also $1/2$. To make our final determination of the conjunction of drawing the first black ball *and* the second black ball, we multiply the two probabilities: $1/2 \times 1/2 = 1/4$.

****Practice Problems: Restricted Conjunction Rule**

Determine the probability of the following set of events using the Restricted Conjunction Rule.

1. What is the probability of getting five heads in a row on five tosses of a coin? What are the odds of this happening?
2. What is the probability of blindly drawing two black jacks from a deck of 52 playing cards when you replace the first card drawn before making the second draw? What are the odds?
3. Imagine an urn containing one white ball, two yellow balls, and one blue ball. What is the probability of drawing a white ball twice on two independent blind selections (i.e., when you replace the first ball drawn before drawing the second ball)? What are the odds?
4. Close friends of Marko and Valeria give the couple 1:2 odds of getting married within the month. What is the probability that the couple will get married within the month *and* you get tails on an honest toss of a coin? What are the odds?
5. What is the probability of rolling two sixes (“boxcars”) with one toss of a pair of six-sided dice? What are the odds?
6. What is the probability of getting a seven and then an even number on two rolls of a six-sided die?
7. What is the probability of getting a number each time on two rolls of a six-sided die?
8. What is the probability of rolling a three on one roll of a six-sided die and drawing an ace from a deck of 52 cards?

Answers:

1. $1/2 \times 1/2 \times 1/2 \times 1/2 \times 1/2 = 1/32$; 1:31
2. $2/52 \times 2/52 = 1/676$; 1:675
3. $1/4 \times 1/4 = 1/16$; 1:15
4. $1/3 \times 1/2 = 1/6$; 1:5
5. $1/6 \times 1/6 = 1/36$; 1:35
6. $0/6 \times 3/6 = 0/6 = 0$ (it’s impossible)
7. $6/6 \times 6/6 = 1/1 = 1$ (it’s guaranteed)
8. $1/6 \times 4/52 = 1/6 \times 1/13 = 1/78$

General Conjunction Rule

We need to handle the probability of two events a little differently when the probability of one is *dependent* on that of the other. For instance, imagine that you need to draw two kings from a deck of 52 playing cards. You draw one and place it in your pocket. The probability of getting that first king was $4/52$, or $1/13$. You get ready to draw a second time, but now the deck has only three kings in it and the deck itself contains only 51 cards. The probability of getting a king now is $3/51$. The two probabilities together (i.e., conjoined) equals $1/13 \times 3/51 = 1/221$. If we

assumed incorrectly that the probability each time was the same (i.e., $1/13$), then we'd get $1/13 \times 1/13 = 1/169$. That's a different (mistaken) answer altogether.

The *General Conjunction Rule* (GCR) is used when we want to know the probability of two or more events when the earlier event has an impact on later events. A formula for this rule looks like this:

$$P(A \text{ and } B) = P(A) \times P(B \text{ given } A)$$

This says that the probability of both A and B occurring equals the probability of A occurring times the probability of B occurring (given that A already occurred). Let's look at some examples.

Imagine an urn holding two white balls, eight green balls, and two purple balls. What is the probability of drawing two green balls when you don't replace the first ball drawn? Note that the second draw is *dependent* on the first, because on the second draw there is one less ball in the urn than on the first draw. The probability of the first draw is determined using the Classical Theory: there are eight possible "winners" and 12 balls total for a probability of $8/12$, or $2/3$. For the second draw, we again use the Classical Theory, noting that there are now seven possible "winners" and only 11 balls total. The probability for this second draw by itself is thus $7/11$. To determine the probability of getting both green balls, we multiply the two probabilities: $2/3 \times 7/11 = 14/33$.

If we had put the first green ball drawn back in the urn before making our second draw, then the first draw would have had no impact on the second draw. The second draw would thus be *independent* of the first, and we could use the Restricted Conjunction Rule: $2/3 \times 2/3 = 4/9$.

GCR is needed when the probability of one event is dependent on another; RCR can be used when the outcomes of two (or more) events are independent of each other. Note, though, that GCR may be used in any conjunction calculation (RCR may be easier to use, however, when the events are independent). Let's go back to the urn problem immediately above, and once again replace the first green ball drawn before making our second draw. We *can* use GCR here instead of RCR. The probability of drawing the first green ball is $2/3$. We'll now replace it. Given the first probability, and given that we just replaced the ball, the probability of drawing a second green ball is also $2/3$. So we get $2/3 \times 2/3 = 4/9$. So, students *could*—if they wished to know the minimum needed to get by with calculating the probability of conjoined events—learn only the General Conjunction Rule. It's called a general rule because it may be used to determine the probability of the conjunction of both dependent and independent probabilities.

We need more examples!

What is the probability of drawing two black queens from a normal deck of cards, assuming no replacement of the first card drawn? Well, the first probability is $2/52$, or $1/26$. That's because there are two black queens in the deck of 52 cards. For the second draw, there is only one black queen left, and only 51 cards in the deck. The probability of that second draw is thus $1/51$. We

then multiply the two probabilities to calculate the probability of the two conjoined events: $1/26 \times 1/51 = 2/1326 = 1/663$.

Imagine a deck with nothing but face cards (kings, queens, and jacks) of all four suits (clubs, hearts, spades, and diamonds). What is the probability of drawing three jacks when you do not replace each card drawn? The probability of the first draw is $4/12$, or $1/3$. We now place the jack over to the side, and draw another card. The probability of getting a second jack is $3/11$. We set that jack aside, and draw a third time. The probability of getting a third jack (given that we've drawn two previously) is $2/10$, or $1/5$. We now multiply the three probabilities: $1/3 \times 3/11 \times 1/5 = 3/165 = 1/55$.

Imagine an urn containing two brown balls, one pink ball, and seven orange balls. What is the probability of drawing a brown ball (without replacing it afterwards) and then a pink ball? To figure this out, we begin by determining the probability of the first draw. There are two "winning" brown balls amidst a total of ten balls. The probability of drawing a brown ball at this point is thus $2/10$, or $1/5$. We now set that brown ball aside and reach in for our second draw. We now have nine balls in the urn with one pink "winner" possible. The probability of drawing the pink ball at this point is $1/9$. We then multiply the two probabilities and get our final result: $1/5 \times 1/9 = 1/45$.

****Practice Problems: General Conjunction Rule**

Determine the probability of the following events using the General Conjunction Rule.

1. Imagine an urn with three red balls, two blue balls, and one green ball. What is the probability of drawing two red balls on two blind draws, without replacement?
2. Imagine the same urn as in problem #1 above. What is the probability of drawing three red balls on three blind draws, without replacement?
3. Imagine the same urn as in problem #1 above. What is the probability of drawing a green ball and then a red ball, without replacement?
4. Imagine the same urn as in problem #1 above. What is the probability of drawing a red and then a purple ball on two blind draws, without replacement?
5. Once again, imagine the same urn as in problem #1 above. What is the probability of drawing a ball on each of two blind draws, without replacement?
6. Imagine an ordinary deck of 52 playing cards. What is the probability of drawing a black king and then a red card on two blind draws, without replacement?
7. Three logic professors are relaxing at hotel bar after a long day attending a philosophy conference on symbolic logic. They drink too much, and stumble their way one after the other attempting to go back to their individually assigned rooms. There are only three rooms (each presently unlocked) in this hotel, and there is an even chance of each man lurching his way to any one of the rooms. (The female logicians at the conference are whooping it up elsewhere having what they're calling "Ladies' Night.") Once each inebriated man gets to a hotel room, he locks the door behind him and falls unto his bed in a stupor. The men leave the bar one by one, each stumbling around looking for an open room. What is the probability that each will end up in his assigned room?

Answers:

1. $3/6 \times 2/5 = 2/10 = 1/5$

2. $3/6 \times 2/5 \times 1/4 = 2/40 = 1/20$

3. $1/6 \times 3/5 = 3/30 = 1/10$

4. $3/6 \times 0/5 = 0/30 = 0$ (there are no purple balls in the urn, so such a sequence of draws is impossible)

5. $6/6 \times 5/5 = 30/30 = 1$ (it's guaranteed that you'll draw *some* ball of one color or another on each draw)

6. $2/52 \times 26/51 = 52/2652 = 13/663$

7. The probability of the first guy making it to his room is $1/3$. Let's assume he gets there. The second guy now has a probability of $1/2$, as there is one "winning" room and he has two options left (the first guy locked his door and is now passed out on his assigned bed). Given that all this takes place, the third guy has only one room left: his room. So the probability of his getting into that one is $1/1$, or 1. We multiply all three probabilities together for the final result: $1/3 \times 1/2 \times 1/1 = 1/6$.

Restricted Disjunction Rule

If we want to know the probability of two events *both* taking place ("conjoined" as it were), we'd use a *conjunction* rule. But sometimes we want to know the probability of event A happening *or* event B happening. Here we need a *disjunction* rule. There are two such rules we can use, with the first being a little easier, but applying to a rather restricted set of circumstances.

The *Restricted Disjunction Rule* (RDR) is used when we want to know the probability of one or another event occurring, and when the events are *mutually exclusive*. Two events are mutually exclusive if and only if they cannot both take place (although it might end up that neither takes place). Either occurs, but not both. Examples of mutually exclusive events include rolling one die and getting either a two or a three, drawing either a king or queen on one blind draw from a deck of cards, and getting heads or tails on a toss of a coin. In each case, you can get one result, but not both (and sometimes neither).

Whereas for the probability of the conjunction of two events we *multiply* the probability of each event, for disjunction calculation we *add* the two probabilities. The formula for RDR looks like this:

$$P(A \text{ or } B) = P(A) + P(B)$$

This says that the probability of either A or B occurring is equal to the probability of A occurring plus the probability of B occurring. As always, examples will help.

What is the probability of rolling an even number or a three on one honest roll of a six-sided die? To determine this, we first figure out the probability of getting an even number. There are three "winners" (i.e., two, four, and six), so the probability is $3/6$. The probability of getting a three is $1/6$. So, the probability of getting an even number *or* a three is $3/6 + 1/6 = 4/6 = 2/3$.

What is the probability of drawing a red king or a spade on one blind draw from a deck of 52 playing cards? It doesn't really matter which probability we determine first, so let's just start with the red king. There are two "winners" (i.e., two red kings: the king of diamonds and the king of hearts), so the probability of drawing one is $2/52$. There are 13 spades in the deck, so the probability of drawing one of them is $13/52$. The probability of drawing one or the other (and we can't get both with one draw, so they are mutually exclusive) is $2/52 + 13/52 = 15/52$.

Imagine we have an urn with two red balls, one blue ball, four yellow balls, and three green balls. On a blind draw we select one ball. What is the probability that ball will be green or red? The probability of selecting a green ball is $3/10$, and the probability of selecting a red ball is $2/10$. So adding those two probabilities together will give us the probability of selecting one or the other: $3/10 + 2/10 = 5/10 = 1/2$. That should be intuitive, as the red and green balls together make up half the number of balls in the urn, so we should have a $1/2$ chance of getting one or the other.

Imagine the same urn as above. What is the probability of selecting on one blind draw a red, blue, or yellow ball? Here we have three mutually exclusive events, as we can draw only one ball. The probability of getting a red ball is $2/10$; the probability of getting a blue ball is $1/10$; the probability of getting a yellow ball is $4/10$. Adding them up, we get the probability of getting one of them: $2/10 + 1/10 + 4/10 = 7/10$.

****Practice Problems: Restricted Disjunction Rule**

Determine the probability of the following mutually exclusive events. Use the Restricted Disjunction Rule.

1. What is the probability of drawing an ace or an even numbered card with one blind draw from a normal deck of 52 playing cards?
2. What is the probability of selecting on one blind draw a face card, a ten or, a red two from a deck of 52 playing cards?
3. What is the probability of rolling an even or odd number with a six-sided die?
4. Imagine an urn containing two red balls, four black balls, and three white balls. What is the probability of drawing either a white or black ball on one blind draw?
5. Imagine an urn with one yellow ball, two blue balls, two green balls, three red balls, and two purple balls. What is the probability on one blind draw of selecting either a yellow, blue, or green ball?
6. We watched ten prisoners at County Jail eat lunch, and three freely chose to eat cheeseburgers (which are served every day). We also watched 20 other County Jail inmates eat lunch, and of that group three freely chose to eat tuna sandwiches (which are served every day). Inmates at County Jail *must* eat lunch, but may choose only one item for lunch. Given this information, what is the probability that a County Jail inmate will eat either a cheeseburger or tuna sandwich for lunch?
7. Janet is practicing poker by herself in her hotel room. She takes a deck of 52 playing cards and draws four cards: the ace of hearts, the two of diamonds, the three of clubs, and the four of hearts. She's about to draw one more card, but pauses to determine the probability of drawing another ace to get a pair of aces, *or* any five to get a straight. What is that probability?

Answers:

1. $4/52 + 20/52 = 24/52 = 6/13$

2. $12/52 + 4/52 + 2/52 = 18/52 = 9/26$

3. $3/6 + 3/6 = 6/6 = 1$ (it's guaranteed)

4. $3/9 + 4/9 = 7/9$

5. $1/10 + 2/10 + 2/10 = 5/10 = 1/2$

6. Using the Relative Frequency Theory, we determine the probabilities of County Jail inmates eating cheeseburgers or tuna sandwiches for lunch: $3/10$ and $3/20$ respectively. We then use RDR to get the probability of inmates eating either item for lunch: $3/10 + 3/20 = 6/20 + 3/20 = 9/20$.

7. The probability of getting a second ace is $3/48$ (since there are three aces left in the deck that now has 48 cards in it). The probability of getting any five is $4/48$ (since there are four fives in the remaining deck of 48 cards). On one draw Janet can't get an ace *and* a five (they are mutually exclusive), so she uses RDR to get the probability of drawing *either* of the desired cards: $3/48 + 4/48 = 7/48$.

General Disjunction Rule

The Restricted Disjunction Rule works for situations in which two potential events are mutually exclusive, like rolling a two or a three with one roll of a die. The events are mutually exclusive because you can't get both numbers on one roll. But some pairs of potential events are *not* mutually exclusive; that is, one or the other, or both might occur. For instance, let's say you are about to draw a card from a full deck of playing cards and will "win" if you get either an ace or a spade. You could win with an ace, or a spade that's not an ace, or the ace of spades. Drawing an ace does not exclude you from drawing a spade; you might get both. To calculate the probability of drawing an ace or a spade (or both), we use the *General Disjunction Rule* (GDR). GDR calculates the probable occurrence of either of two or more independent events whether or not they are mutually exclusive. (If you highlight textbooks, you might want to highlight that last sentence.) Since RDR is easier to work with, it's usually best to simply use it when the two events are mutually exclusive. We need to use GDR when the two events are not mutually exclusive.

To understand the idea behind GDR, continue to imagine that you hope to draw an ace or spade (as above) from a full deck of cards. Either card will be a winner (and of course so will the ace of spades). We might be tempted to add up the two possibilities of getting an ace or a spade to get a final calculation. There are four aces, so the probability of getting an ace is $4/52$. There are 13 spades, so the probability of getting a spade is $13/52$. But merely to add these two probabilities (getting $17/52$) would be a mistake, because one of the spades is an ace, and we'd be counting it twice. So, we need to account for this "overlap" caused by aces and spades *not* being exclusive of each other, and subtract the ace-of-spades probability (i.e., the probability of drawing an ace *and* a spade on one draw: $13/52 \times 4/52 = 1/52$) from the sum of the ace and spade probabilities. Our calculation will then look like this: $13/52 + 4/52 - 1/52 = 16/52 = 4/13$.

This example illustrates the GDR formula:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \times B)$$

This formula says that the probability that either of two independent events—mutually exclusive or not—equals the probability of one plus the probability of the other minus the probability they both occur together.

Let's look at another simple example. What is the probability of getting at least one tails on two tosses of a coin? We “win” if we get tails on the first toss, the second toss, or on both tosses. The tosses and their results are independent, as the first toss has no impact on the second. Also, the result of tails is not exclusive, as we might get tails on either toss, *or on both*. Looking at the formula, let's consider the first toss as event A, and the second toss as event B. The probability of getting at least one tails will thus be calculated like this: $1/2 + 1/2 - (1/2 \times 1/2) = 1 - 1/4 = 3/4$. And that result should seem fairly intuitive, as there are three ways to win amid the four possible coin-toss scenarios:

H-H loses
H-T wins!
T-H wins!
T-T wins!

For another example, consider the logician's favorite piece of pottery, the urn. This urn contains three red balls, three yellow balls, two green balls, and two black balls. What is the probability of selecting a red ball on a blind draw when you get two draws and you have to replace the first ball drawn before drawing the second time? (These latter conditions keep the events independent.)

We can win here by drawing a red ball on the first try, on the second try, or on both tries. Selecting red balls each time is thus *not* mutually exclusive. The probability of a getting a red ball on the first draw is $3/10$, and since we replace the ball before drawing again, the probability of getting a red ball on the second draw is also $3/10$. Since these events are independent and not mutually exclusive, we use GDR: $3/10 + 3/10 - (3/10 \times 3/10) = 6/10 - 6/100 = 60/100 - 6/100 = 54/100 = 27/50$.

The crowd wants one more example! What is the probability of rolling a pair of dice and getting at least one one? One die could come up one; the other could come up one, or both might come up one giving you “snake eyes.” The events are independent, as the roll of one die does not impact the result of the other roll. And since the disjunction is inclusive (i.e., not exclusive), we'll use GDR to calculate the probability here. The probability of getting a one for one toss of one die is $1/6$. So, GDR tells us to calculate things this way: $1/6 + 1/6 - (1/6 \times 1/6) = 2/6 - 1/36 = 12/36 - 1/36 = 11/36$.

GDR works whether the events are mutually exclusive or not, but RDR is easier to use when the events are exclusive. Consider rolling a die once again, this time hoping to get a one or a two. Since we're rolling the die only once, we can't get both numbers, so the results are exclusive: it is impossible for both to occur. The GDR formula can work here, but it's easier to use RDR.

Using GDR, we get the following calculation (keep in mind that getting a one *and* a two on one roll of a die is impossible, and thus has a probability of zero): $1/6 + 1/6 - (\text{one and two}) = 2/6 - 0 = 2/6 = 1/3$. Since the events are mutually exclusive, we'd be warranted in using the simpler RDR to make the following briefer calculation: $1/6 + 1/6 = 2/6 = 1/3$.

****Practice Problems: General Disjunction Rule**

Determine the probability of the following independent events, noting that they are not mutually exclusive. Use the General Disjunction Rule.

1. What is the probability of tossing a coin twice and getting at least one heads?
2. What is the probability of drawing at least one jack on either of two draws from a normal deck of 52 playing cards, assuming you replace the first card before drawing the second?
3. You roll a pair of dice. What is the probability that at least one will turn up with an even number?
4. Imagine an urn with two green balls, three brown balls, and five orange balls. You draw twice, replacing the first ball before making the second draw. What is the probability of getting at least one green ball?
5. Given the urn in problem #4 above, what is the probability of drawing at least one brown ball, given two draws and replacing the first ball before making the second draw?
6. Assuming that the odds of the Seahawks winning their next football game are 3:2, and that the odds of the Sounders winning their next soccer game are 1:2, what is the probability that either will happen?
7. Five women attending a philosophy conference on symbolic logic are spending the later part of an evening at a women's dance club. Three scantily clad guys are dancing on the stage, and Sarah, one of the five women, shouts, "I know those guys! One is a blithering idiot, but is really fun to hang out with; the other two studied logic in college." She goes on to say that the two brighter lads would surely want to talk about symbolic logic with the women after their dance routine. She says the odds are 3:1 that Thomas will want to do so, and 2:1 that Fabio will want to. Of the remaining four women, only Barbara is sober enough to calculate the probability that either Thomas or Fabio (or both) will want to join them after their dance routine to talk about symbolic logic. What is that probability?
8. Janet and her four philosopher friends eventually leave the dance club and go to a nearby bar to play poker. Janet tries to explain some of the rules to them, and illustrates what she says by drawing four cards randomly from a shuffled deck. She deals herself two kings, an ace, a four, and a six. She discards the four and six and keeps her two kings and the ace. Taking this as an opportunity to show how well logicians can do probability calculations, she asks the four women what the probability is of her getting at least one more king upon drawing two cards (without replacement). The exhausted but contented women all ably come to the same conclusion. What is it?

Answers:

1. $1/2 + 1/2 - (1/2 \times 1/2) = 1 - 1/4 = 3/4$
2. $4/52 + 4/52 - (4/52 \times 4/52) = 1/13 + 1/13 - (1/13 \times 1/13) = 26/169 - 1/169 = 25/169$
3. $1/2 + 1/2 - (1/2 \times 1/2) = 1 - 1/4 = 3/4$
4. $2/10 + 2/10 - (2/10 \times 2/10) = 1/5 + 1/5 - (1/5 \times 1/5) = 2/5 - 1/25 = 10/25 - 1/25 = 9/25$

5. $3/10 + 3/10 - (3/10 \times 3/10) = 6/10 - 9/100 = 60/100 - 9/100 = 51/100$

6. We first convert odds to probability: 3:2 is $3/5$, and 1:2 is $1/3$. Given the Seahawks and the Sounders could both win, we use GDR for our calculation: $3/5 + 1/3 - (3/5 \times 1/3) = 9/15 + 5/15 - 3/15 = 11/15$.

7. We begin by using the Subjectivist Theory to determine the probability of Thomas and Fabio each joining the ladies in critical dialectic. The odds for Thomas's future presence are 3:1, so the probability of his joining the ladies is $3/4$. The odds for Fabio conferring with the philosophers on matters pertaining to symbolic logic are 2:1, which makes the analogous probability $2/3$. Since either or both might wish to chat with the women, we use GDR: $3/4 + 2/3 - (3/4 \times 2/3) = 9/12 + 8/12 - 6/12 = 17/12 - 6/12 = 11/12$. The women are delighted at Barbara's findings and expect the evening will progress marvelously.

8. There are 47 cards left in the deck, and within it there are a total of two "winning" kings. So, the probability of getting a "winner" on either or both draws (and she *could* get a winner on both draws, making the results independent but not mutually exclusive) is calculated with GDR: $2/47 + 2/47 - (2/47 \times 2/47) = 188/2209 - 4/2209 = 184/2209$.

Negation Rule

The *Negation Rule* is easy to understand, and is often useful when the conjunction or disjunction rules either won't work or would be difficult to apply. The idea behind this rule is to figure out the probability of an event *not* occurring, and then subtract that probability from 1 to get the probability of the event occurring. Assuming that the event will either occur or not occur, the probability of each adds up to 1 (remember that a probability of 1 means an event is guaranteed to occur, just like a probability of 0 means that the event is impossible). The formula for the Negation Rule is

$$P(A) = 1 - P(\text{not-}A)$$

This says that the probability of an event A is 1 minus the probability that A does not occur.

A fairly simple illustration of the use of the Negation Rule appeals to coin tosses. What would be the probability of getting at least one heads on three tosses of a coin? We could "win" by getting heads on the first toss, the second, the third, the first and second, the first and third, the second and third, or on all three tosses. Whew! That would be a somewhat complex set of disjunctions to calculate. But, there is only one way to "lose," and that's by getting tails three times independently in a row (i.e., tails *and* tails *and* tails). We can use RCR to determine that easily: $1/2 \times 1/2 \times 1/2 = 1/8$. So the probability of "losing" is $1/8$. Using the Negation Rule, we can now easily determine the probability of "winning": $1 - 1/8 = 7/8$.

For another example, imagine an urn containing two red balls, two blue balls, two green balls, and four white balls. What is the probability of drawing at least one red, blue, or green ball on two tries, when you replace the first ball before drawing the second time? Well, we could use GDR as the events are independent and not mutually exclusive (you might draw a "winning" ball either or both times), but it would be a fairly complicated calculation. We can instead easily determine the probability of "losing," that is, of drawing a white ball on each draw (which is the

only way we can “lose” here). To calculate getting a white ball and then another white ball, we use RCR: $4/10 \times 4/10 = 2/5 \times 2/5 = 4/25$. Now we use the Negation Rule to get the probability of “winning”: $1 - 4/25 = 25/25 - 4/25 = 21/25$.

The Negation Rule also makes it easier to calculate the probability of either of two events occurring that are *dependent*. Consider the same urn described immediately above. What would be the probability of drawing at least one red, blue or green ball on two blind draws when you do *not* replace the first ball drawn before selecting the second ball? Again, the only way to “lose” here is to draw a white ball and then another white ball. We can use GCR this time to calculate with relative ease this probability of “losing.” Note that after drawing a “losing” white ball on the first draw that there are only three white balls left among a total of nine balls. GRC gives us the following calculation: $4/10 \times 3/9 = 2/5 \times 1/3 = 2/15$. So, we stand a $2/15$ chance of “losing.” We now use the Negation Rule to determine the probability of “winning”: $1 - 2/15 = 15/15 - 2/15 = 13/15$. Cooly cool!

****Practice Problems: Negation Rule**

Using the Negation Rule as part of your calculation, determine the probability of the following events.

1. What is the probability of getting at least one tails on four tosses of a coin?
2. What is the probability of getting at least one two on three rolls of a six-sided die?
3. Imagine an urn with one black ball, five yellow balls, and two green balls. What is the probability of drawing at least one black or yellow ball given two draws when you do *not* replace the first ball drawn before making the second draw?
4. Consider the same urn as in problem #3 above. What is the probability of drawing at least one black or yellow ball given two draws when you *do* replace the first ball drawn before making the second draw?
5. (a) What is the probability of drawing at least one king from a deck of 52 playing cards, if you are given two chances to do so, and you must replace the first card drawn before drawing the second? (b) What is the probability if you do *not* replace that first card before drawing the second?
6. Only three racehorses—Bellevue Slew, Administariat, Woman O’ War—are competing at Emerald Downs in two races. The odds of winning for each horse in the first race are Bellevue Slew 1:2, Administariat 1:5, and Woman O’ War 1:1. For the second race, the odds are, respectively, 3:2, 1:4, and 1:4. Given these odds, what is the probability that either Bellevue Slew or Administariat will win at least one race? What are the odds of this happening? Assume that the first race and its results have no impact on the second race and its results.
7. Three male and five female philosophy professors wake up with hangovers while attending a conference on symbolic logic. Blurry-eyed and aching, each stumbles haltingly to the hotel café for coffee. Once there, the waitress says that the café has a new pricing option for their \$1 cups of coffee. Each customer can roll three six-sided dice, and if at least one die shows a six, then the coffee is free; if not, then the customer pays \$2 for a cup of coffee. The men all jump at the chance, figuring it’s a fun, even bet and, frankly, they’re in too much pain to think about it. Among the women, Janet is more cautious and wants to calculate the probability of getting a six

with a little more precision than generated by some hung-over guy's blurred intuition. What is the probability of getting at least one six on a roll of three dice? Is this an even bet?

Answers:

1. $P(H \text{ and } H \text{ and } H \text{ and } H) = 1/2 \times 1/2 \times 1/2 \times 1/2 = 1/16$; $P(\text{at least one } T) = 1 - 1/16 = 15/16$.
2. $P(\text{not-2 and not-2 and not-2}) = 5/6 \times 5/6 \times 5/6 = 125/216$; $P(\text{at least one } 2) = 1 - 125/216 = 91/216$.
3. The only way to "lose" is to draw two green balls. We find the probability of that by using GCR (since the second draw is not independent): $2/8 \times 1/7 = 2/56 = 1/28$. We now use the Negation Rule to determine the probability of "winning": $1 - 1/28 = 27/28$.
4. Again, the only way to "lose" is to draw two green balls. We find the probability of that by using RCR (since the second draw *is* independent): $2/8 \times 2/8 = 4/64 = 1/16$. We now use the Negation Rule to determine the probability of "winning": $1 - 1/16 = 15/16$.
5. (a) There are 48 cards in the deck that are *not* kings; the only way to "lose" here is to draw a non-king twice. For the first scenario, we replace the first card drawn before making the second draw. Since this second draw is independent of the first, we can use RCR to determine the probability of drawing a non-king: $48/52 \times 48/52 = 12/13 \times 12/13 = 144/169$. We now use the Negation Rule to determine the probability of "losing" (i.e., of getting something other than a non-king, i.e., a king): $1 - 144/169 = 25/169$. (b) For the second scenario, we *do not* replace the first card drawn before drawing the second, so here we use GCR to determine the probability of "losing" (i.e., drawing two non-kings): $48/52 \times 47/51 = 2256/2652 = 188/221$. Now we use the Negation Rule to calculate the probability of "winning" (i.e., getting a king at least once): $1 - 188/221 = 33/221$.
6. First we use the Subjectivist Theory, and translate the odds into probabilities. Each horse has two races and two probabilities: the probabilities of Bellevue Slew winning are $1/3$ and $3/5$; Administariat's are $1/6$ and $1/5$; Woman O' War's are $1/2$ and $1/5$. The only way we can "lose" is for Woman O' War to win both races. Since the two races are independent, we can use RCR to determine the probability of "losing": $1/2 \times 1/5 = 1/10$. Now we use the Negation Rule to determine the probability of "winning": $1 - 1/10 = 9/10$. The odds—given this probability—are 9:1.
7. The bet is not even, although it may initially seem like it is. Adding up all the possible combinations of outcomes for the three rolled dice, it turns out that more than half are combinations *not* containing a six. It would be difficult to calculate all the combinations that *do* contain a six, but it's much easier to figure out how many *do not*: each die would need to turn up with a number other than six, and each die has a $5/6$ probability of doing that. So, for each die to come up with a non-six, the probability would be $5/6 \times 5/6 \times 5/6 = 125/216$. This would be the probability of "losing." The probability of "winning" is found via the Negation Rule: $1 - 125/216 = 91/216$, which is less than a 50% chance. Janet was right to be cautious.

Combining the Rules

We've already combined use of more than one rule. We did so repeatedly by using a conjunction or disjunction rule to determine the probability of an event *not* taking place, and then using the Negation Rule to determine the probability that it would. Often, we'll need to use one or more rules multiple times to make a final calculation. We might have cause to determine the

probability of a conjunction *and* a second conjunction, a conjunction *and* a disjunction, a disjunction *or* another disjunction, and so on. We thus might use a combination of RCR, GCR, RDR, and GDR. We might even do that to lead up to a use of the Negation Rule. Just *think* of the fun! Let's look at some examples.

What is the probability of drawing two aces with replacement from a deck of 52 cards *and* rolling a two with a six-sided die? To determine the probability of independently drawing two aces, we use RCR: $4/52 \times 4/52 = 1/13 \times 1/13 = 1/169$. To determine the probability of rolling a two, we simply use the Classical Theory: $1/6$. To determine the probability of getting *both*, we use RCR (since drawing the aces and rolling the two are independent): $1/169 \times 1/6 = 1/1014$.

What is the probability of rolling an even number with a six-sided die *and* getting at least one red card given two draws with replacement of the first card before the second draw? There are three "winners" for the die roll, so the probability is $3/6 = 1/2$. We will draw at least one red card as long as we don't draw two black cards. Let's calculate *that* using RCR (the first card drawn is replaced, so the second result is independent of the first): $1/2 \times 1/2 = 1/4$. We now use the Negation Rule to calculate the probability of getting at least one red card: $1 - 1/4 = 3/4$. Finally, we use RCR to determine the probability of *both* events occurring: $1/2 \times 3/4 = 3/8$.

For the next three examples, consider the following scenario. Philosophy department peers of the three men and five women attending a regional symbolic logic conference agree on the following odds regarding what will happen to their close, conference-attending friends: the odds are 3:2 that Sarah will win a prize for best paper presented at the conference, 3:1 that Wanda will fall hopelessly in love with an exotic male dancer, 5:1 that Janet will want to play poker with some of the dancers, 1:2 that Betty will want to talk about modal logic, 1:3 that Nancy will want to quit her job teaching and become a restaurant chef, 7:2 that Mateo will ask Betty to work with him to publish an article about logic, 1:4 that Craig will sleep through the second half of the conference, and 4:1 that Pedro will try to convince the seven others that he should be the next department chair.

(i) What is the probability that either Sarah will win a prize for best paper presented at the conference or Wanda will fall in love with an exotic male dancer, *and* both Craig will sleep through the second half of the conference and Mateo will ask Betty to work with him to publish an article about logic?

First, we translate the odds to probabilities. For the events pertaining to Sarah, Wanda, Craig, and Mateo respectively, we get $3/5$, $3/4$, $1/5$, and $7/9$. Next we calculate the probability of the Sarah-*or*-Wanda events. Both might happen and one does not depend on the other, so we'll use GDR: $3/5 + 3/4 - (3/5 \times 3/4) = 27/20 - 9/20 = 18/20 = 9/10$. Next we calculate the probability of the Craig-*and*-Mateo events using RCR (because one event has no impact on the other): $1/5 \times 7/9 = 7/45$. Finally, we want the probability of both of these results taking place, and since they are independent of each other, we'll use RCR: $9/10 \times 7/45 = 63/450$.

(ii) What is the probability of all the guy events occurring, *and* the Wanda or Betty event?

What we have here is a big, lumbering conjunction problem. The italicized *and* tell us that. Since the conjuncts are independent of each other, we'll make our final calculation using RCR. For all the guy events happening we'll use RCR. Since the Wanda and Betty events are not exclusive, we'll use GDR to determine the probability of either (or both) taking place. First, of course, we convert the accepted odds into probabilities. This is so much fun!

For the guy events, we use RCR to get $7/9 \times 1/5 \times 4/5 = 28/225$. For the Wanda-or-Betty events, we use GDR to get $3/4 + 1/3 - (3/4 \times 1/3) = 9/12 + 4/12 - 3/12 = 10/12 = 5/6$. Using RCR to get the probability of the conjunction of these two results, we get $28/225 \times 5/6 = 140/1350 = 14/135$.

(iii) What is the probability that the Janet and Nancy events will both take place, *or* that the Craig and Pedro events will both take place?

The Janet and Nancy events are independent, so we can use RCR to determine the probability of both occurring. So too with the Craig and Pedro events. Once we get each pair sorted out and a probability for each conjunction, we'll use a disjunction rule to determine the probability of either result occurring. Since both results can happen (they are independent and not mutually inclusive), we need to use GDR. The whole process will look like this:

$$[P(J) \times P(N)] + [P(C) \times P(P)] - \{[P(J) \times P(N)] \times [P(C) \times P(P)]\}$$

Yuk. Let's work through it, though. Determining the probabilities from the odds for each, we plug them in for the Janet, Nancy, Craig, and Pedro events, and get this:

$$(5/6 \times 1/4) + (1/5 \times 4/5) - [(5/6 \times 1/4) \times (1/5 \times 4/5)]$$

Allowing our pre-college math skills to kick in, we are looking at...

$$5/24 + 4/25 - (5/24 \times 4/25) = 9/25 - 20/600 = 216/600 - 20/600 = 196/600 = 49/150$$

****Practice Problems: Combining Probability Rules**

Determine the probabilities called for in each problem below. You will likely need to use a combination of rules in each case.

1. What is the probability of getting two heads on two tosses of a coin, *and* drawing a red jack or a black card on one blind draw from deck of 52 cards?
2. What is the probability of rolling a number less than three on one roll of a normal six-sided die *and* getting heads on two honest tosses of coin? What are the odds?
3. What is the probability of getting at least one six on a roll of two six-sided dice, *and* drawing a queen from a deck of 52 cards?
4. Imagine two urns. The first urn contains two red balls, one green ball, and two yellow balls. The second urn contains three orange balls, two blue balls, and five purple balls. When drawing one ball from each urn, what is the probability of selecting either a red or green ball from the first urn, *and* either an orange or blue ball from the second urn?

5. Consider again the two urns from problem #4. Draw two balls from each urn. What is the probability of selecting red and yellow balls (*with* replacement of the first ball) from the first urn, *or* orange and purple balls (*without* replacement of the first ball) from the second urn?
6. Consider once again the two urns in problem # 4. Draw one ball from the first urn and two balls (*with* replacement) from the second urn. What is the probability of selecting a green or yellow ball from the first urn, *and* both orange and purple balls from the second urn?
7. Yet again, consider the two urns in problem #4. Draw two balls from each of the urns. What is the probability of selecting either a red or green ball or both from the first urn (*without* replacement), *or* either a blue or purple ball or both from the second urn (*without* replacement)?

Answers:

1. $(1/2 \times 1/2) \times (2/52 + 26/52) = 1/4 \times 28/52 = 1/4 \times 7/13 = 7/52$
2. $2/6 \times 1/2 \times 1/2 = 1/12$; 1:11
3. $[(1/6 + 1/6) - (1/6 \times 1/6)] \times 4/52 = 11/36 \times 1/13 = 11/468$
4. For the draw from the first urn, we use RDR: $2/5 + 1/5 = 3/5$. For the draw from the second urn, we use the RDR: $3/10 + 2/10 = 5/10$. To determine the probability of the conjunction of these to independent results, we use RCR: $3/5 \times 5/10 = 15/50 = 3/10$.
5. For the independent draws from the first urn, we use RCR: $2/5 \times 2/5 = 4/25$. For the draws (the second of which is dependent on the first) from the second urn, we use GCR: $3/10 \times 5/9 = 15/90 = 1/6$. Next we use GDR to determine the result of the independent and inclusive disjunction of these two results: $4/25 + 1/6 - (4/25 \times 1/6) = 24/150 + 25/150 - 4/150 = 49/150 - 4/150 = 45/150 = 3/10$.
6. For the draw from the first urn, we use the RDR: $1/5 + 2/5 = 3/5$. For the draw from the second urn, we use the RCR: $3/10 \times 5/10 = 15/100 = 3/10$. To determine the probability of the conjunction of the two independent results, we use RCR: $3/5 \times 3/10 = 9/50$.
7. For both pairs of urn draws, we face a second draw that is dependent on the first. Since these are disjunction calculations, they would be quite complex. So, instead, let's determine the probability of losing on each draw, and then use the Negation Rule in each case to determine the probability of "winning" each draw. We'll conclude by using GDR to determine the probability of "winning" with the first, or second (or both) urns. For the first urn scenario, the only way to "lose" is to draw two yellow balls, so we'll use GCR (we do not replace the first ball drawn): $2/5 \times 1/4 = 2/20 = 1/10$. To determine the probability of "winning" regarding the first urn, we now use the Negation Rule: $1 - 1/10 = 9/10$. For the second urn scenario, the only way to "lose" is to draw two orange balls, so again we'll use the GCR (because, again, we do not replace the first ball drawn before making the second draw): $3/10 \times 2/9 = 6/90 = 2/45$. Now we use the Negation Rule to determine the probability of winning": $1 - 2/45 = 43/45$. To win overall, we need to win with either the first urn, or the second urn, or both. Thus we use the GDR with the "winning" results of each pair of urn draws in mind: $9/10 + 2/45 - (9/10 \times 2/45) = 405/450 + 20/450 - 18/450 = 407/450$.

****Practice Problems: More Probability Calculations**

Use the Classical Theory, Relative Frequency Theory, Subjectivist Theory, or any combination of probability rules to perform the following calculations. Answers can be a fraction or a decimal number.

1. What is the probability of drawing a face card from a deck of 52 playing cards?
2. We observe 250 men go into a gym, and 12 of them use the rowing machine. What is the probability that a man entering the gym will use the rowing machine?
3. What is the probability—given the observations from problem #2, that a man entering the gym will *not* use the rowing machine?
4. The people most knowledgeable about the Seattle Mariners' chances of winning this year's World Series give them 1:25 odds of doing so. On this basis, what is the probability of their winning the upcoming World Series?
5. Consider an urn with two red balls and three green balls. What is the probability of selecting a white ball on one blind draw?
6. What is the probability of getting an ace or a jack from a deck of cards on one blind draw?
7. What is the probability of getting at least one ace on two draws from a deck of cards when the first card is replaced before the second is drawn?
8. What is the probability of getting two kings on two draws from a deck of cards, without replacement after the first draw?
9. What is the probability of getting a face card on a single draw from a deck of cards?
10. What is the probability of getting an even number or a three on any one of three rolls of a single die?
11. The odds are 2:3 that Bill will get an A on his English test. The odds are 2:1 that Sue will get an A on her English test. What is the probability that both Bill and Sue will get As on their English tests?
12. Refer to problem #6. What is the probability that either Bill or Sue (or both) will get an A on the test?
13. What is the probability of getting at least one tails on six tosses of a coin?
14. In a study of 250 baseball players, five developed severe elbow problems. In a study of 500 baseball players, ten developed a bone spur. What is the probability—based on these two studies—of a baseball player developing both severe elbow problems and a bone spur?
15. What is the probability of selecting at least one red ball on two draws from an urn containing two red balls, three white balls, and two green balls, when the first ball is replaced before the second selection?
16. Given the urn in problem #10, what is the probability of selecting either a red or a white ball (or both) on either of two draws, when the first ball is not replaced before the second draw?
17. Imagine two urns. The first urn contains three blue balls, one yellow ball, and two purple balls; the second urn contains four blue balls, two green balls, and one purple ball. You draw two balls from each urn, but must replace the first ball drawn from the first urn before the second draw; and you must *not* replace the first ball drawn from the second urn before the second draw. What is the probability of drawing a total of four blue balls?
18. Imagine the pair of urns in problem #12. What is the probability of drawing either a blue or yellow ball from the first urn on one blind draw, or of drawing two green balls in a row from the second urn when you replace the first green ball drawn before making the second draw into the second urn?
19. Imagine the pair of urns in problem #12. You get one blind draw into each. (a) What is the probability of your drawing a pink ball from the first urn or a green ball from the second? (b) What would be the probability if that question contained “and” instead of “or”?

20. Philosophy professors Wanda and Janet meet with Senator Sunny Shine to urge her to support state funding for student loans. They have known Sunny for years, working with her on gardening projects and activist bike rides. They believe the odds are 3:1 that Sunny will agree with them. There is also a $1/2$ chance that Sunny will invite them to go to the beach with her that afternoon. What are the odds that Sunny will support funding for student loans *and* invite Wanda and Janet to go to the beach with her?

Answers:

1. Use the Classical Theory. There are 12 face cards in a deck of 52: $12/52 = 3/13$.
2. Using the Relative Frequency Theory, we get $12/250 = 6/125$ or 0.048.
3. If the probability of a man using the rowing machine is $6/125$, then the probability of his not using it will be $1 - 6/125 = 119/125$ or 0.952.
4. $1/26$ or 0.38
5. $0/5 = 0$ (since there are no white balls in the urn, it's impossible to draw one)
6. $2/13$ or 0.15
7. $25/169$ or 0.15
8. $1/221$ or 0.0045
9. $3/13$ or 0.23
10. $26/27$ or 0.96
11. $4/15$ or 0.27
12. $4/5$ or 0.8
13. $63/64$ or 0.98
14. $1/2500$ or 0.0004
15. $24/49$ or 0.49
16. $20/21$ or 0.95
17. $1/14$ or 0.071
18. $102/147$ or 0.69
19. (a) $2/7$ or 0.29, (b) 0
20. Odds of 3:1 convert to a probability of $3/4$. Supporting student loans and inviting people to a beach are independent events, so we can use the RCR: $3/4 \times 1/2 = 3/8$ or 0.375. The odds would thus be 3:5.